Trade and Wage Inequality

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Abstract

Most of the models attempting to give an account of trade induced symmetric increase in wage inequality have abandoned the factor price equalization (FPE) framework.
In this paper we identify a plausible route through which trade might increase wage inequality in both the trading countries in a standard factor price equalization framework. We develop a very simple two-sector model with one constant returns and another increasing returns to scale sector. Agents have identical preferences given by a quasi-linear utility function. This notionally captures the divide between basic and fancy goods. There are two types of productive factors skilled and unskilled labour and they differ with respect to their occupational options. Skilled labour can work both in the skill using sector and also in unskilled job. Whereas unskilled labour is tied down to unskilled job. The model holds possibility of multiple equilibria and under reasonable parameterization skill premium increases in both the countries following trade.

1. Introduction

This paper is yet another attempt at theoretically reconciling the empirical observation that wage premium has increased across board: both in skill scarce and skill abundant countries. We propose a very simple model of trade with a constant and an increasing returns to scale sector and with goods differing in their income elasticities. We depart from the usual trade theoretic mode of treating skilled and unskilled labour as substitutible factors. In a very stylized way we introduce the notion that skilled labour has a larger spectrum of occupational options than unskilled labour in the sense that skilled labour can work both in skilled and unskilled jobs, deciding what they do by the rewards that are held by the two options, whereas unskilled labour is necessarily tied down to an unskilled job. A presumption that we do not think needs any intellectual labour to defend. As we show this feature interacting with increasing returns to scale gives rise to myriad possibilities of multiple equilibria, and very naturally renders an avenue through which skill premium can rise in both countries following trade. Notably, this we show in a Factor price equalization (henceforth FPE) framework .

Symmetric movements in wage gap is not consistent with 2 X 2 Heckscher-Ohlin-Samuelson (HOS) model, which would predict relative skill wages to go up in the skill abundant country and come down in the skill scarce one following trade.

Jones (1999) in a reconciliatory move proposes an interesting variant of the H.O.S. model with 3 goods and 2 factors (skilled and unskilled labour) with the goods uniquely ranked in terms of intensities. The good with middle ranked intensity is produced in both the countries and one with the highest and lowest skill intensity are produced in the developed North and the less developed South respectively. The South exports the good with middle intensity. Trade liberalization by North leads to an improvement in terms of trade for the South leading to a rise in demand for skilled labour in the South and in the North, lower import tariff reduces the domestic price of the middle good and which in turn is the unskilled labour intensive good for the North. Consequently unskilled wage rate in North goes down, leading to a rise in the wage gap. This model, thus can account for the symmetric movement in the wage gap. But also note, if North were to export the middle intensive good and south reduced tariff on the middle intensive good wage inequality would fall in both the countries. Though once again relative wage movements are symmetric, whether inequality rises or falls crucially depends on the trade pattern. Interesting variants on similar theme has been worked out in several other papers. Feenstra and Hanson (1996) in an oft cited paper redesigns the Dornbusch et al.(1980) model by adding a third factor capital. In their model a single manufacturing output is assembled from a continuum of intermediate inputs. Such inputs are produced by skilled labour, unskilled labour, and capital. In equilibrium South produces and exports a range of inputs and North does the rest. A rise in the stock of capital in the South shifts the intermediate intensity goods from the developed North to the under developed South raising relative demand for skilled labour in both the countries. Thus symmetrically increasing the wage gap. What is also crucial to note is that most of these models abandon the FPE framework. Trefler and Zhu (2001) closely builds up on the Feenstra, Hanson insight. In their model similar product shifting from North to South is initiated by technological catch up in the South. Marjit and Acharyya (2003) in an interesting contribution suggests yet another route through which trade might symmetrically increase the skill premium,where the channel is through intermediate goods trade. They consider a world where North exports a skill intensive intermediate good to the South on which South imposes a tariff. This intermediate good in turn along with skilled labour is used to produce a final good. A cut in tariff by the south increases the international price of the intermediate goods raising skill rewards in the North. Assuming there is no Metzler Paradox, domestic price of intermediates fall in the South and thus it is cheaper to produce the final good. Since this sector uses skilled labour competition tends to push up the demand for them and skill wage goes up in South as well.

These models, are essentially stepped in the tradition of standard competitive markets and constant returns to scale (CRS) technology. Returns to scale it seems is a natural point to depart. As Krugman (1981) has shown that trade under increasing returns to scale (IRS) might lead to co-movements in absolute factor prices, antithetically to stolper samuelson theorem. But even then in Krugman (1981) the relative factor prices follow the same pattern as HOS model would predict.

Similarly in Ethier (1982), for a small open economy Stolper Samuuelson theorem remains valid, even with IRS in production, to the extent the equilibrium is Marshall stable. These models thus cannot account for symmetric movements in the wage gap across countries.

We explore the potential of IRS in giving a very plausible explanation of rising wage inequality. The basic insight is very simple and as follows: If the demand conditions are such that skilled labour cannot be fully employed in the skill using IRS sector then part of the skilled labour force goes to join the unskilled job thus skill labour in general can merely earn the unskilled wage. Trade via intermediate goods can generate sufficient external economies such that supply prices in the IRS sector falls significantly and is then accomodated by the demand to operate at a level that fully utilizes skill labour in the skill using sector. Thus skill and unskilled wage rates diverge. Trade thus allows to fully utilize hitherto *under utilized* skilled labour. This we identify to be closely in the spirit of the thesis proposed by Hla Myint some three decades back. Myint in a somewhat different context identified trade as *vent for surplus*, the idea being that trade opened a vent or an outlet for a resource that had been previously under-utilized. This was an apt description of the trajectory of development driven by agricultural exports in the second half of the nineteenth century that transformed the economies of Canada and the United States. Our context is surely different, in that we show, when size of the market limits the scale of operation of a skill intensive sector, trade through the external economies can sufficiently depress average costs to an extent that a scale of production that fully utilizes the available skilled labour, can be accomodated by the demand. Hence trade opens up the opportunity of fully utilizing skill resources that has till then remained under- utilized.

The model uses the Dixit –Stiglitz specification to formalize the IRS sector and is like any other standard two sector model except for the fact that we explicitly introduce the notion that skilled and unskilled labour faces differential employment options in that skilled labour can work both in skilled and unskilled job whereas unskilled labour is tied down to the unskilled job. This apparently innocent looking assumption, as we will see, gives rise to myriad possibilities of multiple equilibria.

In what follows, Section 2 lays down the model, Section 3 solves for the autarky, Section 4 is a digression on the trade equilibrium and the last section conclude.

2. Model

The Economy: Basic Description

The economy is populated with two distinct types of agents. We chose to call them skilled and unskilled labour. Each type of labour is endowed with one unit of labour time, which they supply inelastically. Total amount of skilled and unskilled labour endowment in the economy is given by $\overline{H}$ and $\overline{L}$ respectively. We depart from the usual trade theoretic mode of characterizing skilled and unskilled labour where they enter into the production function as substitutible inputs just as capital and labour enters into an usual production function (see Jones(1999), Feenstra and Hanson(1996), Marjit and Acharyya (2003)) . In our model we assume that skilled labour can perform both unskilled and skilled job and they chose the one which holds higher reward. Unskilled labour on the other hand is tied down to the unskilled job and is not suited for the skilled work. This assumption, we believe captures in a very stylized manner a real life situation. And that this is real life, we believe does not require much persuasion. This otherwise innocent looking assumption, as we will see, has serious implications for our model. If the skilled wage rate w_s falls below the unskilled wage rate w_u all skilled workers join the unskilled labour force so it must be true that $w_s \geq w_u$.

Production

The economy produces two goods: 1 and 2, Where X_1 and X_2 are the outputs respectively. Good 1 is produced using unskilled labour under constant returns to scale technology. We chose units such that one unit of unskilled labour is required to produce one unit of good 1. Thus the price of good 1 equals the unskilled wage rate w_u. We also chose good 1 to be the numeraire. This implies w_u is equal to unity. Good 2 on the other hand is produced using differentiated intermediate inputs. The production technology for X_2 follows Dixit-Stiglitz (1977) specification and is given by

$$X_2 = \left[\sum_{i=1}^{n} y_i^{\rho} \right]^{1/\rho} \qquad \text{where } 0 < \rho < 1 \tag{1}$$

Where y_i is the input of intermediate good *i*. Intermediate goods are imperfect substitutes. ρ measures the degree of differentiation of intermediate inputs. Given the number of intermediate inputs the production function (1) exhibits constant returns to scale but there is increasing returns to higher degree of specialization as measured by the number of intermediate varieties *n*. These economies are external to the firm but internal to the industry i.e., X_2 producers take *n* as given. As in Ethier (1982), we assume that all intermediate goods have identical cost functions. The cost of producing the quantity *x* of a given variety of intermediate input is $C_x = (a + bx)\, w_s$, where *a* and *b* are the fixed and marginal requirements of skilled labour respectively. The presence of fixed cost gives rise to internal economies of scale at the firm level. An individual producer of X_2 maximizes profits subject to the production function and considering *n* to be parametrically given. This gives rise to the inverse input demand function for each intermediate input (see Helpman & Krugman (1985))

$$y_i = \frac{(q_i)^{-\sigma} \sum_{i=1}^{n} q_i y_i}{\sum_{i=1}^{n} q_i^{1-\sigma}} \qquad (2)$$

where q_i is the price of the *ith* intermediate input and $\sigma = \frac{1}{1-\rho}$ is the elasticity of substitution between any pair of intermediate inputs. Assuming large number of intermediate good producers, such that strategic behaviour is ruled out on their part, it can be easily shown that σ is the elasticity of demand faced by the intermediate producers. Thus each producer of intermediate inputs equate marginal revenue to marginal cost

$$q_i\left(1 - \frac{1}{\sigma}\right) = bw_s$$

Taking note of the fact that $\sigma = \frac{1}{1-\rho}$

$$\Rightarrow q_i = \frac{bw_s}{\rho} \qquad (3)$$

Thus prices of intermediate goods are a constant mark up over the marginal cost. With identical technology all firms charge the same price for intermediate goods $(q_i = q)$. Free entry in production of intermediate inputs drives down profits to zero (the chamberlinian large group case). Thus the operating surplus must be just enough to cover the fixed cost

$$\frac{q}{\sigma} x_i = a w_s \tag{4}$$

This also implies that output x_i is the same for all producers, $(x_i = x)$.

Dividing equation (4) by (3) we get,

$$x = \frac{a\rho}{b(1-\rho)} \tag{5}$$

with this symmetry $(x_i = x)$ and noting that demand supply equilibrium in intermediate goods market imply $y_i = x$ for all i, equation (1) collapses to

$$X_2 = n^{\alpha} x \qquad \text{Where } \alpha \equiv \frac{1}{\rho} > 1 \tag{6}$$

Further, note that (5) implies output per firm (x) is a constant. Thus equation (6) imply that any expansion of X_2 would be in terms of increased n, and this, as has already been noted, implies increasing returns to scale at the industry level in X_2 production.

If in equilibrium n is the effective number of produced varieties, then the total amount of skilled labour used in the production of intermediate goods is given by

$$H = n\,(a + bx) \tag{7}$$

Noting that x is a constant this in turn imply that all changes in n and thereby in the output of X_2 is brought about singularly by changes in H.

Preferences

All agents share the same Quasi-linear utility function given by

$$U = C_1^{\beta} + C_2 \qquad \text{Where } 0<\beta<1 \tag{8}$$

The quasi-linear form notionally captures the idea that agents might even go without good 2. And in this sense this good is not a basic one. As we will see we might have an equilibrium where sector 2 production does not come into being.
First order conditions for utility maximization imply

$$\beta C_1^{\beta-1} = \lambda \tag{9}$$

$$1 = \lambda p \tag{10}$$

Where p is the relative price of good 2 and λ is the associated Lagrange multiplier.
Equation (8) and (9) imply

$$p = \frac{1}{\beta} C_1^{1-\beta} \tag{11}$$

3. Autarky

The general equilibrium supply curve

Noting that zero profit condition prevails in the production of final output X_2 ;

$$pX_2 = nqx \tag{12}$$

Where the LHS is the total revenue and the RHS is the total cost.
Substituting for X_2 from equation (6), equation (12) boils down to

$$p = n^{1-\alpha} q \tag{13}$$

Using equations (3), (5) and (7), equation (13) can be rewritten as

$$p = ZH^{1-\alpha} w_s \tag{14}$$

Where $Z \equiv \left[\frac{1-\rho}{a}\right]^{1-\alpha} \frac{b}{\rho}$

Equation (14) is nothing but the price and average cost equality. This is our fundamental supply side equation and it essentially follows Ethier (1982). But we intend to tailor this equation bit differently for the purpose of our model.
Let us now focus on the factor market. Note that until the moment the whole of skilled labour has been absorbed in intermediate goods sector, skilled labour can be employed in

this sector at the wage rate of unity (i.e. $w_s = 1$, the wage rate prevailing in sector 1). Simply because the residual skilled labour remains employed in sector 1 and earn the unskilled wage rate unity. This therefore imply that in this relevant regime equation (14) holds with $w_s = 1$

$$p = ZH^{1-\alpha} \tag{15}$$

The moment skilled labour has been completely employed in intermediate goods sector equation (14) reduces to

$$p = Z\overline{H}^{1-\alpha} w_s \tag{16}$$

Where $\overline{H}$ is the total amount of skilled labour available in the economy.
In equation (15) w_s remains frozen at unity and H, thereby the output of good 2, adjusts to bring the price and average cost to equality. Increase in H increases the output of good 2 and there being increasing returns to scale, depresses the average cost and therefore the price. In equation (16) H remains frozen at $\overline{H}$ and output of good 2 does not change and any increase in price translates into an increase in skilled wages.
These two regimes corresponding to equation (14) is depicted as the curve SUM in figure 1. The downward sloping section SU of this curve depicts equation (15). Here w_s remain fixed at unity and any increase in H depresses p. It can readily be checked that this section of the curve is asymptotic to the p axis and is also convex. To see this note that in equation (15) as $H \to 0, p \to \infty$. Furthermore from equation (15),

$$\frac{dp}{dh} = (1-\alpha)ZH^{-\alpha} < 0 \tag{17}$$

and

$$\frac{d^2 p}{dh^2} = -\alpha(1-\alpha)ZH^{-(\alpha+1)} > 0 \tag{18}$$

Note that point U corresponds to a price $p = Z\overline{H}^{(1-\alpha)}$. This is the minimum price at which good 2 can be sold. At U skilled labour is employed at the wage rate unity and total amount of skilled labour $\overline{H}$ has been absorbed in intermediate goods production so the output of good 2 is at its maximum and noting that there being increasing returns to scale this imply that average cost of production and hence price of good 2 is at its minimum.
On the other hand, to the left of U on the segment SU, though the skilled wage is at unity,

skilled labour employed (H) in sector 2 is less than $\overline{H}$ and therefore output of good 2 is less than that at U. Hence average cost and thereby price of good 2 on this segment is higher than that at U. On the segment UM which corresponds to equation (16) skilled labour has been completely absorbed in intermediate production so out put of good 2 is at its maximum and there cannot be any further expansion of the same. Any rise in p is therefore translated into a rise in w_s to bring average cost in line with p , as can be seen from equation (16). Thus on the segment UM $w_s \geq 1$ with $w_s = 1$ at U and $w_s > 1$ at all other points.

General equilibrium Demand Curve

On the demand side equation (11) can be integrated with the factor market equilibrium to arrive at an expression for the general equilibrium demand. First note that for all $H < \overline{H}$ skilled wage $w_s = 1$. Thus with both skilled and unskilled wage equal to unity all agents skilled and unskilled should consume the same amount of good 1 at any given p , given the fact that the utility is quasi linear and good one is the basic good. Thus $C_1 = \frac{(\overline{L} + (\overline{H} - H))}{(\overline{L} + \overline{H})}$ where the neumerator is the total production of good 1, and the denominator gives the total number of people consuming good 1. To see that the neumerator is the total amount of good 1 produced in the economy, note that with H being the amount of skilled labour employed in the intermediate goods sector, $(\overline{H} - H)$ is left to work in sector 1 over and above $\overline{L}$ unskilled labour who can only work in sector 1. Thus noting that production of one unit of good 1 requires one unit of labour, $(\overline{L} + (\overline{H} - H))$ is the total amount of good 1 produced in the economy. Inserting this expression of C_1 in equation (11) we arrive at

$$p = \frac{1}{\beta}\left(\frac{\overline{L} + \overline{H} - H}{\overline{L} + \overline{H}}\right)^{1-\beta} \qquad (19)$$

This demand expression can thus be depicted on the p, H plane as shown by the curve DC in figure 1. Now note that the argument we rendered to derive this curve was premised on the assumption $H < \overline{H}$, which implied $w_s = w_u = 1$. We now argue that the same demand relation (equation (19)) remains valid at $H = \overline{H}$ or to put it otherwise the terminal point C on the curve DC which from equation (19) corresponds to the demand price $p = \frac{1}{\beta}\left[\frac{\overline{L}}{\overline{L}+\overline{H}}\right]^{(1-\beta)}$ is truly the demand price at $H = \overline{H}$. Note that at $H = \overline{H}$ skilled wage w_s can either be equal to or greater than unity. If it is unity then both skilled and unskilled labour consumes same amount of good 1 and the above argument goes through and equation (19) remains valid at $H = \overline{H}$. We now argue that even if $w_s > 1$ equation (19) remains valid. To see this first note that at $H = \overline{H}$ all skilled people are working in the intermediate goods sector, but given the quasi-linear utility they must be consuming good 1. This entails that unskilled labour trades some of its good 1. Carrying the argument little further this would imply that unskilled labour consumes some of good 2, or to put it differently is saturated in consumption of good 1. Now, given the quasi-linear structure of the utility, if unskilled labour is saturated in consumption of good 1 then at any common price it must be true that skilled labour with higher wages must also be saturated in consumption of good 1 and their consumption levels of good 1 must be the same. With this in view the previous argument follows and equation (19) remains valid at $H = \overline{H}$.

It can readily be checked that $\frac{dp}{dh}$ and $\frac{d^2p}{dh^2}$ are both negative.

$$\frac{dp}{dh} = -\frac{(1-\beta)}{\beta(\overline{L}+\overline{H})}\left[\frac{\overline{L}+\overline{H}-H}{\overline{L}+\overline{H}}\right]^{-\beta} < 0 \tag{20}$$

$$\frac{d^2p}{dh^2} = -\frac{(1-\beta)}{(\overline{L}+\overline{H})^2}\left[\frac{\overline{L}+\overline{H}-H}{\overline{L}+\overline{H}}\right]^{-(1+\beta)} < 0 \tag{21}$$

Hence the curve DC in figure 1 is concave and downward sloping.

Autarkic Equilibrium

We now have all the resources to determine the autarkic equilibrium. The supply side curve SUM and the demand side curve DC determines the equilibrium of the economy. Three alternative possibilities can arise as shown in figures 1,2 and 3. In figures 1 and 2 the economy exhibits multiple equilibria at points D, E and C and at D, E and V respectively. Under reasonable presumptions of Marshallian adjustment (as is customary to assume in the increasing returns to scale literature (see Ethier(1982)) one can see that point E is an unstable equilibrium in both these cases with the demand curve intersecting the supply curve from below. So in figure 1, D and C and in figure 2, D and V are the two stable equilibria. At D in both figures sector 2 production never takes off and in the vicinity of point D supply price exceeds demand price. Thus under Marshallian adjustment, small attempts at increasing production of good 2 (which is same as increasing H) would not work. With supply price exceeding the demand price the economy comes back to point D.

At C in figure 1 and at V in figure 2, sector 2 is operative. But as is evident there is a crucial point of difference in the two cases shown in figures 1 and 2. At the stable equilibrium C in figure 1, skilled labour has been completely employed in the skill using intermediate goods sector and noting that the segment UM of the supply curve corresponds to $w_s > 1$, skilled wage is greater than the unskilled wage. On the other hand at V in figure 2 though the skill using sector 2 is operative, skilled labour has not been completely absorbed in intermediate goods production and noting that the segment SU corresponds to $w_s = 1$, both skilled and unskilled wages are equal. And no wonder this happens. With skilled labour not being completely absorbed in intermediate goods sector some of the skilled workers remain working in the unskilled job in sector 1 earning a wage of unity, consequently the economy wide skilled wages are tied down to the level of unskilled wages at unity. We chose to call an equilibrium of this kind as shown by point V in figure 2, an *under-utilization equilibrium.* This we believe has a semblance to the argument put forward by Myint, where he was making a case in favour of international trade as a *vent for surplus.* As we will show later, international trade can deliver a country from such an *under-utilization equilibrium* to full utilization of its skilled labour.

It is straightforward to see that the difference in the two cases shown in figures 1 and 2 lies in the fact that in figure 1 the demand curve intersect the supply curve stretch UM at $H = \overline{H}$ and in figure 2 the demand curve lies below the UM stretch at $H = \overline{H}$. Alternatively in figure 1(2) the demand price at $H = \overline{H}$ which from equation (19) is given by $\frac{1}{\beta}\left[\frac{\overline{L}}{\overline{L}+\overline{H}}\right]^{(1-\beta)}$, is greater (less) than the minimum supply price of good 2 at $H = \overline{H}$ (which corresponds to the point U), which from equation (16) is given by $Z\overline{H}^{1-\alpha}$. Thus strong scale economies or high α favours the case shown in figure 1. In figure 2; which corresponds to a case of low α, the supply price, even at the largest possible scale of operation of sector 2 ($H = \overline{H}$) is not sufficiently low as to be matched by a corresponding demand price (see that point U lies above C), consequently the economy cannot fully absorb the skilled work force in the skill using sector.

Figure 3 corresponds to a case where the supply price is uniformly higher than the demand price. Hence the unique equilibrium is given by point D. This would be the case if the strength scale economies or which is the same as α, is sufficiently low.
In such a case sector 1 is the only operative sector. And all wages, skilled and unskilled would be unity.
The observations made above are summarized in the following proposition.

Proposition 1. *The economy exhibits multiple equilibria . The following three situations exhausts all the possibilities.*

a) *The economy has two stable equilibria (as shown in figure1). One in which sector 2 is non-operative and skilled wage is equal to unskilled wage and another with an operative sector 2 with skilled labour completely absorbed in that sector and with skilled wage higher than the unskilled wage. It is evident that a necessary and sufficient condition for such a situation to arise is given by*

- $\frac{1}{\beta}\left[\frac{\overline{L}}{\overline{L}+\overline{H}}\right]^{(1-\beta)} > Z\overline{H}^{1-\alpha}$.

b) *The economy has two stable equilibria (as shown in figure 2).One in which sector 2 is non-operative and skilled wage is equal to unskilled wage and another with an operative sector 2 with skilled labour partially absorbed in that sector and with skilled wage remaining equal to the unskilled wage. Necessary and a sufficient condition for such a situation to arise is given by*

- *There exists* $0 < H < \overline{H}$ *such that* $\frac{1}{\beta}\left[\frac{\overline{L}+\overline{H}-H}{\overline{L}+\overline{H}}\right]^{(1-\beta)} > ZH^{1-\alpha}$.

And

$$\frac{1}{\beta}\left[\frac{\overline{L}}{\overline{L}+\overline{H}}\right]^{(1-\beta)} < Z\overline{H}^{1-\alpha}$$

c) *The economy has an unique equilibrium in which sector 2 production never takes off (as shown in figure 3). All skilled and unskilled workers are employed in sector 1 and they earn the same wage unity. Necessary and a sufficient condition for such a situation to arise is given by*

$$\frac{1}{\beta}\left[\frac{\overline{L}+\overline{H}-H}{\overline{L}+\overline{H}}\right]^{(1-\beta)} < ZH^{1-\alpha} \quad \forall H \in (0,\overline{H}]$$

4. Trade

Let two countries having the above structure engage in commodity trade. We call them home and foreign (whenever needed, we denote them by h and f respectively).

Countries are similar preference and technology wise but can possibly differ in their quantities of factor endowments.

We assume all goods (final goods 1 and 2, and also the intermediate goods) to be freely tradable.

With commodity trade equalizing good 1 prices in home and foreign, it must be the case that unskilled wages are equalized in both the countries.

Now let us focus on the market clearing conditions for intermediate goods.

$$x^h = y_h^h + y_f^h \qquad (22a)$$

$$x^f = y_h^f + y_f^f \tag{22b}$$

where x^h and x^f are the supplies of a representative brand of intermediate input of home and foreign country respectively, and y_i^j denotes the amount of intermediate input produced in the j th country and used by the i th country producers of good 2. Thus the LHS in equations (22a-b) denotes the total world supply and the RHS denotes the total world demand of intermediate inputs respectively.

Now from the demand equation (2) and noting that zero profit condition prevails in the production of good 2, and that trade equalizes price of commodity 2 in both the countries, the set of equations (22) reduces to

$$x^h = \frac{(q^h)^{-\sigma} pX_2^h}{n^h (q^h)^{1-\sigma} + n^f (q^f)^{1-\sigma}} + \frac{(q^h)^{-\sigma} pX_2^f}{n^h (q^h)^{1-\sigma} + n^f (q^f)^{1-\sigma}} \tag{24a}$$

$$x^f = \frac{(q^f)^{-\sigma} pX_2^h}{n^h (q^h)^{1-\sigma} + n^f (q^f)^{1-\sigma}} + \frac{(q^f)^{-\sigma} pX_2^f}{n^h (q^h)^{1-\sigma} + n^f (q^f)^{1-\sigma}} \tag{24b}$$

Where q^j now denotes the price of a representative brand of intermediate input produced in the j th country, X_2^j is the output of good 2 produced in the j th country and n^j is the number of varieties produced in the j th country.

It is clear from equation (5) that with identical technology, $x^h = x^f$. Therefore the set of equations (24) imply $q^h = q^f$. Now noting that intermediate good prices are set as a constant mark-up over skilled wages (see equation (3)), skilled wages are also equalized across countries.

Therefore the following proposition is immediate.

Proposition 2. *Free trade in final and intermediate goods equalizes skilled and unskilled wages in both the countries.*

With price of intermediate goods produced in both the countries now equal (i.e. $q^h = q^f$), it must be true that a country will be using the same amount of each brand of intermediate input whether produced at home or foreign, in the production of final good 2. Which imply

$$y_h^h = y_h^f \text{ and } y_f^f = y_f^h$$

Now with zero profit condition prevailing in the final good 2 production

$$pX_2^j = n^h q^h y_j^h + n^f q^f y_j^f \ : \ j = h, f \tag{25}$$

Recalling that $q^h = q^f$ and $y_j^h = y_j^f : \ j = h, f$, and using the production function given in equation (1), equation (25) reduces to

$$p = (n^h + n^f)^{1-\alpha} q^h \tag{26}$$

This is evidently the trade counterpart of equation (13).

As in equation (14), equation (26) reduces to

$$p = Z(H^h + H^f)^{1-\alpha} w_s \tag{27}$$

Where H^h and H^f are the amount of skilled labour employed in intermediate goods production in the home and in the foreign country respectively. Comparing equations (13) and (26) it is clear that trade in intermediate goods by making available a larger number of varieties of intermediate inputs have depressed the supply price of good 2 for any given q.

With factor prices equalized through trade, the trading equilibrium can be solved just as was done under autarky, but now by considering the world as an integrated economy. Thus on the demand side we have the counterpart of equation (19)

$$p = \frac{1}{\beta}\left(\frac{\overline{L}^w + \overline{H}^w - H^w}{\overline{L}^w + \overline{H}^w}\right)^{1-\beta} \tag{28}$$

Where $\overline{L}^w = \overline{L}^h + \overline{L}^f$, $\overline{H}^w = \overline{H}^h + \overline{H}^f$ and $H^w = H^h + H^f$.

Equation (27) is now the supply side relation depicted by the SUM curve and equation (28) is the demand side relation depicted as the DC curve in figures 4,5 and 6. As under autarky, these two relations determine the trade equilibrium[1].

[1] Note that in the stable equilibrium shown by point V in figure 5 the total amount of skilled labour employed in the intermediate goods sector world-wide (H^w) is determined. But it is important to note that the amount of skilled labour devoted to intermediate goods production for each country (H^h and H^f)separately remain indeterminate. Any allocation of H^h and H^f is consistent with this equilibrium as long as $H^w = H^h + H^f$.

Trade as a" vent for surplus" and the possibility of symmetric rise in wage inequality thereof

Let us start with the most straightforward case in which trade leads to an increase in skill premium in both the countries. And in this, we will also try to argue that trade plays a role of a *vent for surplus* in the spirit of the thesis proposed by Hla Myint.

Let us assume that the parameters of the model are such that both the economies, home and foreign, begin from a situation where the autarkic equilibrium is of the kind depicted by point V in figure 2. That is to say, both the countries have an operative sector 2 but skilled labour remains *under-utilized*. And thereby in both the countries skilled and unskilled wages are equal to unity. A situation of this order is evidently characterized by

$$\frac{1}{\beta}\left[\frac{\overline{L}^i}{\overline{L}^i+\overline{H}^i}\right]^{(1-\beta)} < Z(\overline{H}^i)^{1-\alpha}$$

and there exists $0 < H^i < \overline{H}^i$ such that

$$\frac{1}{\beta}\left[\frac{\overline{L}^i+\overline{H}^i-H^i}{\overline{L}^i+\overline{H}^i}\right]^{(1-\beta)} > Z(H^i)^{1-\alpha} \quad ; \; i=h,f \tag{29}$$

Now with opening up of trade it can always be possible that the trading equilibrium is of the kind depicted as point C in figure 4. Clearly such an equilibrium ensures a higher wage for skilled workers than unskilled workers.

Note that under trade the minimum supply price, which corresponds to point U in figures 4, 5 and 6, is given by

$$p = Z(\overline{H}^h+\overline{H}^f)^{1-\alpha} \tag{30}$$

Thus the minimum supply price under autarky in both the countries, as given by the RHS of (29) is greater than what would be achieved under trade, given by equation (30).

For the trade equilibrium to be of the kind depicted as point C in figure 4, the demand price given by equation (28) evaluated at $H^w = \overline{H}^w$, has to be greater than the minimum supply price given by equation (30). Putting the arguments together, the following proposition follows

Proposition 3. *If* $\frac{1}{\beta}\left[\frac{\overline{L}^i}{\overline{L}^i+\overline{H}^i}\right]^{(1-\beta)} < Z(\overline{H}^i)^{1-\alpha}$; $i = h, f$ *and* $\frac{1}{\beta}\left(\frac{\overline{L}^w}{\overline{L}^w+\overline{H}^w}\right)^{1-\beta} > Z(\overline{H}^h+\overline{H}^f)^{1-\alpha}$ *then the first inequality suggests that the autarkic equilibrium for both the countries can be an under-utilization equilibrium(point V in figure 2) with equal skilled and unskilled wages and the second inequality suggests that the trade equilibrium can be characterized by full utilization of skilled labour in the skill using sector with skilled wage higher than the unskilled wage in both the countries. Thus trade can lead to symmetric increase in wage inequality (skilled to unskilled wage ratio) in both the countries.*

Thus we have a very plausible theory of trade led rise in skill premium in both the trading countries. The mechanism is intuitively pretty clear. Trade in intermediate goods, generate externalities which reduces average cost of producing good 2 in both the countries. If to begin with the autarky, the minimum average cost (which corresponds to the full utilization of skilled labour in the intermediate goods sector and skilled wage of unity) of producing good 2 is higher than the demand price evaluated at full absorption of skilled labour in the skill using intermediate goods sector, skilled labour cannot be fully absorbed in the skill-using sector. Part of the skilled work force therefore remains engaged in sector 1. Skilled wage thus remains tied down to the level of unskilled wage. Trade by bringing down the minimum average cost to a level that can be accomodated by the demand price, helps absorb skilled labour in the skill-using sector fully. Thereby skilled and unskilled wages diverge. What is crucial in the mechanism involved is that trade helps utilize fully, hitherto under-utilized skilled labour. Noticeably we can show here a case of trade induced rise in skill premium, symmetrically incident on both the countries, remaining within the FPE framework which most of the models in this genre of the literature abandon.

Yet another case could be, both countries having an equilibrium of the kind depicted as point C in figure 1 in autarky and the trade equilibrium is of the kind depicted as point C in figure 4. Thus both under autarky and trade these countries are fully utilizing the skilled labour in the skill-using sector. Even under such a situation inequality could rise in both the countries following trade. To see this note that skilled wage under autarky in this situation can be solved from equation (16) and equation (19) evaluated at $H = \overline{H}$. Yielding

$$w_s^i = \frac{\frac{1}{\beta}\left[\frac{\overline{L}^i}{\overline{L}^i + \overline{H}^i}\right]^{1-\beta}}{Z(\overline{H}^i)^{1-\alpha}} \quad ; \ i = h, f \tag{31}$$

Where w_s^i denotes the autarkic skilled wage in the ith country.

Similarly the solution for skilled wage under trade for an equilibrium like point C in figure 4 would be given by solving equations (27) and (28) evaluated at $H^h = \overline{H}^h$, $H^f = \overline{H}^f$ and $H^w = \overline{H}^w$. Thus

$$w_{sT} = \frac{\frac{1}{\beta}\left[\frac{\overline{L}^w}{\overline{L}^w + \overline{H}^w}\right]^{1-\beta}}{Z(\overline{H}^h + \overline{H}^f)^{1-\alpha}} \tag{32}$$

Where w_{sT} denotes the skilled wage under trade.

Comparing equations (31) and (32), we can see that on count of the denominator (which captures the external economies generated through intermediate goods trade) trade pushes up skilled wages in both the countries. But noting that the neumerator in equation (31) is increasing in $\frac{\overline{L}}{\overline{H}}$, and that $\frac{\overline{L}^w}{\overline{H}^w}$ lies between $\frac{\overline{L}^h}{\overline{H}^h}$ and $\frac{\overline{L}^f}{\overline{H}^f}$ skilled wage in a country relatively scarce in skilled labour (i.e. higher $\frac{\overline{L}}{\overline{H}}$) can potentially suffer a fall in skilled wages. And for a skilled labour abundant country both on the neumerator and the denominator count trade pushes up skilled wage. Thus if strength of external economies generated through trade is strong enough to outweigh the fall in skilled wage due to factor endowment difference in a skill scarce country, wage inequality will rise in both the countries. These points are summarized in the following proposition.

Proposition 4. *If both the countries operate at the full utilization equilibrium under autarky and under free trade and if* $\frac{\overline{L}^h}{\overline{H}^h} > \frac{\overline{L}^f}{\overline{H}^f}$ *then a necessary and sufficient condition for trade to increase wage inequality in both the countries is given by*

$$\frac{\left[\frac{\overline{L}^h}{\overline{L}^h+\overline{H}^h}\right]^{1-\beta}}{(\overline{H}^h)^{1-\alpha}} < \frac{\left[\frac{\overline{L}^w}{\overline{L}^w+\overline{H}^w}\right]^{1-\beta}}{(\overline{H}^h+\overline{H}^f)^{1-\alpha}}.$$

5. Conclusion

We develop a simple two sector (with a constant and an increasing returns to scale sector) two factor trade model explicitly introducing the notion that skilled and unskilled labour faces differential employment options, in that skilled labour can work both in skilled and unskilled job whereas unskilled labour is tied down to the unskilled job. The demand pattern is generated out of a quasi-linear utility function. Thus one of the good is basic in the sense that it is always consumed in positive quantities for any positive income. The other good may not be consumed at all. We show that such a model can, in an intuitively appealing way, account for trade driven wage inequality. Remaining within the FPE framework (which most of the models in the relevant literature abandon) we are able to show that trade can symmetrically raise wage inequality in both the trading countries.

The model holds interesting possibilities of multiple equilibria. Of which an equilibrium can be either characterized by a non-operative skill using IRS sector or an operative IRS sector but not fully absorbing the available skilled labour or an operative IRS sector fully absorbing the available skilled labour.

If to begin with at the autarky, the minimum average cost (which corresponds to the full utilization of skilled labour in the IRS sector and with skilled wage equal to the unskilled wage) of producing the IRS good is higher than the demand price, skilled labour cannot be fully absorbed in the skill-using IRS sector. Part of the skilled work force remains engaged in the CRS sector, which uses unskilled labour. Skilled wage thus remains tied down to the level of unskilled wage

Trade by bringing down the minimum average cost to a level that can be accomodated by the demand price, helps absorb skilled labour in the skill-using sector fully. Thereby skilled and unskilled wages diverge. What is crucial in the mechanism involved is that trade helps full utilization of hitherto under-utilized skilled labour.

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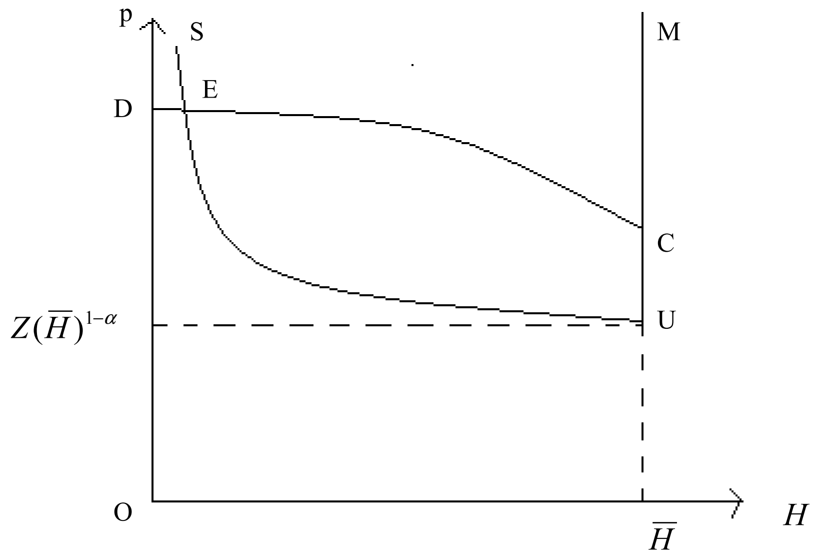

Figure 1

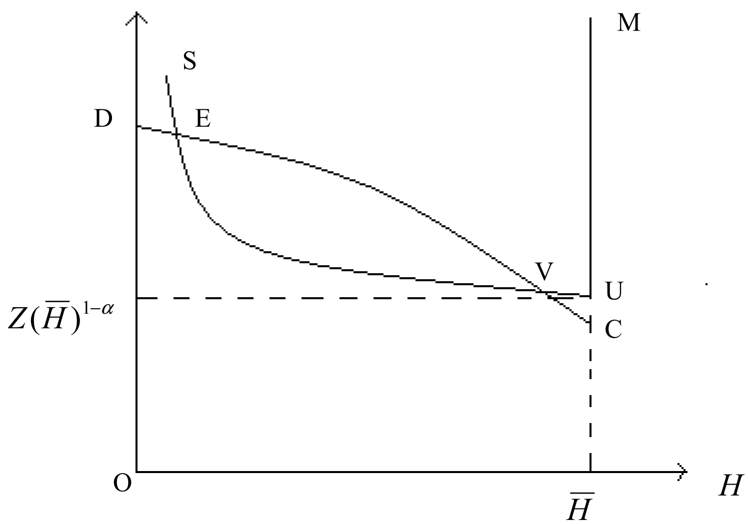

Figure 2

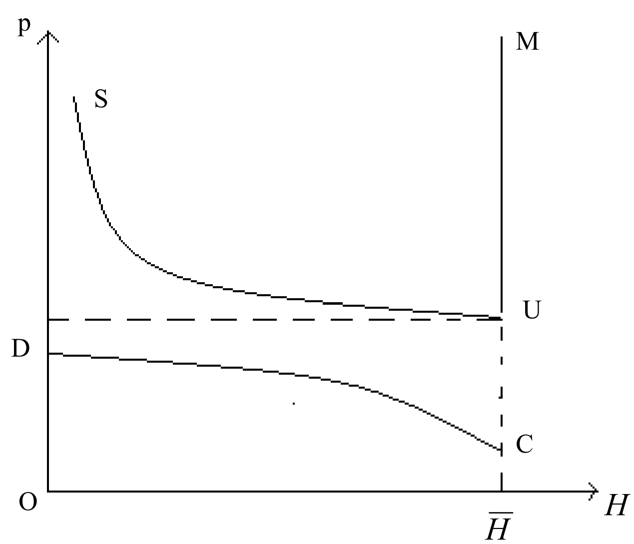

Figure 3

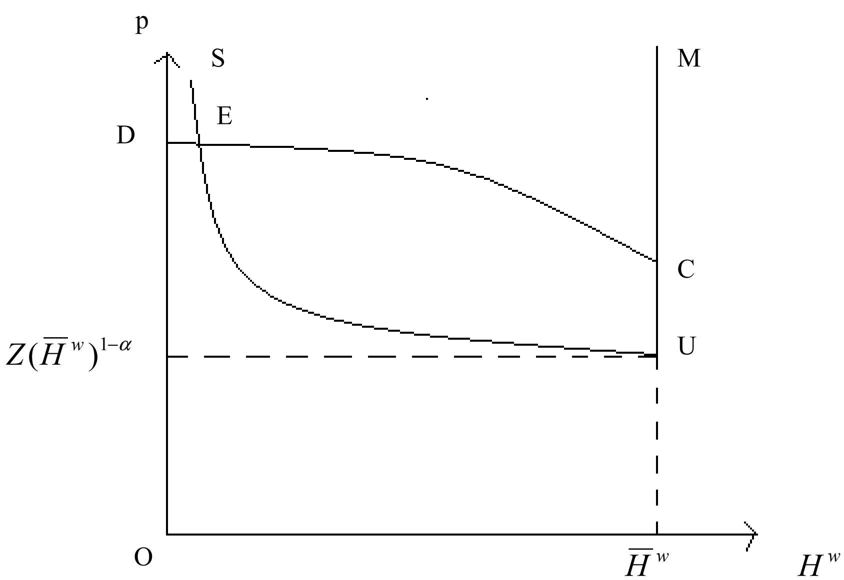

Figure 4

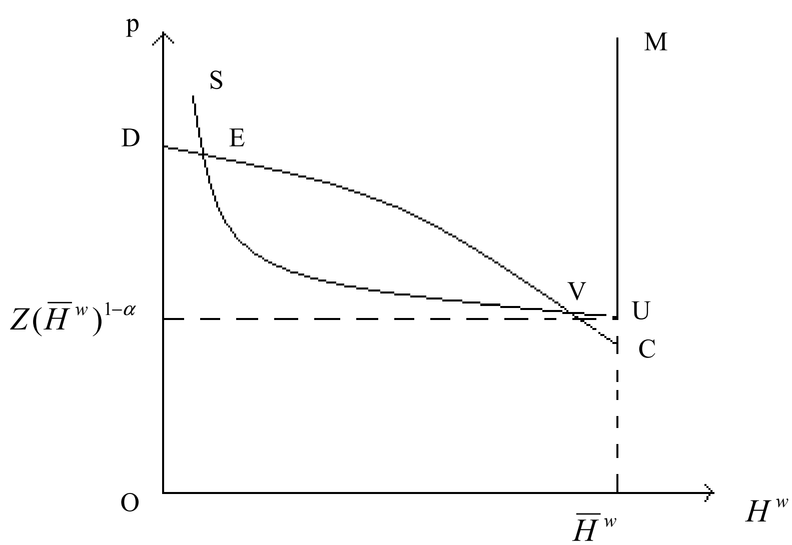

Figure 5

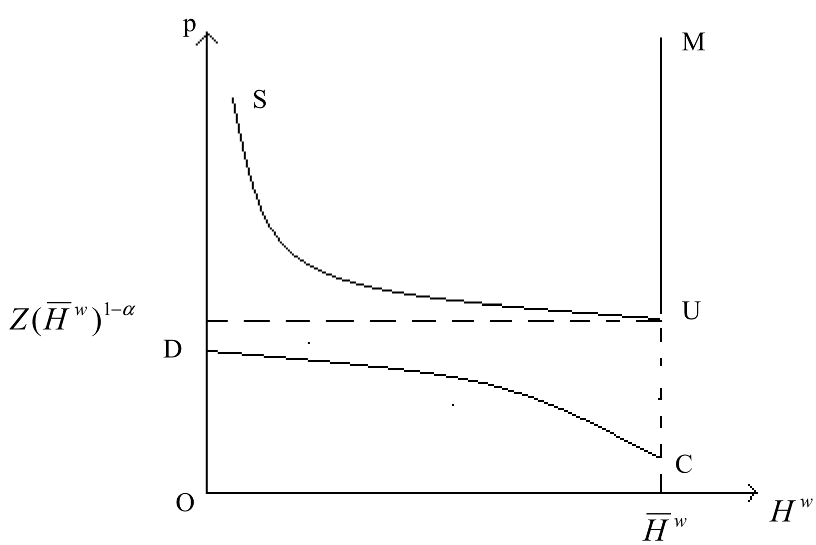

Figure 6