Is India better off today than 15 years ago ? A robust multidimensional answer*

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Abstract

This paper provides a robust normative evaluation of the spectacular growth episode that India has experienced in the last 15 years. Specifically, the paper compares the evolution, between 1988, 1996 and 2001 of the distribution of several important individual attributes on the basis of ethically robust dominance criteria. The individual attributes considered are real consumptions (measured at the individual level), literacy rate, infant mortality and violent crime rates (all measured at the district levels). District level variables are interpreted as (local) public goods which, along with consumption, are assumed to contribute to individual well-being. The robust criteria used are generalizations, to more than two attributes, of the first and second order dominance criteria of Atkinson and Bourguignon (1982) and are known to correspond to the unanimity of utilitarian value judgements taken over a specific class of individual utility functions. The main result of

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the empirical analysis is that all utilitarian rankings of distributions of the four attributes who assume that individual utility functions satisfy the assumptions of second order dominance aggree that India is better off in 2002 than in 1996. Furthermore, if one removes crime from the list of attributes, the dominance is shown to apply steadily over the whole period and to be of first order on the period 1988-1996.

1. Introduction

In the last fifteen years, the Indian economy has grown at an average of 6% per year (about 3% per capita). This spectacular growth, which seems to be connected to the liberalization reforms introduced in the late eighties and early nineties, has immensely modified the lives of the billion individuals living in this country. The object of this paper is to provide a *robust* normative appraisal of this modification. Specifically, we are providing a robust answer to the basic question: Is India a better place to be now than it was fifteen years ago ?

There has been for sure numerous attempts to providing answers to this question in the recent literature. Many of these attempts have been concerned with the impact of the Indian growth episode on pecuniary poverty and/or inequality (see e.g. Datt & Ravallion (2002), Deaton & Drèze (2005) and the various contributions contained in the collective volume of Deaton & Kozel (2005)). Yet, interesting as they are, most of these attempts have suffered from *two* basic insufficiencies.

First they have focused on specific poverty (e.g. headcount ratio, poverty gap, squared poverty gap, etc.) or inequality (typically Gini coefficient) indices. Poverty analysis based on a specific poverty index is fragile because it rides heavily on the choice of a poverty line, choice that is known to be very difficult (see e.g. Lipton & Ravallion (1998)). Inequality analysis based on a specific index suffers from the same lack of robustness with respect to the choice of the index (i.e. would the conclusions obtained from comparing Gini coefficients remain valid for the Coefficient of variation or for the Theil index ?).

The second, and our view most important, limitation of the existing attempts to normatively appraise the recent spectacular growth in India is that they have taken a *unidimensional* perspective of focusing only on *pecuniary* variables. Yet it has long been recognized (see for instance Kolm (1977), Atkinson & Bourguignon (1982), Atkinson & Bourguignon (1987), Rawls (1971), Sen (1987) and Sen (1992)) that monetary income or consumption is not the only individual attribute that is relevant for normative evaluation. Attributes such as health, education, protection against crime and pollution (to mention just a few) are also important contributors to individual well-being and the distributions of these attributes, along with that of pecuniary consumption, is of key importance for the normative evaluation of the development path of a country. While this multidimensionality of economic development is becoming increasingly acknowledged, it has failed so far to give rise to successful empirical implementations. With some recent exceptions (see e.g. Crawford (2005), Duclos *et al.* (2006) and Gravel *et al.* (2005)) much applied work on multidimensional normative appraisal that is done nowadays aggregates the various individual attributes into a single Human Development Index (HDI) and looks at the distribution of this one-dimensional index. Yet this approach obviously suffers from the arbitrariness of the aggregation procedure.

The insufficient development of the theory of multidimensional normative measurement, as compared to its unidimensional cousin, contributes certainly for no small part to the scarcity of studies that perform multidimensional normative evaluation. The theory of one-dimensional normative measurement has reached its full fruition for quite a long time. The theory rides on a remarkable equivalence, first established by Hardy *et al.* (1952), and popularized, among economists, by Sen (1973) (see also Kolm (1969) and Dasgupta *et al.* (1973)) between *four* plausible answers to the basic question: When can we say that a distribution A of one attribute between n individuals is unambiguously better than another distribution B ? Assuming that the total amount of the attribute to be distributed is the same in both distributions, the four equivalent answers to this question are:

(1) When A could be obtained from B by *a finite sequence of bilateral Pigou-Dalton transfers* between individuals.

(2) When A would be ranked above B by all utilitarian planners who assume that individuals convert income into utility by the same *non-decreasing* and *concave* utility function.

(3) When poverty, as measured by the *poverty gap*, is lower in A than in B no matter what the poverty line is.

(4) When the *Lorenz curve* associated to A lies everywhere above that corresponding to B.

These equivalences have been generalized to distributions involving different total quantities of the attribute and/or different numbers of individuals[1]. Any of the four answers provides an ethically robust conclusive ranking of one-dimensional distributions with non-crossing Lorenz curves. This ranking, it should be emphasized, shows that there is no difference between poverty and inequality reduction when one requires poverty comparisons (performed by the poverty gap) to be robust to the choice of poverty lines. Poverty decreases for all poverty lines if and only if inequality decreases for all Pigou-Dalton sensitive inequality indices. It is, for this reason, somewhat surprising that this robust and well-established one dimensional dominance analysis has not been much used in the normative appraisal of the Indian experience. A possible reason for this could be that it does not lead, in the Indian case, to a conclusive ranking. Yet, as shown below, this does not seem to be case for the distributions of individual consumption observed over the period 1988-2002.

While we do not have at our disposal such a theoretical foundation for performing multidimensional normative evaluations, the (slow) progress that have been made in the last twenty years on this question do not make us completely deprived. Following the important contribution by Atkinson & Bourguignon (1982) and Atkinson & Bourguignon (1987), and the less noticed one of Bourguignon (1989), as well as recent works by Fleurbaey *et al.* (2003) or Gravel & Moyes (2006), we dispose of a few operational dominance criteria that rank alternative distributions of *two* attributes in the same way as would all utilitarian social planners believing that individuals convert attributes into utility by the same function satisfying specific properties. Furthermore, for some of these criteria (e.g. the first Atkinson & Bourguignon (1982) criterion and that of Bourguignon (1989)), we know from Gravel & Moyes (2006) the underlying elementary transformations (multidimensional analogues of Pigou-Dalton transfers) that correspond exactly to them.

Hence we have the theory and the methods for appraising, in an ethically robust matter, the impact of the spectacular growth that India has experienced on the distribution of well-being amongst Indians through the evolution of the distribution of several attributes. The attributes considered in this paper are individual consumption (as obtained from the Na-

[1] Distributions with different numbers of individuals can be made comparable by applying the so-called Dalton population principle according to which the replication of the same distribution an arbitrary number of time does not change its distributional characteristics. Distributions with different total amount of the attribute can be unambigously compared just as those with the same total amount if *increments* of attribute are added to the bilateral transfers in answer (1) and if the Lorenz curve is replaced by the *generalized Lorenz* one (see Shorrocks (1983)) in answer (4).

tional Sample Survey NSS) of India in the rounds 1988, 1996 and 2002) and three attributes measured at the level of the district of residence of each household (as provided by NSS data): literacy, infant mortality and violent crimes. We interpret each of these three attributes as a local public good. For instance, the district literacy rate can be interpreted as the probability that an individual living in the district encounters someone who is literate. This is obviously a plausible indicator of the "quality" of the (district) environment in which the individual lives. Similarly the infant mortality rate that prevails in a district can be interpreted as the probability that a decision to have a child results in the decease of the child before the age of five. This probability is meant to be a *gross output* of the health system of the district, output which depends upon both the information available to prevent infant mortality (by having regular medical examination during and after the pregnancy for instance) and the quality of hospitals and doctors. Finally, the fraction of the district population that has been the victim of a violent crime is obviously an indicator of the "public safety" that prevails in the district and is a clear contributor to individual well-being.

The main conclusion of the analysis is that the joint distribution of district literacy, district infant mortality and individual consumption in India in 2002 dominates at the second order that of 1996 and 1988 and that the distribution of 1996 dominates that of 1988 at the first order. Hence, in a rather robust sense, there has been a steady improvement of social welfare in India on the period 1988-2002. This is a *strong* dominance result since it is based, at least for 1988-1996, on a first order and three-dimensional argument. In a nutshell, all anonymous and Paretian welfarist social planners who assume that individuals convert district infant mortality, district literacy and individual consumption into well-being by the same utility function satisfying very mild properties, recalled below, would agree to say that India has been steadily improving over the considered period. In the case of individual consumption and district literacy, where the data enables a distinction between the rural and the urban part of India, the dominance conclusion is shown to be true at the overall Indian level as well as in the urban and rural sublevel. The only attribute whose introduction breaks, sometimes, the dominance verdict is crime, whose average level has been increasing between 1988 and1996, before starting a descent from 1996 to 2002. Yet, if one limits the analysis to the 1996 to 2002 period, one obtains the result that the joint distribution of all four attributes in 2002 dominates that of 1996 at the second order. While somewhat less strong than the previous ones, this dominance also contributes to make one relatively optimistic with respect to the appraisal of the recent growth episode on the Indian distribution of well-being.

The plan of the rest of the paper is as follows. In the next section, we present the theoretical criteria used to perform the comparisons. Section 3 discusses the data, the statistical methodology and the results of the comparisons and section 4 provides some conclusion.

2. Presentation of the criteria

2.1. One-dimensional setting

We recall first the well-known criteria used to compare distributions of one *cardinally measurable* attribute (income) across n households.[2] The assumption of cardinal measurability of the attribute is important for the interpretation of the methodology adopted here (see e.g. Allison & Foster (2004) for one-dimensional comparisons of distributions of health indicators not assumed to be cardinally measurable).

Let x and $y \in \mathbb{R}_+^n$[3] be two distributions of the attribute between the n households indexed by i ($i = 1, ..., n$). We interpret x_i and y_i as the amount of the attribute received by household i in distributions x and y respectively. For every vector z in $\mathbb{R}^n$, we denote by $z_{(.)} = (z_{(1)}, ..., z_{(n)})$ the ordered permutation of z such that, for all $i = 1, ..., n-1$, $z_{(i)} \leq z_{(i+1)}$.

Much of the comparisons performed in this section are based on the symmetric utilitarian criterion. Let $U : \mathbb{R}_+ \rightarrow \mathbb{R}$ be a particular utility function that transforms the attribute into individual well-being. For the utility function U, the symmetric utilitarian criterion ranks x above y if and only if $\sum_{i=1}^{n} U(x_i) \geq \sum_{i=1}^{n} U(y_i)$. As shown in D'Aspremont & Gevers (1977) (see also Denicolò (1999) for a recent and concise proof of this), utilitarianism is the only Pareto inclusive and anonymous way to aggregate individual utilities into a social ranking when individual utility is assumed to be cardinally measurable and when utility differences are assumed to be inter-

[2] We focus the discussion on the case where the number of households is the same. As is well-known, cases where the number of individuals differ between distributions can be transformed into cases with the same number of individuals after appropriate replications of the distributions.

[3] The assumption for the attribute to be measured by a non-negative number is not essential.

personally comparable.[4] The symmetric requirement that individuals use the same function to convert the attribute into well-being seems genuine to the unidimensional nature of the analysis. If two individuals were different in their ability to convert attributes into well-being, this difference should be accounted for and included in the analysis, which would then become multidimensional.

Obviously, the assumption that the social planner has all the required information to measure utility cardinally and to perform interpersonal comparisons of utility differences that justifies the use of utilitarianism is a strong one. A more acceptable assumption, which lies at the heart of the dominance approach, is to assume that the social planner is willing to measure utility cardinally and to perform interpersonal comparisons of utility differences, but does not know which exact function to use. It only knows that the function satisfies some basic properties and, being careful, it only accepts to make a definite ranking of two distributions when the symmetric utilitarian criterion ranks them in the same fashion for all the utility functions satisfying the properties.

We define formally as follows the concept of utilitarian dominance for a class $\mathbb{U}$ of utility functions.

Definition 1. ***(Utilitarian dominance)****. We say that x utilitarian dominates y for the class of functions $\mathbb{U}$, denoted $x \succeq_{\mathbb{U}} y$, if and only if $\sum_{i=1}^{n} U(x_i) \geq \sum_{i=1}^{n} U(y_i)$ for all utility functions U in $\mathbb{U}$.*

The following two classes of utility functions are typically considered in unidimensional analysis:

$\mathbb{U}^{U1} = \{U : \mathbb{R}_+ \to \mathbb{R}$ such that U is increasing$\}$ and

$\mathbb{U}^{U2} := \{U : \mathbb{R}_+ \to \mathbb{R}$ such that U is increasing and concave$\}$.[5]

[4] Utilitarianism provides one theoretical justification for the dominance criteria considered herein. As shown in Gravel & Moyes (2006), it is also possible to justify the criteria by appealing more generally to anonymous and utility-inequality averse welfarist ethics.

[5] A function $g : A \to \mathbb{R}$ where $A \subseteq \mathbb{R}^k$ for $k = 1, 2, ..$ is increasing if $a \geq b \Rightarrow g(a) \geq g(b)$ and is concave if $g(\lambda a + (1-\lambda)b) \geq \lambda g(a) + (1-\lambda)g(b)$ for every a and b in A and any $\lambda \in [0, 1]$. Clearly the property of concavity of the utility function requires the arguments of the utility function to be cardinally measurable (as the property of concavity is not preserved if an arbitrary monotonic transformation is applied to any one of the function's argument).

A well-known accomplishment of one-dimensional normative evaluation theory has been to provide easy-to-implement statistical criteria that are equivalent to alternative notions of utilitarian dominance. For the sake of completeness, we recall what are these criteria and equivalences.

Definition 2. *(**Quantile dominance**). We say that x quantile dominates y, denoted $x \succeq_Q y$, if $x_{(i)} \geq y_{(i)}$ for all i.*

In words, x quantile dominates y if the ith poorest individual in x has at least as much of the attribute as the ith poorest individual in y. Clearly, this kind of dominance can never be observed between two distributions of the same total quantity of the attribute that have distinct ordered vectors.

Definition 3. *(**Headcount Poverty dominance**). We say that x headcount poverty dominates y, denoted $x \succeq_{Hp} y$, if $\#\{i : x_i \leq t\} \leq \#\{i : y_i \leq t\}$ for every possible poverty threshold $t \in \mathbb{R}_+$.*

In words, x headcount poverty dominates y if the *number* of individuals whose income are below some poverty line is lower in x than in y *no matter what is the poverty line*. This criterion is nothing else than a discrete version of the first order stochastic dominance one (see e.g. Hadar & Russell (1974) for a classical statement).

Definition 4. *(**Generalized Lorenz dominance**). We say that x generalized Lorenz dominates y, denoted $x \succeq_{GL} y$ if, for every $k = 1, ..., n$, it is the case that $\sum_{i=1}^{k} x_{(i)} \geq \sum_{i=1}^{k} x_{(i)}$.*

In words, x generalized Lorenz dominates y if the total quantity of the attribute possessed by the k poorest individuals in x is at least as large as the corresponding quantity possessed by k poorest individuals in y. The numbers $\sum_{i=1}^{k} x_{(i)}$ and $\sum_{i=1}^{k} y_{(i)}$ are the values of the ordinates of the generalized Lorenz curves (see Shorrocks (1983)) for x and y, respectively, that correspond to the abscissa k.

Definition 5. *(**Poverty gap dominance**). We say that x poverty gap dominates y, denoted $x \succeq_{PG} y$ if, for every poverty threshold t, it is the case that $\sum_{i=1}^{n} \max(t - x_i, 0) \leq \sum_{i=1}^{n} \max(t - y_i, 0)$.*

In words, x poverty gap dominates y if, no matter what is the poverty threshold, a lower quantity of the attribute is needed in x than in y to eliminate totally the poverty defined by the threshold. This criterion is a discrete version of 2nd-order stochastic dominance.

In the following proposition, we summarize the well-known equivalences that exist between these criteria and utilitarian dominance.

Proposition 1. *For every two distributions* x *and* $y \in \mathbb{R}^n_+$, $x \succeq_{\mathbb{U}^{U1}} y \Leftrightarrow x \succeq_Q y \Leftrightarrow x \succeq_{Hp} y$ *and* $x \succeq_{\mathbb{U}^{U2}} y \Leftrightarrow x \succeq_{GL} y \Leftrightarrow x \succeq_{PG} y$.

2.2. Multidimensional setting

While the analysis is conducted herein with four attributes (individual consumption, district infant mortality, district literacy and district crime), most of the theoretical results on multidimensional dominance have been derived for two attributes only. This is especially true of the results that identify the *elementary transformations* that correspond to the various criteria. These elementary transformations are well-known in the case of one-dimensional analysis. They are increments for 1st order (quantile) dominance and a combination of increments and Pigou-Dalton transfers for 2nd order (poverty gap) dominance. In the case of two-attributes distributions, the elementary transformations that correspond to the criteria presented in this paper are not as well-known and, to the best of our knowledge, have only been derived in Gravel & Moyes (2006) for the first order dominance criterion.[6] As far as we are aware, the elementary transformations that correspond to criteria that enable the ranking of distribution of more than two attributes have not been identified.

In this section we state the criteria used to perform arbitrary k-dimensional comparisons as well as their equivalence with utilitarian dominance notions.

Assume therefore that there are k attributes. A distribution z of the k attributes is described as a $k \times n$ matrix of non-negative numbers[7] which

[6] In Gravel & Moyes (2006), one finds also another criterion that happens to lie, in terms of discriminatory power, between first and second order dominance. This criterion, only defined in the case of two attributes, is not used herein.

[7] We maintain the assumption that the quantities of each attribute is non-negative even though it is not essential.

we write as:

$$z = \begin{bmatrix} z_{11} & z_{21} & \cdots & z_{n1} \\ z_{12} & z_{22} & \cdots & z_{n2} \\ \cdots & \cdots & \cdots & \cdots \\ z_{1k} & z_{2k} & \cdots & z_{nk} \end{bmatrix}$$

where, for every $i = 1, ..., n$ and $j = 1, ..., k$, z_{ij} represents the amount of attribute j received by individual i in the distribution z. We let z_i denote the vector of attributes received by individual i and z_j denote the distribution of attribute j in the population

Using, for the reasons mentioned in the previous section, utilitarian dominance as the basic normative criterion for comparing alternative distributions of the two attributes, our task is to propose plausible properties that individual utility could satisfy when it is assumed to be a function of four attributes. To define these properties, it is convenient, but not necessary, to assume that the utility function is differentiable with respect to its 2 argument to the required degree. For every function H of k variables ($k \geq 2$), we denote by $H_j(z)$ its jth partial derivative evaluated at the $k-$dimensional vector z. With this notation, the class of utility functions that are considered are the following.

$$\mathbb{U}^{M1} = \{U : \mathbb{R}^k_+ \to \mathbb{R}\text{: } (-1)^{\#H} U_{h_1 h_2 \ldots h_{\#H}}(z) \leq 0 \ \forall z \in \mathbb{R}^k_+ \text{ and } H \subseteq \{1, 2, ..., k\} \text{ with } H = \{h_1, ..., h_{\#H}\}\}.$$

$$\mathbb{U}^{M2} = \mathbb{U}^{M1} \cup \{U : \mathbb{R}^k_+ \to \mathbb{R} : (-1)^{\#H \cup J} U_{h_1 h_2 \ldots k_{\#K} j_1 j_2 \ldots j_{\#J}}(z) \leq 0 \ \forall z \in \mathbb{R}^k_+ \text{ and all subsets } H = \{h_1, ..., h_{\#H}\} \text{ and } J = \{j_1, ..., j_{\#J}\} \text{ of } \{1, 2, ..., k\}\}$$

As their one-dimensional counterpart in $\mathbb{U}^{U1}$, functions in $\mathbb{U}^{M1}$ have the property of being *increasing* with respect to every attribute. This property emerges from the formal definition of $\mathbb{U}^{M1}$ by taking $H = \{j\}$ for every $j \in \{1, ..., k\}$. Yet, in addition to this one-dimensional property, functions in $\mathbb{U}^{M1}$ satisfy other conditions that specify the way by which the marginal utility of every attribute varies with the level of the other attributes. These conditions reflect assumptions made on the substitutability/complementarity between any two attributes, and the way by which this pairwise substitutability/complementarity varies with the level of the other attributes, and the way by which this cross-attribute variation of the substitutability/complementarity between attributes vary with other attribute, and so on, until one exhausts the list of attributes. Specifically, we are assuming here that any two attributes are substitute to each other and therefore, that the marginal utility of one attribute is decreasing with respect to any other attribute (condition $U_{ij}(z) \leq 0$, obtained from the formal definition of $\mathbb{U}^{M1}$ by considering $H = \{i, j\}$ for every i, $j \in \{1, ..., k\}$).We are also assuming that the *decrease* in marginal utility of an attribute with respect to another is *itself* decreasing with respect to any other attribute (condition $U_{hij}(z) \geq 0$) and that this decrease in the decrease of the marginal utility of one attribute is also decreasing with respect to the other remaining attribute, and so on. Unless, one assumes additive separability of the individual utility function between the attributes, it is important that one specifies minimally the *connections* that exists between these attributes. In the class $\mathbb{U}^{M1}$, we connect, in the fashion described in the set $\mathbb{U}^{M_1}$, all *first order* own derivatives

In addition to imposing properties on cross-dimensional behavior of the first own derivatives, the class $\mathbb{U}^{M2}$ impose analogous properties on the cross-dimensional behavior of the *second order* own derivatives, assumed to be negative just like their standard one-dimensional counterpart. The properties on the cross-dimensional behavior of the second own derivatives are obviously more difficult to understand intuitively. They roughly say that the decrease in the marginal utility of each attribute should be decreasing with respect to another attribute, and that this decrease should be also decreasing with respect to another attribute, and so on. All in all, functions in $\mathbb{U}^{M2}$ satisfy the properties that the impact of anything that happens in one or several dimensions should be decreasing with respect to the other dimensions. As for the class $\mathbb{U}^{M_1}$, the sign of the derivative are alternating with the number of terms involved (negative when there is an even number of terms, positive when the number of terms is odd).

Atkinson & Bourguignon (1982) have proposed, in the case of two attributes, two operational criteria that, as it turns out, are equivalent to the rankings provided by all utilitarian planners who assume that individual utility functions are in $\mathbb{U}^{M1}$ and $\mathbb{U}^{M2}$ respectively. The definitions of these criteria for the k dimensional case are as follows.

Definition 6. ***(multidimensional headcount poverty dominance)*** *Distribution x dominates distribution y for the multidimensional headcount poverty criterion, denoted $x \succeq_{MHp} y$ if, for every list of k poverty lines $(t_1, t_2, ...t_k)$, $\#\{i : (x_{i1}, x_{i2}, ..., x_{ik}) \leq (t_1, t_2, ...t_k)\} \leq \#\{i : (y_{i1}, y_{i2}, ..., y_{ik}) \leq (t_1, t_2, ...t_k)\}$.*

In words, x headcount poverty dominates, in a multidimensional sense, y if, for every list of poverty lines (one such line for every attribute), the number of individuals who are poor with respect to all attributes is lower in x than in y. This criterion is a straightforward generalization of the one-dimensional poverty headcount dominance one where people can be poor with respect to both attributes. It can be noted that if x headcount poverty dominates y in a multidimensional sense, then x headcount poverty dominates y in the one-dimensional sense for every attribute in isolation but that the converse does not hold.

The second criterion, first introduced by Atkinson & Bourguignon (1982) in the case of two-attributes distributions, can be viewed as a generalization of the one-dimensional poverty gap dominance criterion presented above.

Definition 7. (***multidimensional poverty gap dominance***) *Distribution x dominates y for the multidimensional poverty gap criterion, denoted $x \succeq_{MPG} y$, if, for all vector of poverty lines $(t_1, t_2, ..., t_k)$, and all non-empty subsets K of $\{1, ..., k\}$, one has:*

$$\sum_{i=1}^{n} \prod_{j \in K} \max(t_j - x_{ij}, 0) \leq \sum_{i=1}^{n} \prod_{j \in K} \max(t_j - y_{ij}, 0) \tag{2.1}$$

In words, x poverty gap dominates y in the multidimensional sense if, for all lists of poverty lines (one such list for every attribute), and all (non-empty) combinations of attributes, the *product* of the amounts of the attributes that would be necessary to eliminate the poverty defined by the lines is lower in x than in y. Notice carefully that the implementation of the multidimensional poverty gap criterion requires, because of the need to consider $K = \{j\}$ for every j, the usual one dimensional poverty gap criterion to hold on every dimension.

We now provide in the following two propositions, statements and proofs of these equivalences between each of the two operational criteria and their utilitarian dominance counterpart since, to the very best of our knowledge, these are not available in the literature for the general k-dimensional case. Atkinson & Bourguignon (1982) have provided, in the two dimensional case, a proof of one direction of both equivalence and Hadar & Russell (1974) have provided, for the general case, a proof of one direction of the first equivalence.

Proposition 2. *For every two distributions of two attributes x and y,*

$x \succeq_{\mathbb{U}^{M1}} y \Leftrightarrow x \succeq_{Hp2} y.$

Proof. *Necessity.* Assume $x \succeq_{\mathbb{U}^{M1}} y$. Then, we have

$$\sum_{i=1}^{n} U(x_i) \geq \sum_{i=1}^{n} U(y_i) \tag{2.2}$$

for all U in $\mathbb{U}^{M1}$. Consider the family of functions $\Phi^t : \mathbb{R}^k \to \mathbb{R}$, defined, for every $a \in \mathbb{R}_+^k$, by:

$$\begin{aligned} \Phi^t(a) &= 1 \text{ if } a_j > t_j \text{ for some } j \\ &= 0 \text{ otherwise.} \end{aligned}$$

and indexed by t, where $t = (t_1, ..., t_k) \in \mathbb{R}_+^k$ is a given vector of poverty lines. While non-differentiable (and non-continuous), it can be checked that Φ^t belongs to $\mathbb{U}^{M1}$ for every vector t. Indeed Φ^t is mildly increasing in each of the argument (the only increase that can arise from an increase in one of its argument is a discontinuous jump from 0 to 1). Moreover the increase of the function in one argument may not happen if the value of another argument is increased up to above the poverty threshold. Hence the condition on the cross partial derivative is also satisfied. Similarly, it can be checked that all other conditions on the partial derivatives (interpreted as discrete rate of variations) are satisfied. Since Φ^t belongs to $\mathbb{U}^{M1}$, and $x \succeq_{\mathbb{U}^{M1}} y$, we must have, for all vector t of poverty lines:

$$\sum_{i=1}^{n} \Phi^t(x_i) \geq \sum_{i=1}^{n} \Phi^t(y_i)$$

$$\Leftrightarrow$$

$$\#\{i : x_{ij} > t_j \text{ for some } j\} \geq \#\{i : y_{ij} > t_j \text{ for some } j\}$$

$$\Leftrightarrow$$

$$n - \#\{i : x_{ij} > t_j \text{ for some } j\} \leq n - \#\{i : y_{ij} > t_j \text{ for some } j\}$$

$$\Leftrightarrow$$

$$\#\{i : x_{ij} \leq t_j \text{ for all } j\} \leq \#\{i : y_{ij} \leq t_j \text{ for all } j\}$$

which is the definition of multidimensional headcount poverty dominance. *Sufficiency*: For any vector $a \in \mathbb{R}_{+}^{k}$, define the (discrete) densities:

$$\begin{aligned} f^{x}(a) &= \frac{\#\{i : x_i = a\}}{n} \text{ and} \\ f^{y}(a) &= \frac{\#\{i : y_i = a\}}{n} \end{aligned}$$

With this notation, the condition (2.2) can be written as:

$$\int_{0}^{\overline{z}_1} \ldots \int_{0}^{\overline{z}_k} [f^{x}(z) - f^{y}(z)]U(z)d_z \geq 0 \tag{2.3}$$

for some appropriate definition of integration (which could be the Lebesgue one or, if one wants to stick to the discrete setting, the Abel identity formula (see e.g. (Fishburn & Vickson (1978); eq 2.49)) and where $\overline{z}_j$ for $j = 1, ..., k$ is an upper bound for the attribute j in the two distributions. The proof of the sufficiency of multidimensional headcount poverty dominance for utilitarian dominance over the class $\mathbb{U}^{M1}$ can then be obtained by integrating by parts expression (2.3). The result of this integration by part are provided in equation (5.5') in Hadar & Russell (1974) and the statement of the sufficiency of the condition is the content of theorem 5.8 in this paper. ■

Proposition 3. *For every two distributions of two attributes x and y, $x \succeq_{\mathbb{U}^{M3}} y \Leftrightarrow x \succeq_{BPG} y$.*

Proof. Necessity. *Assume $x \succeq_{\mathbb{U}^{M2}} y$ and, therefore, that*

$$\sum_{i=1}^{n} U(x_i) \geq \sum_{i=1}^{n} U(y_i)$$

holds for all U in $\mathbb{U}^{M2}$. Consider the family of functions $\Phi^{tK} : \mathbb{R}^k \to \mathbb{R}$, defined, for every $a \in \mathbb{R}_{+}^{k}$, by:

$$\Phi^{tK}(a) = -\prod_{j \in K} (-\min(a_j - t_j, 0)).$$

and indexed by t and K where $t = (t_1, ..., t_k)$ is a given vector of poverty lines and K is a non-empty subset of the index set $\{1, ..., k\}$. It should *be noted that this function is a proper function of* $\mathbb{R}_+^k$ *even though, for several specifications of* K*, it does not depend at all upon the arguments that correspond to dimensions whose index lies outside* K*. A graphical representation of this function (for the case where* $K = \{1, 2\}$ *and* $t_1 = t_2 = 2$*) is provided on figure 1.*

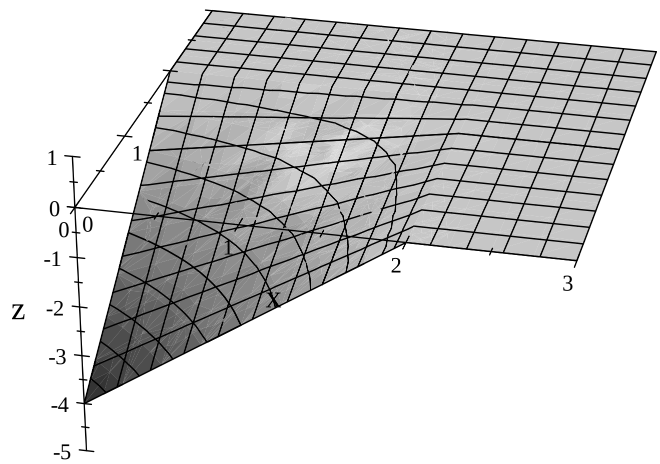

Figure 1

It can be seen that this function, for every $t \in \mathbb{R}^k$ *and* $K \subset \{1, ..., k\}$*, belongs to* $\mathbb{U}^{M2}$*. To see this, consider first the behavior of the function, when viewed this time as a function of the arguments indexed by* K*, in the interior of the set* $\times_{j\in K}[0, t_j]$ *(where the min operator does not apply). In any point* a *in this set, the function* Φ^{tK} *writes:*

$$\Phi^{tK}(a) = -(-1)^{\#K} \prod_{j\in K} (a_j - t_j)$$

On this set, one has:

$$\begin{aligned}\Phi_j^{tK}(a) &= -(-1)^{\#K} \prod_{k\in K, k\neq j} (a_k - t_k) \text{ if } j \in K \\ &= 0 \text{ otherwise}\end{aligned}$$

Since $a_j - t_j < 0$ *is in the interior of the set* $\times_{j\in K}[0, t_j]$, *the sign of* $\prod_{k\in K, k\neq j} (a_k - t_k)$ *is negative if* $\#K$ *is even and positive if* $\#K$ *is odd. Hence* $\Phi_j^{tK}(a) \geq 0$. *Similar arguments can establish that all the derivative properties of the functions in* $\mathbb{U}^{M2}$ *are satisfied. The argument can be adapted, with some care, to the case where the* min *operator enters into the picture. Since* Φ^{tK} *belongs to* $\mathbb{U}^{M2}$ *and* $x \succeq_{\mathbb{U}^{M2}} y$ *holds, we have:*

$$\sum_{i=1}^{n} \Phi^{tK}(x_i) \geq \sum_{i=1}^{n} \Phi^{tK}(y_i)$$

$$\Leftrightarrow$$

$$-\sum_{i=1}^{n} \prod_{j\in K} (-\min(x_{ij} - t_j, 0)) \geq -\sum_{i=1}^{n} \prod_{j\in K} (-\min(y_{ij} - t_j, 0))$$

$$\Leftrightarrow$$

$$\sum_{i=1}^{n} \prod_{j\in K} (-\min(x_{ij} - t_j, 0)) \leq \sum_{i=1}^{n} \prod_{j\in K} (-\min(y_{ij} - t_j, 0))$$

$$\Leftrightarrow$$

$$\sum_{i=1}^{n} \prod_{j\in K} (\max(t_j - x_{ij}, 0)) \leq \sum_{i=1}^{n} \prod_{j\in K} (\max(t_j - y_{ij}, 0))$$

which, applied to every t *and every* K, *is precisely the definition of the multidimensional poverty gap dominance of* y *by* x.
Sufficiency*: We provide the proof in the appendix.* ■

In view of these two propositions, as well as the definitions of $\mathbb{U}^{M1}$ and $\mathbb{U}^{M2}$, it is clear that $x \succeq_{Hp2} y$ implies $x \succeq_{BPG} y$ but that the reverse implication does not hold. Hence multidimensional poverty gap dominance is more discriminant than multidimensional headcount poverty dominance. As usual with dominance analysis, the increase in discriminatory power gained from switching from one criterion to the other must be balanced against the decreasing plausibility of the properties of the individual utility function assumed in the corresponding utilitarian dominance criterion. The class $\mathbb{U}^{M2}$ may seem particularly exhausting in this respect, especially if many attributes (such as literacy, infant mortality or crime) are not cardinally measurable and if, therefore, a second (or larger order) derivative taken with respect to them has no real meaning.

3. Empirical implementation

3.1. Data

In this paper, we compare over time joint distributions of individual consumption expenditure, district level literacy, under 5 mortality rates and violent crime rates. We interpret the latter three variables as local (district) public goods that affect all households living in the district, and that contribute to individuals' well-being.

Household consumption figures are obtained from the 43rd (1987-1988), 52nd (1995-1996) and 58th rounds (2002) of consumption expenditure surveys conducted by National Sample Survey Organization (NSSO)[8]. Individual consumption expenditures have been derived from household consumption expenditures using the Oxford equivalence scale and are in 2002 Rupees[9]. Consumption data have also been made comparable, to the extent possible, in terms of the reference period over which consumption expenditures are recollected by surveyed households. As is well-known (see e.g. Deaton & Drèze (2005) or Himanshu & Sen (2005)) there has been some time inconsistency as to the recall period used in the NSS questionnaires to determine the spending on various group of commodities, especially the durables, clothing and footwear. In 1987-88 data on these goods have been collected using both a 30 days recall period and a 365 days recall period while only a 365 days recall period was used for 2002 data. In order to make data on these two periods comparable, we have used calculations based on the 365 recall period. However, for 1995-96, half the sample is at 365 days

[8] While the 43rd Round is a "thick" round of data collection, the 52nd and 58th are "thin" samples. The choice of the latter two rounds is dictated by the lack of district identifiers in the closest thick round of data collection in 1993-94 and the unavailability of a publicly available "thick" sample data set post 2000s respectively.

[9] Price deflators are the Urban Non Manual Employees price index for urban data and Agricultural Labourers price index for rural ones. Comparisons or pooling between urban and rural data are performed using Deaton (2005) (table 17;3) ideal Fisher index.

recall period, while the other half is at 30 days period. Because of the lack of information, we could not correct this subsample for this. At the all India level[10], our analysis is based on 131,511 individuals in 2002, 203,228 individuals in 1995-96 and 563,931 individuals in 1987-88.

As the district of residence of each individual is provided in NSS data for each period, we have assigned to each individual the literacy, under 5 mortality rate and violent crime rate of the district the individual resides in. Due to subdivisions in the district areas that have taken place in India over the 1981-2001 period, there are more districts in 2001 and 1991 than in 1981. In order to make the comparisons consistent, we have aggregated data for 1991 and 2001 to adhere to the original, and coarser,1981 districts partition.

District literacy rates (fraction of the district population above 7 years old which is literate) have been obtained from the Census, for the census years of 1981, 1991 and 2001. There is an unavoidable problem for the1981 figures because it has expressed literacy rates for the population above the age of 5. Since household level data is not available for the 1981 census, it is not possible to come up the proportion of population above 7 years old who are literate. Moreover, for the years 1991 and 2001, it is not possible either to express literacy as a fraction of the population above 5 years old. Hence the reader should keep in mind that a small part of the significant increase in literacy rates observed in data between 1981 and 1991 may be due to this change in the reference population. Data are available both for the district as a whole and for the rural and urban parts of the district separately.

Under 5 mortality rates (number of children who die before the age of five per thousand birth) data have been calculated, for the same census years, from the Census of India by the International Institute for Population Science. Data are only available at the whole district level and do not enable a distinction between urban and rural population of a district.

Violent crime rates (number of murders, attempted murders, and rapes per million individuals) have been obtained, for the same years as NSS data from the National Crime Record Bureau. We have restricted our attention to the most violent and extreme form of crime to reduce the risk of trend biases due to the evolution of the reporting behavior of the victims of crimes (or their families). It is indeed well-known that crime reporting tends to grow

[10]We have excluded the areas of Jammu Kashmir, as well as all North Eastearn States of India because of suspicion that data gathered in these troubled areas are good as in the rest of the country.

with education and wealth (wealthier and more educated people are more prone to report crime to the police than deprived or less educated ones). Our assumption is that this bias is less important in the case of violent crimes, who tend to be reported to the police no matter what is the wealth or education level of the family of the victim, than for robberies, burglaries, and other form of criminal acts. As for infant mortality, data on crime do not allow us to distinguish between the rural and the urban population of a district.

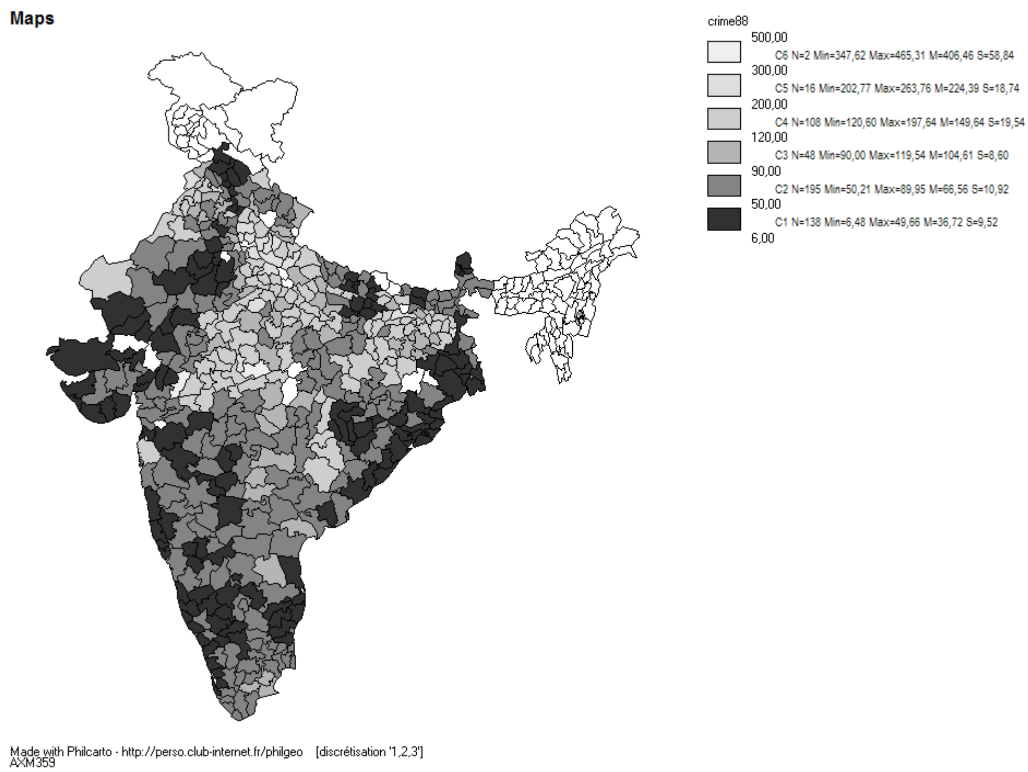

Figure 1a: Violent Crime Rates 1988

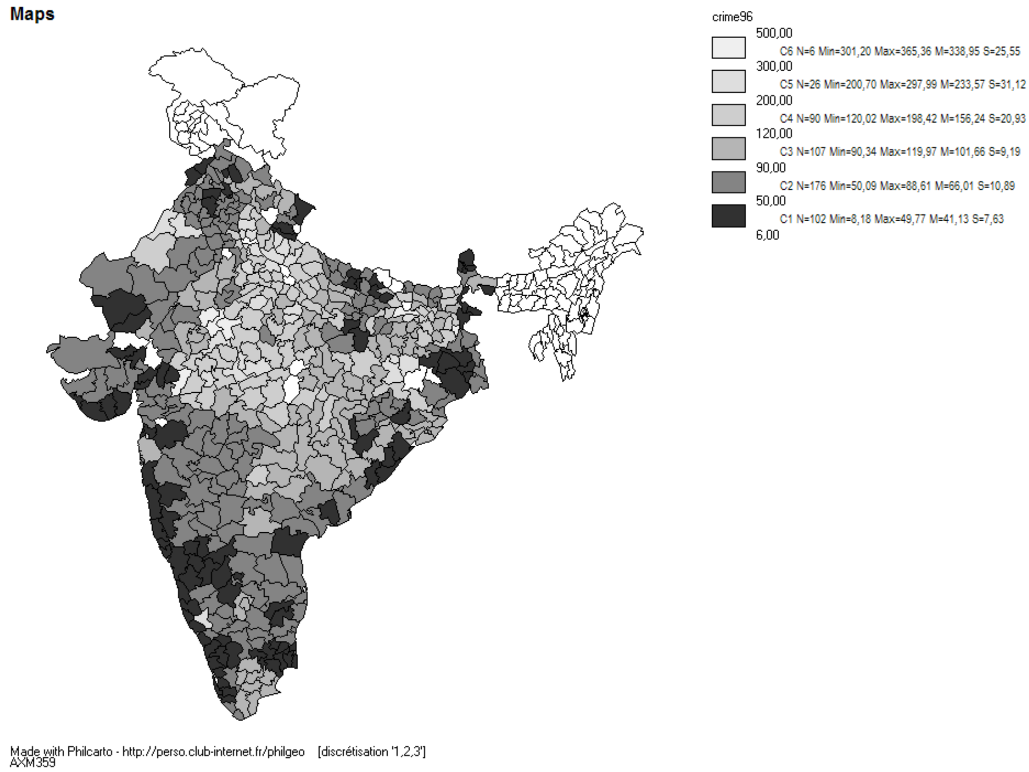

Figure 1b: Violent Crime Rates 1996

The overall district trends in violent crime, under 5 infant mortality and literacy are shown on the maps of figures 1-3 which, for each attribute, indicate a better outcome by a darker shade. As can be seen from the maps, and with the exception of crime rates, which have tend to go up between 1988 and 1996, before going down between 1996 and 2002, the overall trends are rather favorable.

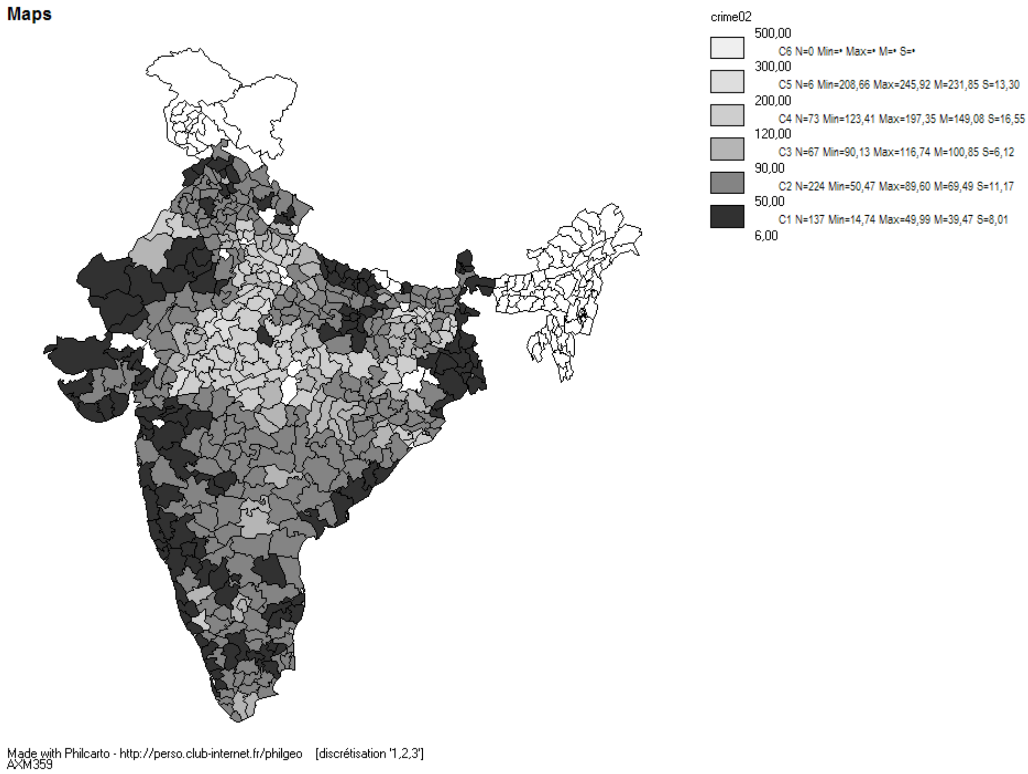

Figure 1c: Violent Crime Rates 2002

Beside overall favorable trends, one can observe that, in all cases, there has been a relative geographic stability in the performance of the various districts in securing local public goods to their citizens, with the south-west coastal districts showing better than average performance and northern interior districts (especially eastern Uttar Pradesh and Bihar) showing lower than average ones. The spatial disparities of Indian districts with respect to the access they give to local public goods that is revealed by this map is, of course, striking.

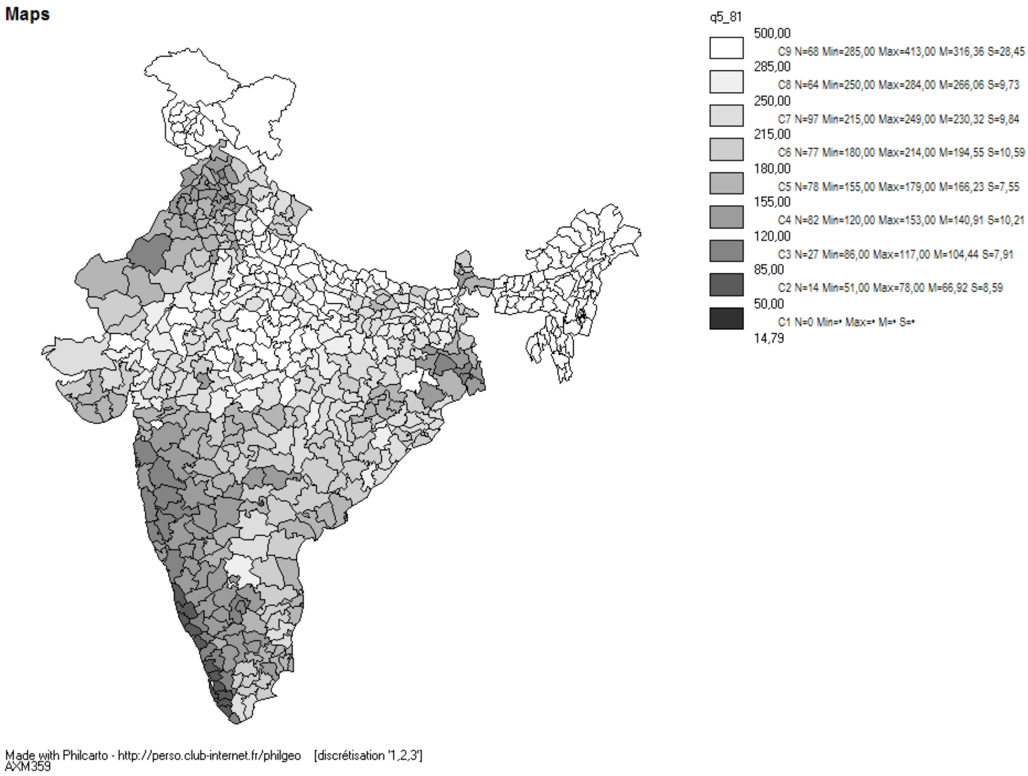

Figure 2a: Under 5 Mortality Rate, 1981.

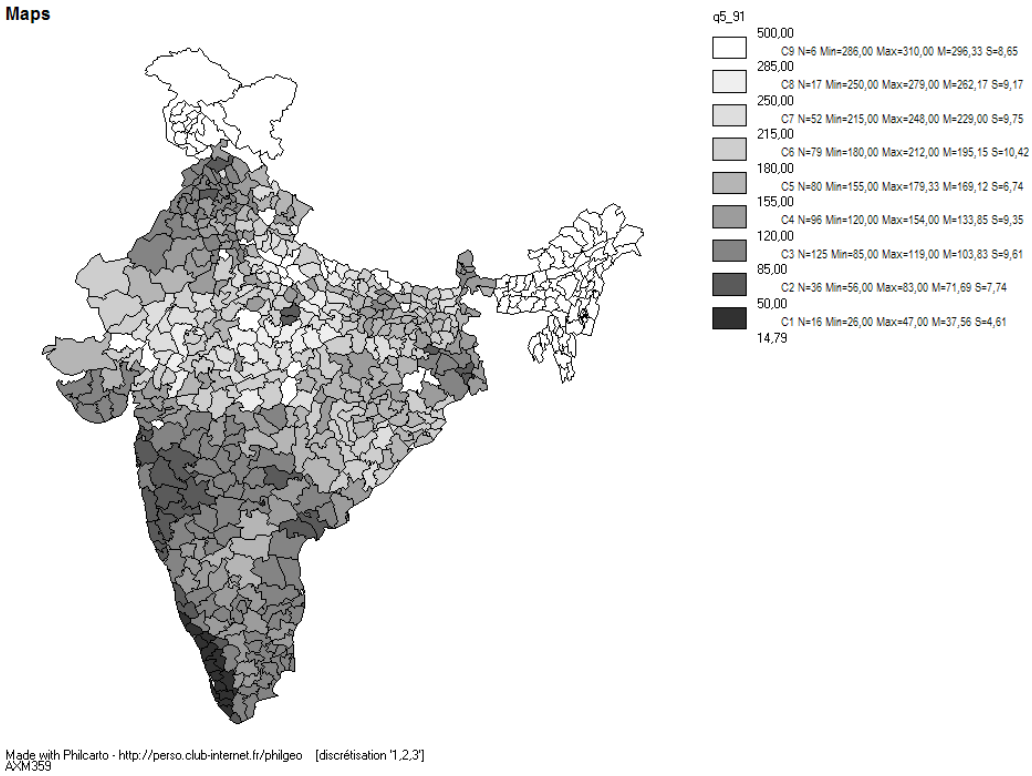

Figure 2b: Under 5 Mortality Rate: 1991

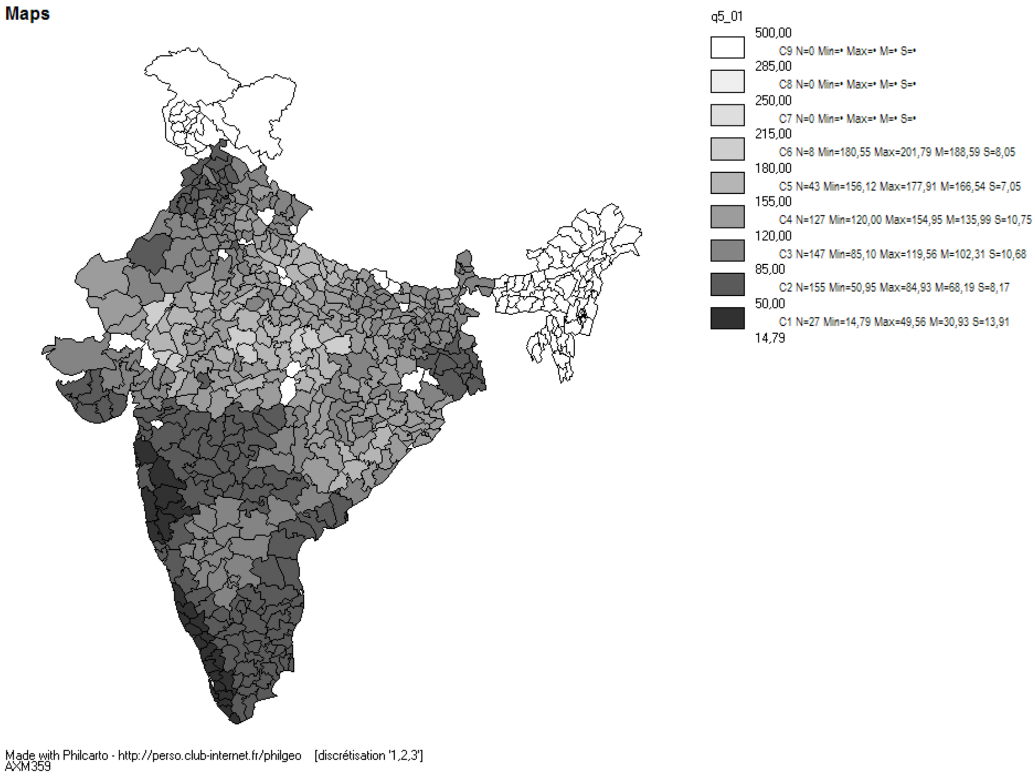

Figure 2c: Under 5 Mortality, 2001.

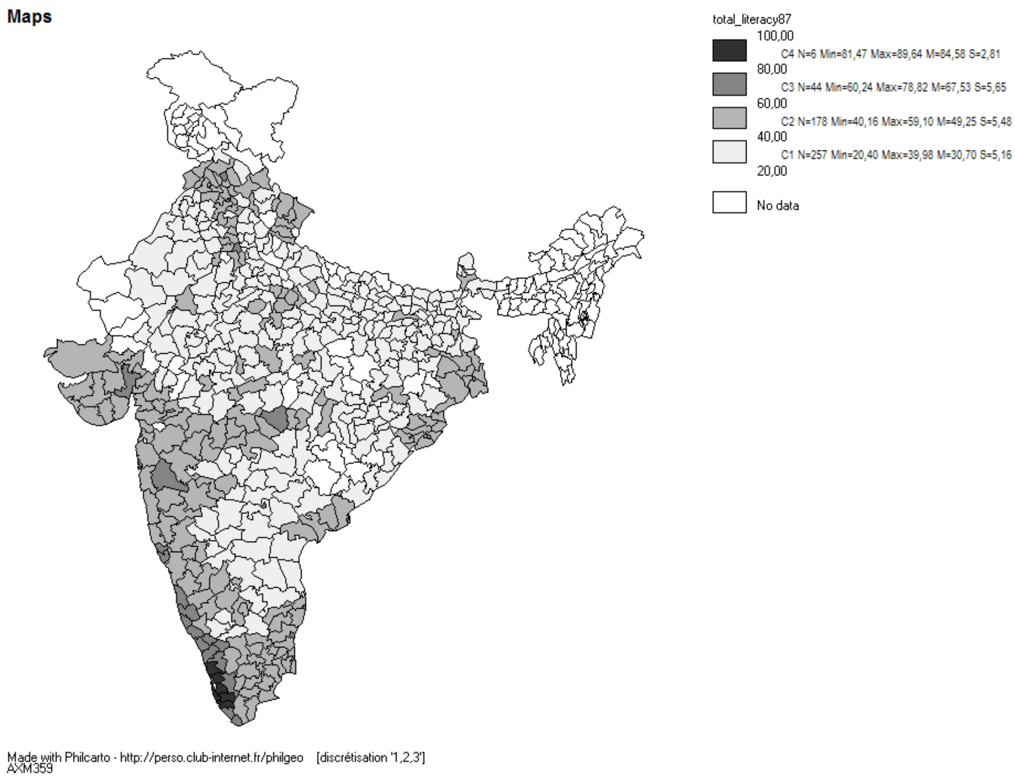

Figure 3a: District Literacy, 1981.

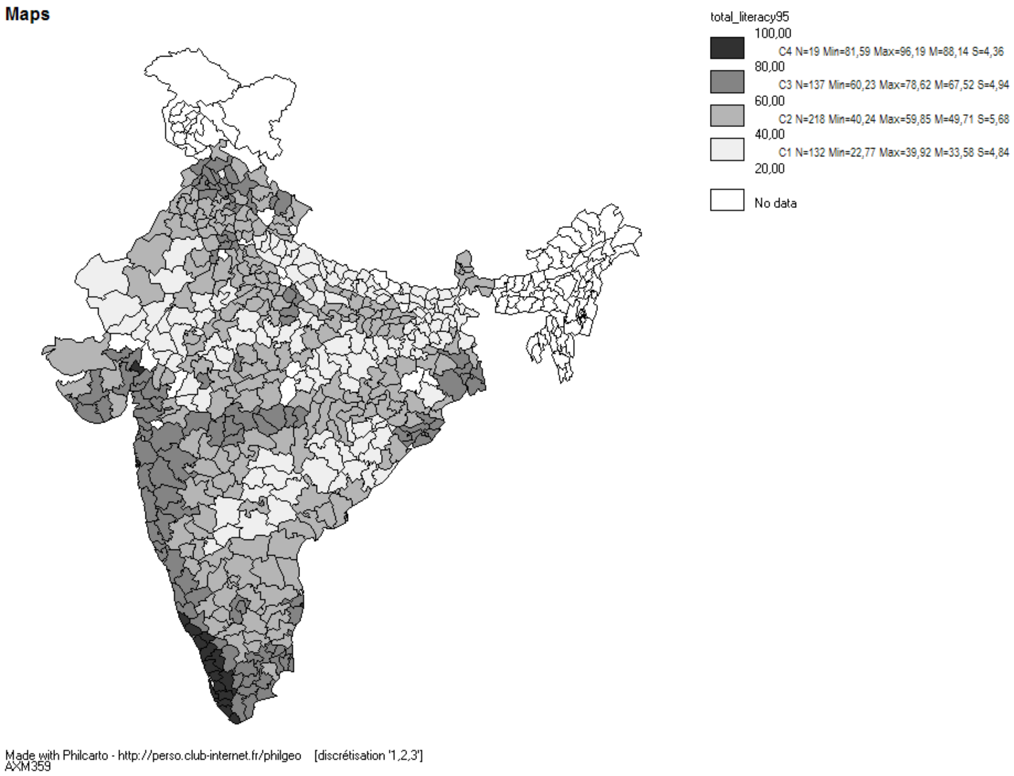

Figure 3b: District Literacy Rate, 1991.

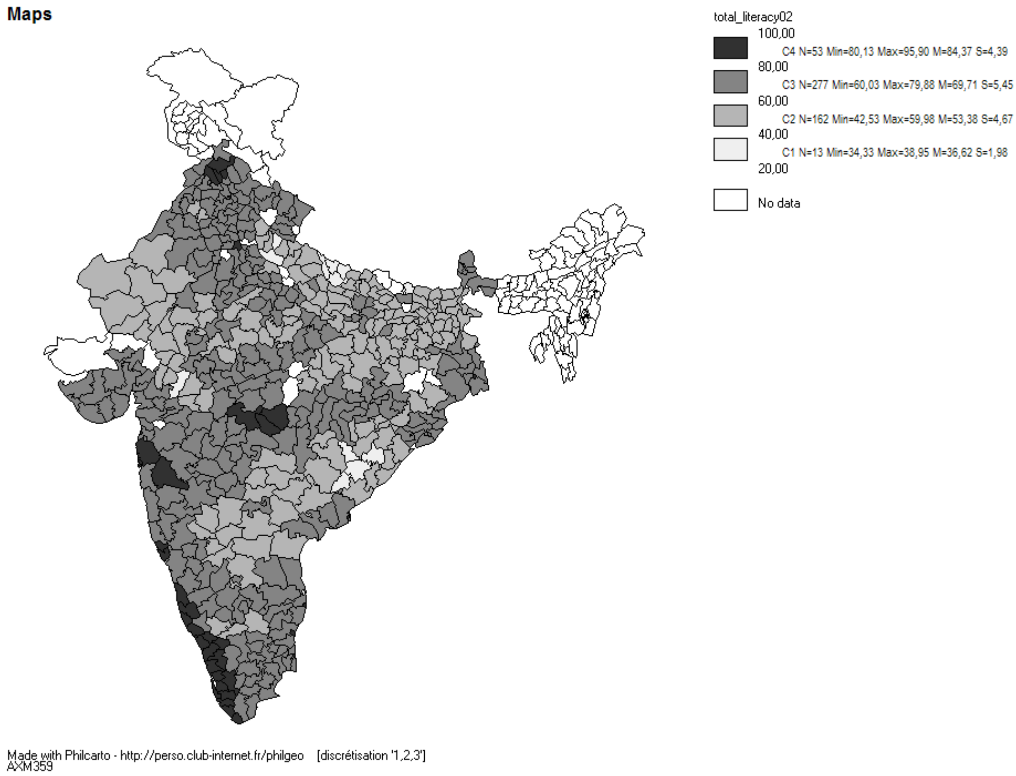

Figure 3c: District Literacy Rate, 2001.

3.2. Statistical methodology

In order to account for the fact that the compared distributions of disposable income are samples drawn from a larger population, we perform statistical inference based on the Union-Intersection (UI) method as initiated by Bishop & Formby (1999). The details of the methodology are provided in appendix B. All comparisons that are presented herein are performed at the 95 % confidence level.

4. One-dimensional comparisons

4.1. Distributions of consumption

Figures 4 and 6 compare the ordered vectors of 10 000 individual consumptions in rural and urban India respectively for the three periods. These 10 000 individual consumptions levels have been selected randomly from the underlying sample distributions.

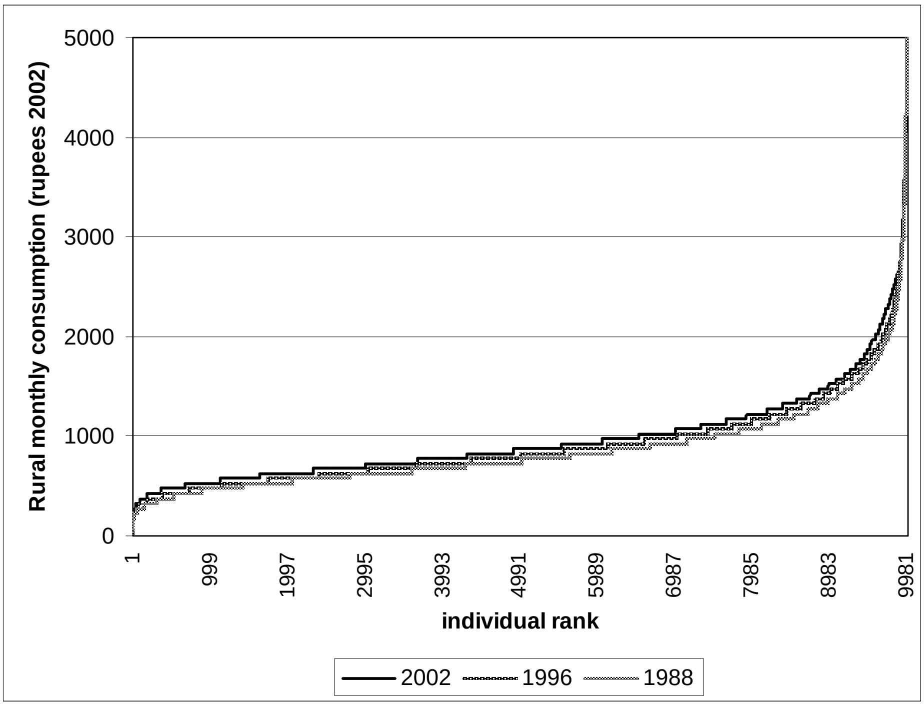

Figure 4: Ordered Vectors of Consumption in Rural India.

As revealed in figure 4, there has been an almost secular rise of rural expenditure over the years. While this is definitely true for individuals with low rankings in the distributions, it is, surprisingly, not true for higher ranked individual. As can be seen, there is some crossing in the right tail of the distributions. We suspect that this is a result of the thinness of the sample in 2002, and the under representation of high income households that is notorious in NSS data.[11] On conducting a *one-dimensional* UI test (the results of all test statistics and critical values are given in Appendix C) on whether 2001 dominates 1996 and 1988, we find that the crossing is "significant" in the sense that the distributions are not-comparable by the Headcount poverty dominance criterion. There are therefore poverty lines for which there are significantly more poor in 2001 than in 1988 and 1996 even though the converse conclusion holds for a vast majority of poverty lines. Indeed, except for implausibly high poverty lines, it appears that 2001 dominates 1996 and 1988 for the headcount poverty criterion. Moreover, as indicated below, the crossing that takes place on the right part of the distributions is not significant when one adopts a *multidimensional* perspective in which the distribution of consumption represents only one marginal distributions. In view of this, it can be said that the fierce debates on the choice of poverty line in India to appraise the impact of growth on pecuniary poverty is not that important in the case of India. No matter how one defines the line, the fraction of the Indian population that falls below it has gone down steadily over the period.

The problem with the right tail of the distribution obviously disappears when one moves at the second order and look at the generalized Lorenz curves, as can be done on figure 5. One-dimensional UI tests confirm what is suggested on the picture, namely that 2001 second order stochastically dominates both 1996 and 1988. Hence, if one uses poverty gap as a measure of poverty, there is no debate whatsoever to have on the appropriate poverty line in order to appraise the poverty trends in India. Poverty has gone down no matter what is the line used to define it.

[11] It is a well-documented fact (see e.g. Banerjee & Piketty (2005)) that the consumption expenditures measured by NSS tend to underestimate the consumption expenditures as defined in National Accounting data and, more importantly for our purpose here, that this downward bias has increased significantly during the nineties. The reason for this increasing underestimation, by NSS, of average consumption expenditure are not fully understood.

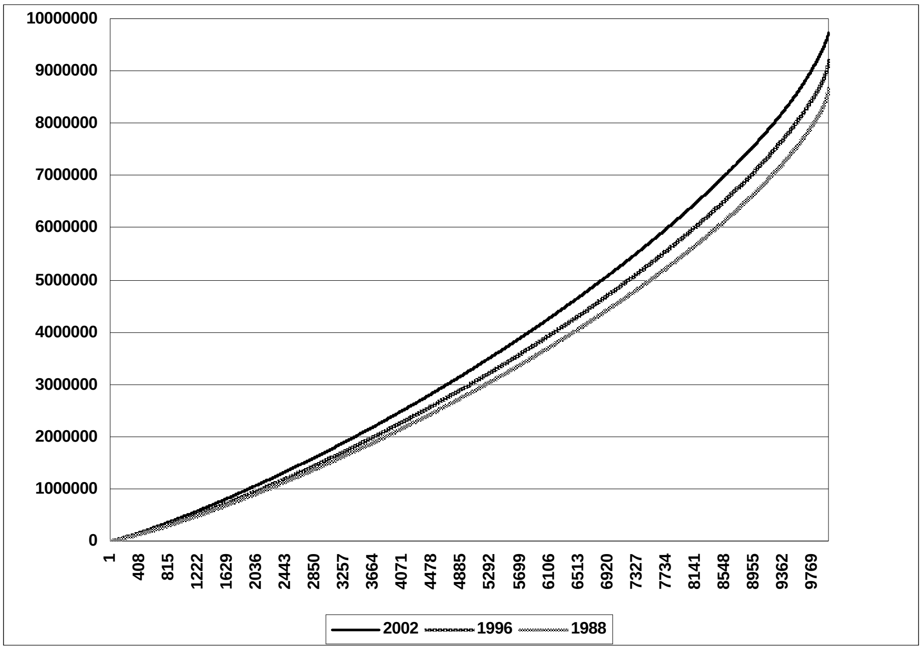

Figure 5: Generalized Lorenz Curves in Rural India.

Urban India shows broadly the same trends. However there seems to be greater improvements between 1988 and 1996 than between 1996 and 2002. This is consistent with the recent policy discussions that have taken place in India about the fact that the reduction of urban poverty has been slower in the recent years. Statistical tests reject dominance of 2002 over 1996 and 1988 because of crossing at the right tail of the distribution for the headcount poverty domination criterion, but accept the verdict of dominance of 2002 over both 1996 and 1988 for the poverty gap domination criterion.

Similar conclusions hold when we pool data at the all-India level. While 1996 dominates 1988 for headcount poverty, 2001 dominates 1996 and 1988 for the poverty gap, or generalized Lorenz, domination criterion.

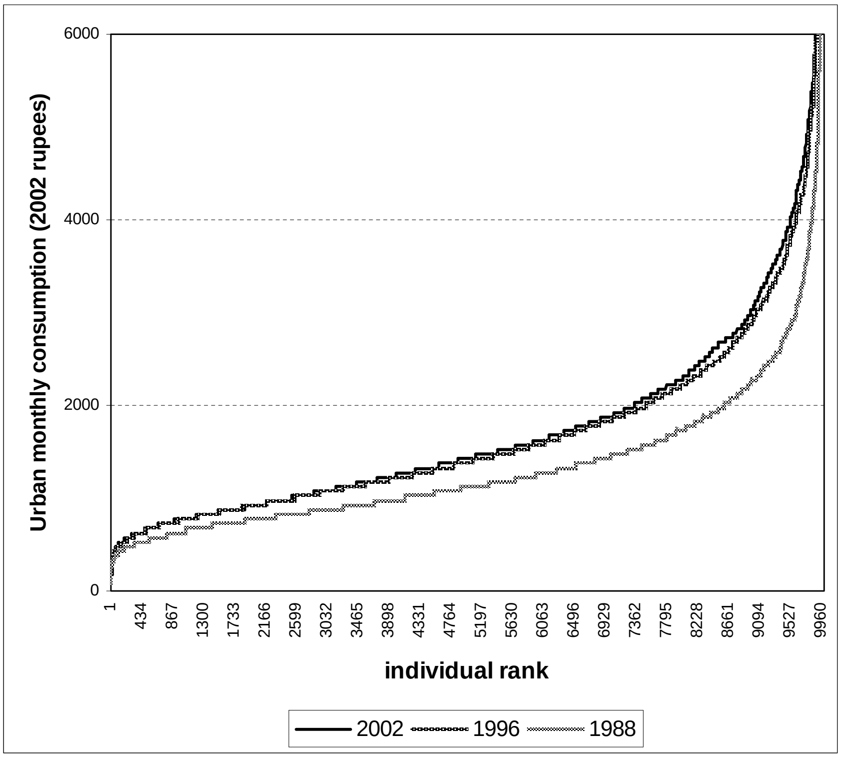

Figure 6: Ordered Vector of Consumption, Urban India.

A major theme of discussion in the normative appraisal of the post-reform Indian economy has been the perceived widening of the urban-rural gap. Is such a widening visible here? Figures 7a, b and c, which compare the ordered consumption vectors in rural and in urban India for the three years provide some answer to this question.

As can be seen, for the early 1988-1996, there seems to be a clear increase in the gap between real consumption of the jth poorest individual living in rural area and his or her equivalent in rural area, no matter what this j is. On the other hand, the gap seems to have remained stagnant and, perhaps, to have been slightly reduced in the subsequent 1996-2002 period. Clearly, this trend echoes the remarks made above with respect to the fact that, as compared with the situation in rural areas, urban poverty has been reduced more in the first 1988-1996 period than in the second 1996-2002 one. As is also clear from the picture, the increase in gap appears to be growing with the rank of the individual in the income distribution.

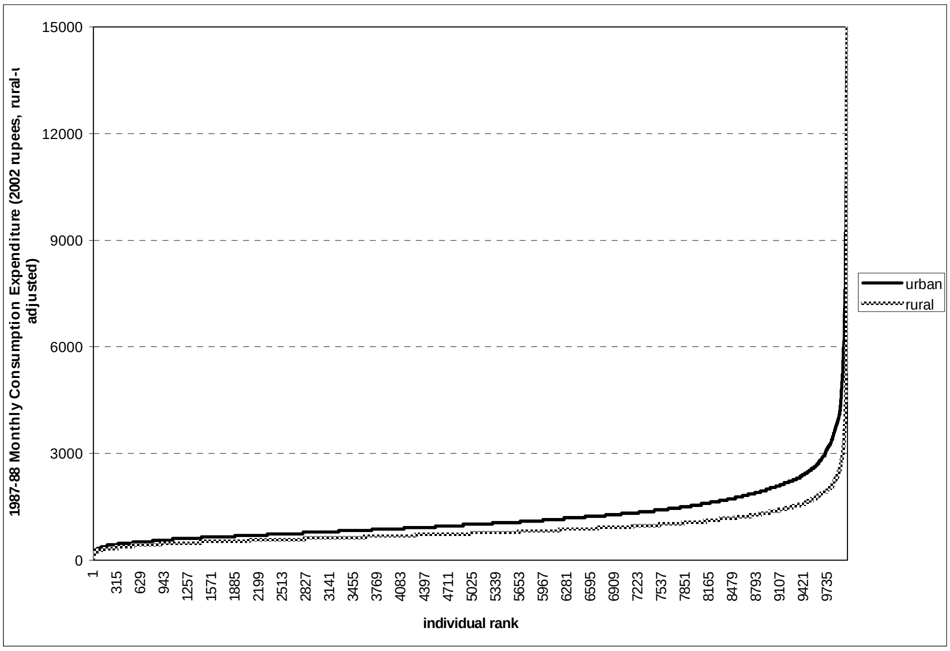

Figure 7a: Ordered Vectors of Consumption in Rural and Urban India, 1987-1988

Non surprisingly, UI tests indicate that, for all three years, the urban distribution dominates the rural population. It would be nice to test for the evolution of this domination over time but we are not aware of the existence of a testing methodology that would enable one to say things such as: "we accept the hypothesis that the urban distribution dominates more strongly the rural one in 2002 than in 1996".

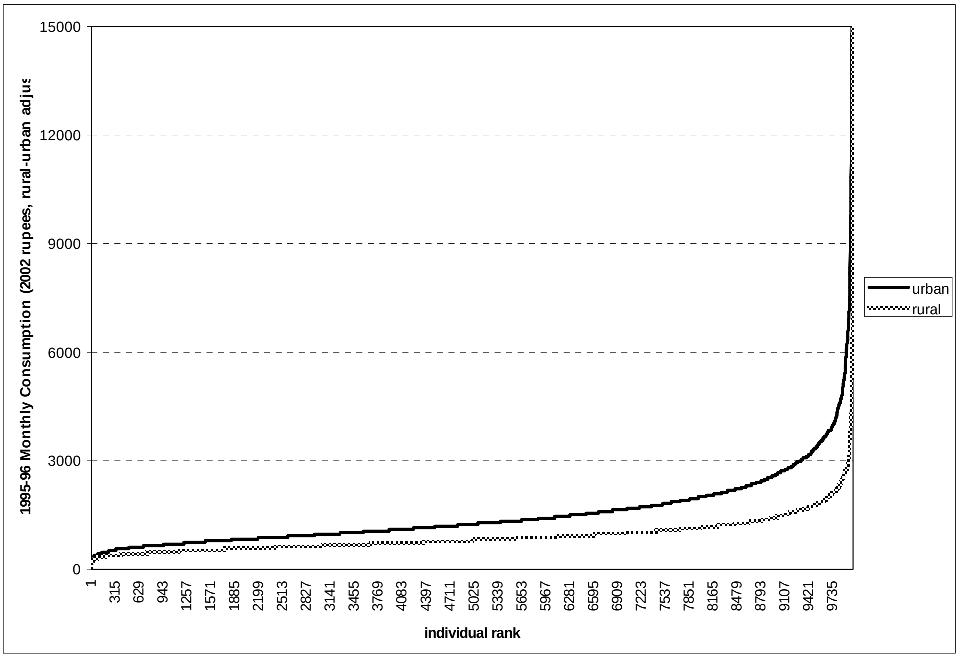

Figure 7b: Ordered Vectors of Consumption in Rural and Urban India, 1995-1996.

4.2. Distributions of district public goods

4.2.1. Literacy

Figures 8 and 9 show ordered vectors of literacy rates in the individuals' districts of residences (with individuals increasingly ordered in terms of the literacy rate of their district of residence) in the rural and urban part of India respectively. As above, the pictures are obtained from a random drawing of 10000 individuals from the empirical distribution.

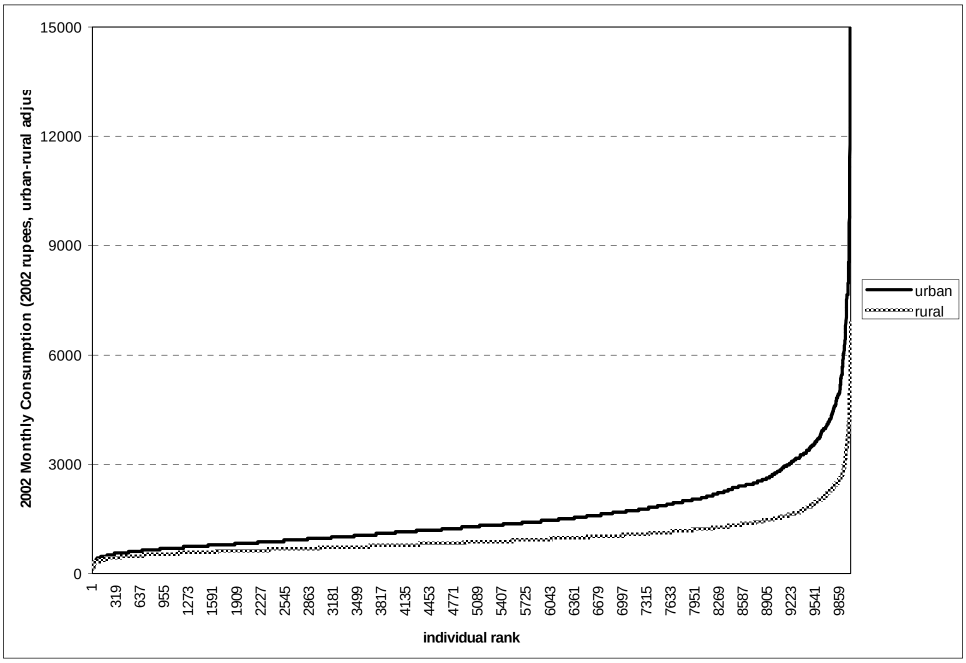

Figure 7c: Ordered Vectors of Consumption in Rural and Urban India; 2002.

As is clear from the two figures, and except for individuals who live in the most literate districts, where the room for improvement is small, there has been a clear increase in district literacy during the whole period for any individual position in the distribution of literacy rates. The small crossing that takes place at the very upper tail of the distributions between the ordered vectors of 1991 and 2001 (for both rural and urban India) turns out to be *not* statistically significant.

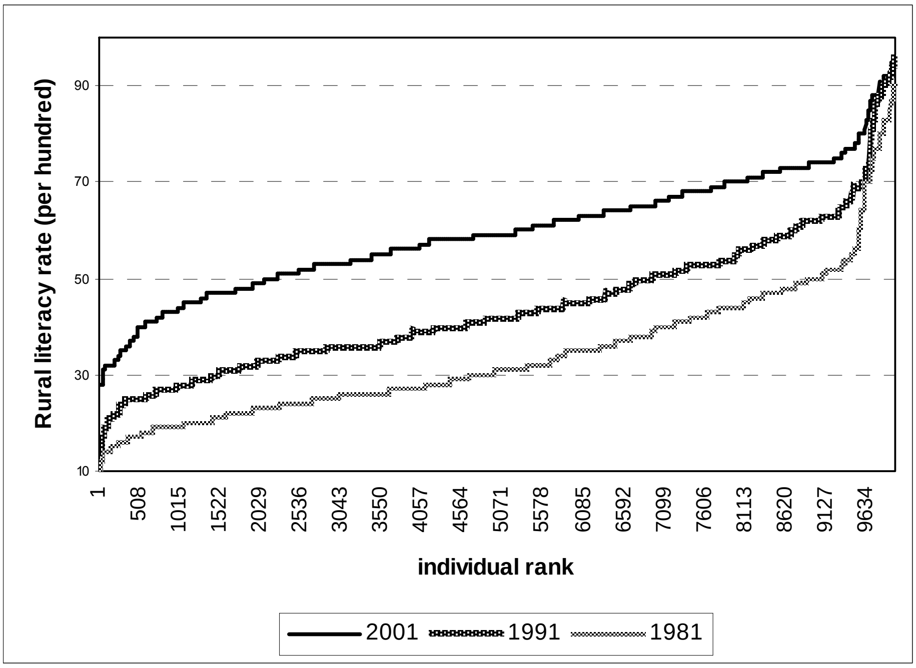

Figure 8: Ordered Vector of Rural Literacy

In a somewhat parallel fashion to what was happening for consumption, progress in literacy have been faster in the first (1981-1991) than in the second (1991-2001) period in urban India while the reverse conclusion holds for rural India. As can be expected, progress have been more important for individuals located in the center of the ordered vectors than for those located at the extreme. As is also clear from the pictures, urban ordered vectors tend to be "flatter" - more equal - than their rural counterparts.

One-dimensional UI tests indicate that the distribution of 2001 head-count poverty dominates that of 1991 and 1981. Similarly the distribution of 1991 dominates 1981.

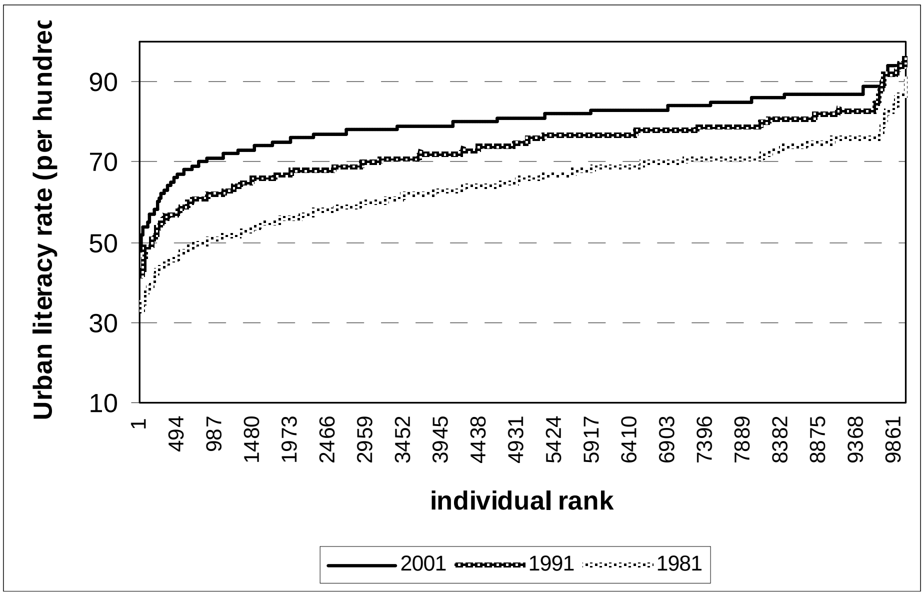

Figure 9, Ordered Vectors of Urban Literacy.

While the comparison of ordered consumption vectors in rural and urban India suggests a small increase in the rural-urban gap, the somewhat opposite conclusion seems to hold for literacy, as illustrated in figures 10a, b and c, which are the analogs, for literacy, to what figures 7a, b and c were for consumptions. One-dimensional UI tests confirm that the urban distribution always dominates the rural distribution.

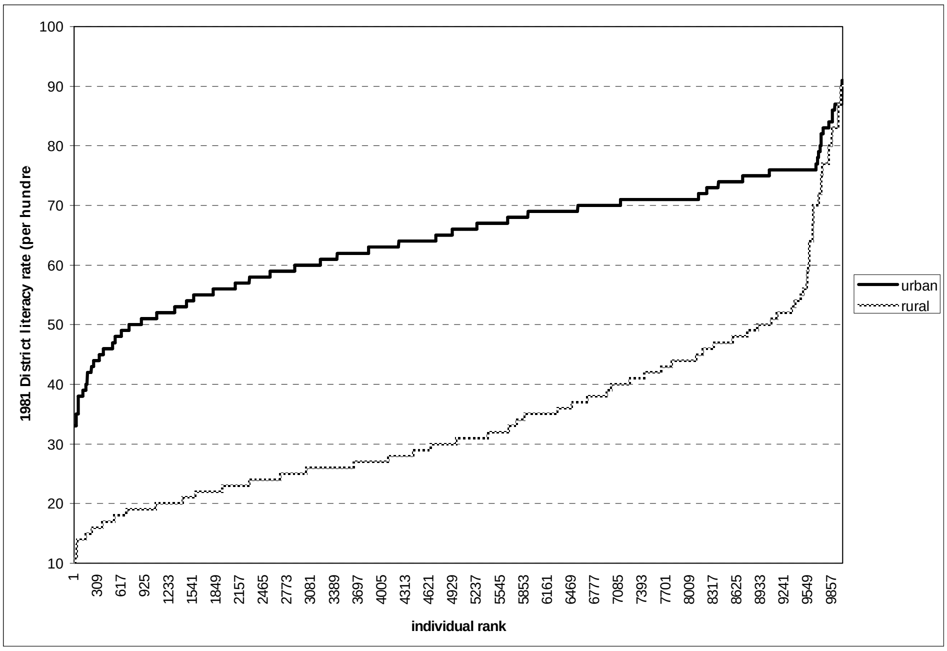

Figure 10a: Ordered vectors of Literacy in Rural and Urban India, 1981.

4.2.2. Under 5 infant mortality

Figure 11 depicts the (decreasingly) ordered vectors of under 5 infant mortality. As mentioned earlier, data on this local public good do not enable a distinction rural-urban, and the analysis is, for this reason, confined to the whole India level. The picture makes clear the dominance, confirmed by one-dimensional UI tests, of both 1991 and 2001 over 1981 and the dominance of 2001 over 1991.

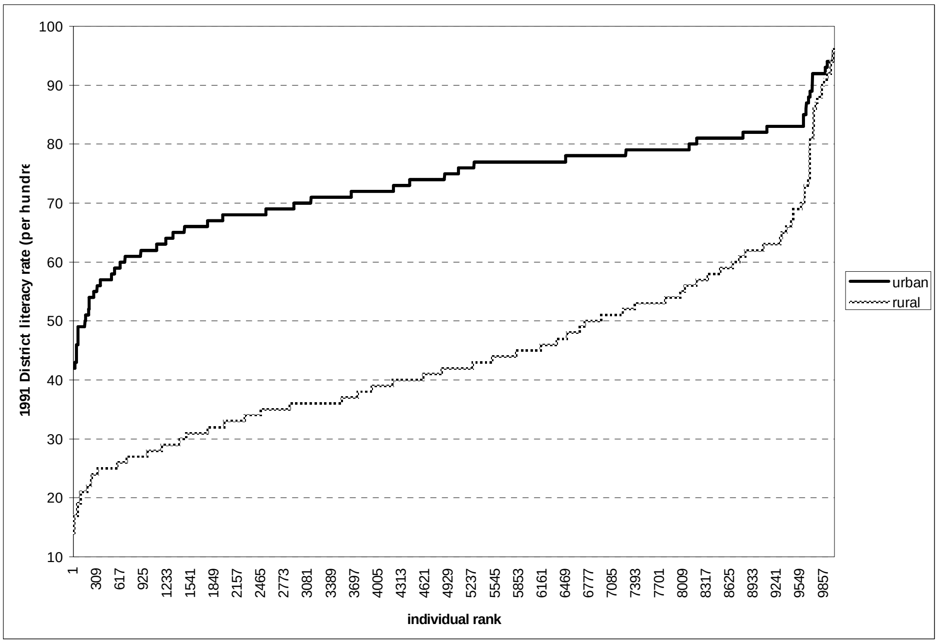

Figure 10b: Ordered Vectors of Literacy in Rural and Urban India, 1991.

4.2.3. Crime

Figure 12 shows similar decreasingly ordered vectors of district violent crime levels. The ordered vectors show that there was has been an increase in crime rate in the safest districts as compared to1988. This increase has been particularly strong in 1996. However from then onwards, crime rates have fallen secularly across most districts. However the safest district in 2002 is still worse off, crime-wise-, than the best off district in 1996 and 1988. This verdict is confirmed by UI tests. We reject the dominance of 2002 over 1988 and 1996 at the first order. For 2002 and 1996, this rejection is essentially due to the deterioration that has taken place in the best districts. If we perform a poverty gap dominance test, we find a dominance of 2001 over both 1988 and 1996. However there is no dominance relationship whatsoever between 1988 and 1996.

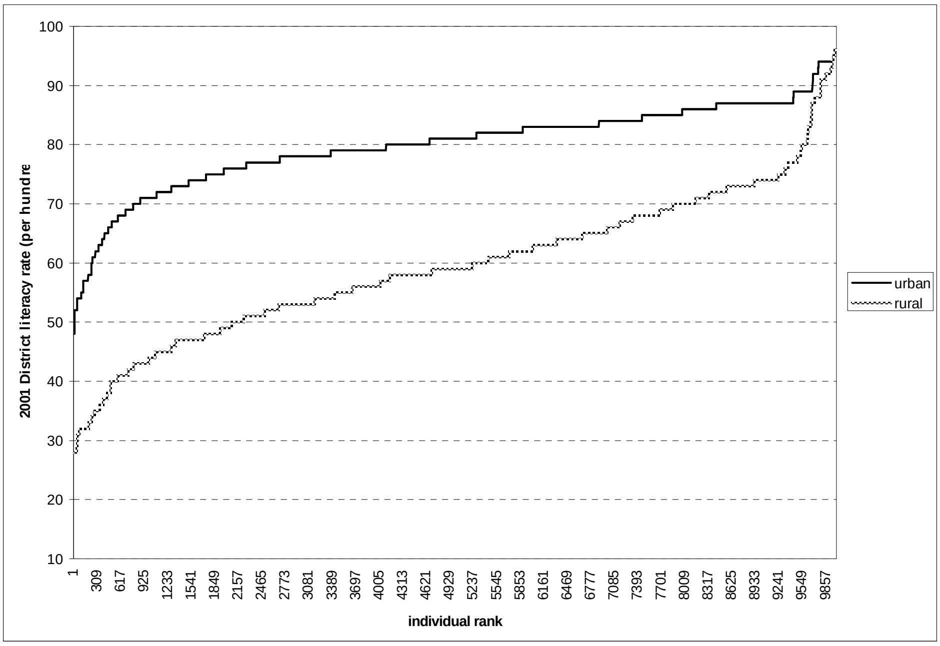

Figure 10c: Ordered Vectors of Literacy: in Rural and Urban India, 2001.

5. Multidimensional comparisons

As is clear from the theoretical definitions, any failure to achieve one-dimensional dominance in one variable in isolation implies a failure to achieve multi-dimensional dominance if this variable is included. This seems to suggest that it would be redundant to test for first order dominance for the joint distribution of expenditure and literacy for the years 2002 and 1996, because, as we have seen above, such a verdict does not hold at the first order for each dimension in isolation. Yet this conclusion is unwarranted when one considers the statistical significancy of the dominance or non-dominance. Indeed, and as explained in appendix B, statistical testing for dominance involves the testing of an hypothesis on the signs of a sequence of (poverty) inequalities. Obviously there are much more inequalities to be tested in a multi-dimensional test than in a one-dimensional ones. Yet the threshold values that each inequality is required to majorize or to minorize in order to be considered of a given sign at a specified level of confidence depends (in absolute value positively) upon the number of inequalities. Hence, it is harder to have an inequality that is statistically significantly negative or statistically significantly positive in a multi-dimensional analysis than in a one-dimensional one.

Figure 11: Ordered Vector of Under 5 Mortality Rates: All India.

Since our motivation for multivariate analysis is to be as inclusive as possible in terms of dimensions, we start with the discussion of the very demanding four-dimensional test. We consider the four variables at the all India level: expenditure, literacy, violent crime rates and under 5 child mortality. The table that follows figure 12 reports the best dominance results (which turn out to be second-order, confirming the one-dimensional result obtained for crime that 2002 dominates 1996 and 1988 only at the second order).[12]

[12]In doing the four dimensional analysis, we have to give up the finer grid considered for the univariate analysis. There are too many points to compare as checks need to made on the set of values given by the cross product of the four variables for all the data points observed in the sample. We can check at some predefined points but the choice of the grid

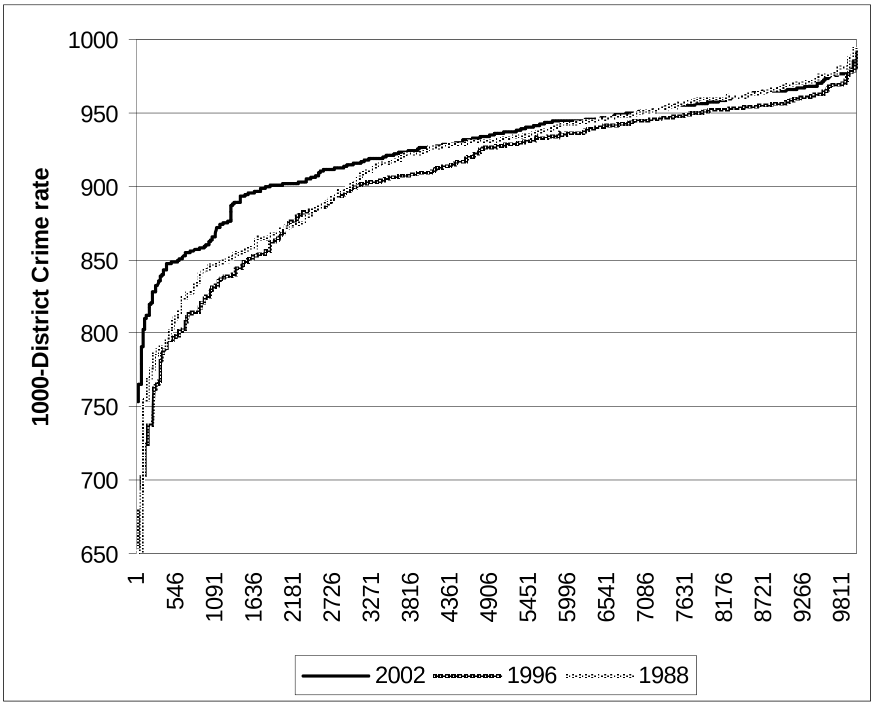

Figure 12, ordered vectors of crime rate

Year	**1988**	**1996**	**2002**
1988	-		
2002	$2002\left(\mathbb{U}^{M2}\right)$	$2002\left(\mathbb{U}^{M2}\right)$	-

These results tell us that all utilitarian planners who assume that Indians transform individual expenditure, district public safety, district literacy and district protection against the risk of loosing one's child into well-being by the same utility functions in $\mathbb{U}^{M2}$ agree to say that India is a better place to be in 2002 than it was in either 1996 or 1988. This is not true for

is arbitrary. Instead we make our grid coarser (without affecting the marginal distribution results). In the case of crime rate and under 5 mortality rate, we round off our variable (1000-the rate) to the closest hundreds value (so 589 becomes 600). We round off the literacy values to the closest tens value (17 becomes 20). We leave the expenditure as it is, because it has already been rounded off. Even with these rounding offs, we had to check for some 180,000 inequalities!!

1996 over 1988 as there was a sharp rise in crime rate during that interval that happens to prevents significantly any hope of getting four-dimensional dominance over that period. We emphasize that this is a *strong* dominance result. Obtaining joint dominance at the second order for four variables is difficult, as there is a very large family of social welfare judgements that need to agree on that. The fact that we obtain it here suggests that there is a strong sense in which one can say that social welfare in India has increased between 1996 and 2002.

Next we test what happens if we drop crime rate, whose blocking power seems to be responsible for the failure of obtaining dominance of 1996 over 1988. The table that indicates the extremely strong dominance verdict for that case is the following.

Year	**1988**	**1996**	**2002**
1988	-		
1996	1996 $\left(\mathbb{U}^{M1}\right)$		
2002	2002$\left(\mathbb{U}^{M1}\right)$	2002$\left(\mathbb{U}^{M1}\right)$	-

There is therefore a clear trend in improvement over the period 1988 and 2002 that happens to be of the first order. Abstracting for crime therefore, all utilitarian planners who believe that Indians transform identically district levels risk of infant mortality and illiteracy encounters and individual consumption into well-being by a function in $\mathbb{U}^{M1}$ (a rather large class) agree to say that social welfare in India has increased steadily over the period.

Next we consider how the results stand if we consider the rural and urban part of the district separately. As noticed above, only expenditure and literacy data enables such a distinction, so that we look for joint dominance in these two variables only. As indicated in the following table, which is valid for both rural and urban India, the verdict of the previous table remains unchanged.

Year	**1988**	**1996**	**2002**
1988	-		
1996	1996 $\left(\mathbb{U}^{M1}\right)$		
2002	2002$\left(\mathbb{U}^{M1}\right)$	2002$\left(\mathbb{U}^{M1}\right)$	-

Another interesting observation that can be taken from this analysis is that over the years, there seems to be no change in welfare reducing correlation between the variables. In our case, all results on the joint distributions appear to be driven strongly by results obtained in the marginal distributions in isolation. For example, it is never the case that there is dominance of the marginal distributions (at say the jth order) but no dominance when we consider the joint distribution. To that extent, it seems that, in India, the evolution of the correlation between the various dimensions, which is brought about by multidimensional analysis, does not play much role in ranking distributions.

6. Conclusion

Is India better off today than 15 years ago ? The answer that we gave in this paper to this question is a (mildly) qualified yes. In view of the importance of India in the world, and the importance of the changes that this country has gone through in the last fifteen years or so, we believe this answer to be of intrinsic general interest. But more importantly, the point of the paper was also to illustrate both the possibility and the fruitfulness of robust multidimensional methodologies for answering questions like this. When one looks at individual consumption, district literacy, district infant mortality and district crime (the later three variables being interpreted as local public goods) either separately, or jointly, there seems to be little doubt that the distribution of well-being in India has improved over the period no matter what are the assumptions made on the function that transform these attributes into well-being, provided that it is in the class $\mathbb{U}^{M2}$. As it turns out, in the case of India, there is not much point in looking at the joint distribution of the attributes as the ranking of the distributions that has been obtained is the one that results from the intersection of all rankings based on every dimension in isolation. This, obviously, could not be guessed at first glance.

We interpret our results as saying that someone who had normative doubts about the direction taken by India in the last fifteen year would need to question these doubts somehow. Of course, we have not considered all individual attributes that are normatively relevant. Environmental indicators are, in particular, lacking and it would be nice to obtain good data on those. Yet we would like to emphasize that, if our results push toward some optimism with respect to the normative direction taken by India in the last fifteen years, they do not *in themselves* say much about the normative appraisal of the liberalization reforms launched in the eighties, and which are believed by some to be partly responsible for the increase in growth observed over the period as compared to the pre-eighties situation. For, in order to normatively appraise such liberalization reforms, one would need to compare the current distribution of the attributes with the (counterfactual) one that would have prevailed now had the reforms not been implemented and had India continued to grow on the pre-eighties path. The analysis in this paper obviously does not provide any answer as to what would be the verdict of this counterfactual comparison.

Appendix A. Proof of the sufficiency part of proposition 3

Proof. We now prove that $\sum_{i=1}^{n}\prod_{j\in K}\max(t_j - x_{ij}, 0) \leq \sum_{i=1}^{n}\prod_{j\in K}\max(t_j - y_{ij}, 0)$ for every $t \in \mathbb{R}_+^k$ and $K \subset \{1, ..., k\}$ is a sufficient condition for

$$\sum_{i=1}^{n} U(x_i) \geq \sum_{i=1}^{n} U(y_i)$$

to hold for all utility functions in $\mathbb{U}^{M2}$. As in the proof of proposition 2, this inequality can be written as:

$$\int_0^{\overline{z}_1} ... \int_0^{\overline{z}_k} [f^x(a) - f^y(a)]U(a)d_z \geq 0 \qquad (.1)$$

with $f^x(a) = \frac{\#\{i:x_i=a\}}{n}$ and $f^y(a) = \frac{\#\{i:y_i=a\}}{n}$ being the discrete joint density corresponding to x and y, and the integration being the appropriate one (for instance the Abel discrete decomposition of (Fishburn & Vickson (1978); eq 2.49)), which we write as an integral, to alleviate the notation). As in the proof of proposition 1, $\overline{z}_j$ is an upper bound for the attribute j that is relevant for the comparison of x and y. Let $\Delta f(a) = f^x(a) - f^y(a)$ for every $a \in \mathbb{R}_+^k$. Furthermore, for any two vectors v and w in $\mathbb{R}_+^k$ and any index set $K \subset \{1, ..., k\}$, we denote by $(v_K; w_{-K})$ the vector in $\mathbb{R}_+^k$ whose coordinate that are indexed by K are as in v and all the other coordinate are as in w. Furthermore, when the number of coordinates is small, we write $(v_{hij}; w_{-hij})$ instead of $(v_{\{hij\}}; w_{-\{hij\}})$. If one integrates by part the left hand side of (.1) once for every integrand, one obtains, after lengthy manipulations :

$$\int_0^{\overline{z}_1} ... \int_0^{\overline{z}_k} \Delta f(a)U(a)d_z = -\sum_{h=1}^{k}\int_0^{\overline{z}_h} \Delta F_h(a_h)U_h(a_h; \overline{z}_{-h})da_h$$

$$+\sum_{h=1}^{k-1}\sum_{i=h+1}^{k}\int_0^{\overline{z}_h}\int_0^{\overline{z}_i}\Delta F_{hi}(a_h,a_i)U_{hi}(a_h,a_i;\overline{z}_{-hi})da_h da_i$$

$$-\sum_{h=1}^{k-2}\sum_{i=h+1}^{k-1}\sum_{j=i+1}^{k}\int_0^{\overline{z}_h}\int_0^{\overline{z}_i}\int_0^{\overline{z}_j}\Delta F_{hij}(a_h,a_i,a_j)U_{hij}(a_h,a_i,a_j;\overline{z}_{-hij})da_h da_i da_j$$

....

....

$$+(-1)^k\int_0^{\overline{z}_1}...\int_0^{\overline{z}_k}\Delta F(a_1,...,a_k)U_{12...k}(a_1,...,a_k)da_1...da_k \qquad (.2)$$

where:
$\Delta F(a)=\int_0^{a_1}...\int_0^{a_k}\Delta f(\alpha)d\alpha_1...d\alpha_k$ denotes the difference in the cumulative distribution, and,
ΔF_{hi} denote the difference in the cumulative joint distribution of the attributes h and i (the value of the other attributes being fixed at their upper bound; a similar interpretation holds for ΔF_h, ΔF_{hij}, etc.)
This expression was obtained in Hadar & Russell (1974) (equation 5.5'). It shows that, if the utility function is in $\mathbb{U}_1$, then the condition $\Delta F(a)\leq 0$ for every $a\in[0,\overline{z}_1]\times...\times[0,\overline{z}_k]$ (headcount poverty dominance for every combinations of poverty lines) is sufficient for the inequality (.1). If we now integrate by part every term of (.2) with respect to every integrand, we get:

$$\int_0^{\overline{z}_1}...\int_0^{\overline{z}_k}\Delta f(a)U(a)d_z = -\sum_{h=1}^{k}[\int_0^{\overline{z}_h}\Delta F_h(a_h)da_h U_h(\overline{z}_1...,\overline{z}_k)$$
$$-\int_0^{\overline{z}_h}\int_0^{a_h}\Delta F_h(\alpha_h)d\alpha_h U_{hh}(a_h;\overline{z}_{-h})da_h]$$

$$+\sum_{h=1}^{k-1}\sum_{i=h+1}^{k}[\int_0^{\overline{z}_h}\int_0^{\overline{z}_i}\Delta F_{hi}(a_h,a_i)da_h da_i U_{hi}(\overline{z}_1,...,\overline{z}_k)$$

$$-\int_0^{\overline{z}_h}\int_0^{\overline{z}_i}\int_0^{a_i}\Delta F_{hi}(a_h,\alpha_i)da_h d\alpha_i U_{hii}(a_i;\overline{z}_{-i})da_i$$

$$-\int_0^{\overline{z}_i}\int_0^{\overline{z}_h}\int_0^{a_h}\Delta F_{hi}(\alpha_h,a_i)d\alpha_h da_i U_{hih}(a_h;\overline{z}_{-h})da_h$$

$$+\int_0^{\overline{z}_h}\int_0^{\overline{z}_i}\int_0^{a_h}\int_0^{a_i}\Delta F_{hi}(\alpha_h,\alpha_i)d\alpha_h d\alpha_i U_{hihi}(a_h,a_i;\overline{z}_{-hi})da_h da_i]$$

$$-\sum_{h=1}^{k-2}\sum_{i=h+1}^{k-1}\sum_{j=i+1}^{k}[\int_0^{\overline{z}_h}\int_0^{\overline{z}_i}\int_0^{\overline{z}_j}\Delta F_{hij}(a_h,a_i,a_j)da_j da_i da_h U_{hij}(\overline{z}_1,...,\overline{z}_k)$$

$$-\sum_{g=h}^{j}\int_0^{\overline{z}_h}\int_0^{\overline{z}_i}\int_0^{\overline{z}_j}\int_0^{a_g}\Delta F_{hij}(\alpha_g;a_{-g})d\alpha_g da_{-g}U_{hijg}(a_g;\overline{z}_{-g})da_g$$

$$+\int_0^{\overline{z}_h}\int_0^{a_h}\int_0^{\overline{z}_i}\int_0^{a_i}\int_0^{\overline{z}_j}\Delta F_{hij}(\alpha_h,\alpha_i,a_j)da_j d\alpha_i d\alpha_h U_{hijih}(a_h,a_i;\overline{z}_{-hi})da_i da_h$$

$$+\int_0^{\overline{z}_h}\int_0^{a_h}\int_0^{\overline{z}_i}\int_0^{\overline{z}_j}\int_0^{a_j}\Delta F_{hij}(\alpha_h,a_i,\alpha_j)d\alpha_j da_i d\alpha_h U_{hijjh}(a_h,a_j;\overline{z}_{-hj})da_j da_h$$

$$+\int_0^{\overline{z}_h}\int_0^{\overline{z}_i}\int_0^{\overline{z}_j}\int_0^{a_j}\int_0^{a_i}\Delta F_{hij}(a_h,\alpha_i,\alpha_j)da_h d\alpha_j d\alpha_i U_{hijji}(a_i,a_j;\overline{z}_{-ij})da_j da_i$$

$$-\int_0^{\overline{z}_h}\int_0^{a_h}\int_0^{\overline{z}_i}\int_0^{\overline{z}_j}\int_0^{a_j}\int_0^{a_i}\Delta F_{hij}(\alpha_h,\alpha_i,\alpha_j)d\alpha_h d\alpha_j d\alpha_i U_{hijjih}(a_h,a_i,a_j;\overline{z}_{-hij})da_j da_i da_h]$$

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$$(-1)^k[-\sum_{g=1}^{k}\int_0^{\overline{z}_g}\Delta H(a_g;\overline{z}_{-g})U_{12...kg}(a_g;\overline{z}_{-g})da_g$$

$$+\sum_{f=1}^{k-1}\sum_{g=h+1}^{k}\int_0^{\overline{z}_f}\int_0^{\overline{z}_g}\Delta H(a_f,a_g;\overline{z}_{-fg})U_{12...kfg}(a_f,a_g;\overline{z}_{-hi})da_f da_g$$
$$-\sum_{f=1}^{k-2}\sum_{g=f+1}^{k-1}\sum_{h=g+1}^{k}\int_0^{\overline{z}_f}\int_0^{\overline{z}_g}\int_0^{\overline{z}_h}\Delta H(a_f,a_g,a_h;\overline{z}_{-fgh})U_{12...kfgh}(a_f,a_g,a_h;\overline{z}_{-fgh})da_f da_g da_h$$

....

....

$$+(-1)^k\int_0^{\overline{z}_1}...\int_0^{\overline{z}_k}\Delta H(a_1,...,a_k)U_{12...k12....k}(a_1,...,a_k)da_1...da_k]$$

where, for every $a\in[0,\overline{z}_1]\times...\times[0,\overline{z}_k]$,

$$\begin{aligned}\Delta H(a) &= \int_0^{a_1}....\int_0^{a_k}\Delta F(\alpha_1,...,\alpha_k)d\alpha_1...d\alpha_k\\ &= \int_0^{a_1}....\int_0^{a_k}(a_1-\alpha_1)....(a_k-\alpha_k)\Delta f(\alpha_1,...,\alpha_k)d\alpha_1...d\alpha_k\end{aligned}$$

Hence, for utility functions satisfying:

$$\begin{aligned}U_i &\geq 0 \text{ for all } i\\ U_{ij} &\leq 0 \text{ for all } i,j \text{ not necessarily distinct}\\ U_{hij} &\geq 0 \text{ for all } h,i,j, \text{ two of which at least being distinct}\\ U_{ghij} &\leq 0 \text{ for all } g,h,i,j, \text{ at most 2 pairs of which being identical}\\ U_{fghij} &\geq 0 \text{ for all } f,g,h,i,j, \text{ at most 2 pairs of which being identical}\\ U_{efghij} &\leq 0 \text{ for all } e,f,g,h,i,j \text{ at most 3 pairs of which being identical}\\ &....\\ U_{11...kk} &\leq 0\end{aligned}$$

the condition that $\Delta H(a) \leq 0$ for all a and that:

$$\begin{aligned}
& \int_{j \in K}^{a} \cdots \int \Delta F_K(\alpha) d\alpha \\
= & \int_{j \in K}^{a} \cdots \int \prod_{j \in K} (a_j - \alpha_j) \Delta f_K(\alpha) d\alpha \\
\leq & \ 0
\end{aligned}$$

for all non-empty $K \subset \{1, 2, ..., k\}$ is sufficient for the inequality (.1) to hold.
■

Appendix B. Statistical Inference

As made clear in section 2, either one or k-dimensional dominance criteria requires the verification of a finite number, m say, of inequalities. Each such inequality can be seen as a statistical hypothesis and the sequence of these inequalities can also be seen as a statistical hypothesis.

To be more specific, suppose that we want to test the hypothesis of the dominance of distribution A over distribution B and consider the following sequences of sub-hypothesis.

$$\begin{aligned}
H_0^i &: \gamma_i^A \geqslant \gamma_i^B \\
H_A^i &: \gamma_i^A < \gamma_i^B \\
\overline{H}_0^i &: \gamma_i^B \geqslant \gamma_i^A \\
\overline{H}_A^i &: \gamma_i^B < \gamma_i^A \\
\text{for } i &= 1, ..., m
\end{aligned}$$

where γ_i^j can be either the headcount poverty or the poverty gap for the distribution j ($j = A, B$) at the poverty line i, $\overline{H}_0^i$ is the null sub-hypothesis that poverty in A for poverty line i is not larger than in B and $\overline{H}_A^i$ is the alternative to the null sub-hypothesis. There are two broad testing strategies that have been proposed in the literature. One is the intersection-union (IU) strategy, initiated by Howes (1994) and Kaur *et al.* (1994) and the other is the Union-Intersection one, advocated among others by Bishop & Formby (1999). A comparison of the two methods is performed by Howes (1994). According to the most conservative Intersection-Union (IU) the *rejection region* of the null hypothesis is the union of K subhypothesis and the *non-rejection region* of the null hypothesis is the intersection of the non-rejection regions of the K subhypothesis. In other words, with this methodology, we accept dominance of A over B if we fail to reject all K null subhypothesis $\overline{H}_0^i$ and we reject dominance if we reject any one of the K null subhypothesis. This is a very conservative test because it requires, in order to get dominance of A over B, that we reject all inequalities that are compatible with a dominance of B over A. This is why Bishop & Formby (1999) have suggested the more liberal Union Intersection (UI) methodology for which the rejection region of the null hypothesis is the intersection of the rejection of K subhypothesis and the non-rejection region of the null hypothesis is the union of the non-rejection regions of the K subhypothesis. Hence, with UI methodology, we accept dominance of A over B if we fail to reject one of the K null subhypothesis $\overline{H}_0^i$ and we reject dominance if we reject all K null subhypothesis.

In this paper, we resort to the more liberal UI methodology to test for the significances of the m inequalities. For this sake, we need to construct a test statistic for the poverty measure γ_i^j used in the methodology. To this aim, let T_i be defined by:

$$T_i = \frac{\widehat{\gamma}_i^A - \widehat{\gamma}_i^B}{\left(\frac{\widehat{\omega}_{ii}^A}{N^A} + \frac{\widehat{\omega}_{ii}^B}{N^B}\right)^{\frac{1}{2}}}$$

where $\widehat{\gamma}_i^j$ is the sample estimate of γ_i^j $(i = 1, ..., K \; ; \; j = A, B)$, $\widehat{\omega}_i^A$ is the variance estimates of $\widehat{\gamma}_i^j$ and N^j the size of the sample drawn from population j, $j = A, B$. The variance estimators are derived in Davidson & Duclos (2000) for the one-dimensional headcount ratio and the poverty gap and in Duclos *et al.* (2006) for their multi-dimensional generalizations according the following formula:

$$\widehat{\omega}_i = \frac{1}{N} \sum_{\{h:y_h<t\}} \left[(t_1 - y_{h1})^{s-1} (t_2 - y_{h2})^{s-1} ... (t_k - y_{hk})^{s-1}\right]^2 - (\widehat{\gamma}_i)^2$$

for k-dimensional poverty (for any $k \geq 1$) where s denote the order of dominance ($s = 1$ for headcount poverty and $s = 2$ for poverty gap).

With these estimators, the UI inference rule is defined by:

$$\begin{aligned} A \text{ dominates } B &\Leftrightarrow \min(T_1, ..., T_K) < -C_\alpha \text{ and } \max(T_1, ..., T_K) < C_\alpha \\ B \text{ dominates } A &\Leftrightarrow \max(T_1, ..., T_K) > C_\alpha \text{ and } \min(T_1, ..., T_K) > -C_\alpha \\ & \quad A \text{ and } B \text{ are not-comparable otherwise} \end{aligned}$$

where C_α is the critical value for a significance level of α (α is the probability of rejecting H_0 when H_0 is true) determined from the Student Maximum Modulus (SMM) distribution provided by Stoline & Ury (1979).

In our empirical implementation, we perform inference tests at 95% confidence.

Appendix C. Details of statistical tests

Rural Expenditure

	Maximum *t*	Critical Value	Verdict	Minimum *t*
First Order Dominance				
2002 Vs 1996	8.3	4.09	REJECT	-43.0
2002 Vs 1988	4.5	4.09	REJECT	-79.8
1996 Vs 1988	1.4	4.09	ACCEPT	-40.0
Second Order Dominance				
2002 Vs 1996	-1.6	4.09	ACCEPT	-45.3
2002 Vs 1988	-4.3	4.09	ACCEPT	-92.8
1996 Vs 1988	-1.8	4.09	ACCEPT	-43.2

Studentized Modulos Distribution with Degrees of freedom 219, ∞

Urban Expenditure

	Maximum *t*	Critical Value	Verdict	Minimum *t*
First Order Dominance				
2002 Vs 1996	6.1	4.19	REJECT	-13.4
2002 Vs 1988	5.6	4.19	REJECT	-73.7
1996 Vs 1988	2.2	4.19	ACCEPT	-83.8
Second Order Dominance				
2002 Vs 1996	1.1	4.19	ACCEPT	-12.6
2002 Vs 1988	-4.1	4.19	ACCEPT	-87.4
1996 Vs 1988	-0.4	4.19	ACCEPT	-98.1

Studentized Modulos Distribution with Degrees of freedom 365, ∞

All India Expenditure

	Maximum *t*	Critical Value	Verdict	Minimum *t*
First Order Dominance				
2002 Vs 1996	8.7	4.2	REJECT	-47.1
2002 Vs 1988	4.25	4.2	REJECT	-104.8
1996 Vs 1988	2.2	4.2	ACCEPT	-71.4
Second Order Dominance				
2002 Vs 1996	--2.2	4.2	ACCEPT	-50.1
2002 Vs 1988	-4.5	4.2	ACCEPT	-124.3
1996 Vs 1988	-1.4	4.2	ACCEPT	-79.5

Studentized Modulos Distribution with Degrees of freedom 387, ∞

Expenditure: Rural-Urban

	Maximum *t*	Critical Value	Verdict	Minimum *t*
First Order Dominance				
2002: Urban Vs Rural	-1.30	4.11	ACCEPT	-158.6
1996: Urban Vs Rural	0.49	4.11	ACCEPT	-231.6
1988: Urban Vs Rural	0.38	4.17	ACCEPT	-268.5

Studentized Modulos Distribution with Degrees of freedom (244, ∞),(241, ∞) and (331, ∞)

Rural Literacy

	Maximum *t*	Critical Value	Verdict	Minimum *t*
First Order Dominance				
2002 Vs 1996	0.21	3.84	ACCEPT	-273.3
2002 Vs 1988	-13.82	3.84	ACCEPT	-625.9
1996 Vs 1988	-16.58	3.84	ACCEPT	-275.8

Studentized Modulos Distribution with Degrees of freedom 84, ∞

Urban Literacy

	Maximum *t*	Critical Value	Verdict	Minimum *t*
First Order Dominance				
2002 Vs 1996	0.37	3.76	ACCEPT	-156.9
2002 Vs 1988	-8.40	3.76	ACCEPT	-438.0
1996 Vs 1988	-10.4	3.76	ACCEPT	-244.7

Studentized Modulos Distribution with Degrees of freedom 60, ∞

All India Literacy

	Maximum *t*	Critical Value	Verdict	Minimum *t*
First Order Dominance				
2002 Vs 1996	-0.24	3.84	ACCEPT	-302.3
2002 Vs 1988	-16.40	3.84	ACCEPT	-710.9
1996 Vs 1988	-20.30	3.84	ACCEPT	-330.1

Studentized Modulos Distribution with Degrees of freedom 82, ∞

Literacy: Rural-Urban

	Maximum *t*	Critical Value	Verdict	Minimum *t*
First Order Dominance				
2002: Urban Vs Rural	0.08	3.79	ACCEPT	-390.1
1996: Urban Vs Rural	-6.35	3.83	ACCEPT	-624.9
1988: Urban Vs Rural	0.82	3.83	ACCEPT	-268.5

Studentized Modulos Distribution with Degrees of freedom (68, ∞),(79, ∞) and (77, ∞)

1000-Under 5 Mortality Rates(q_5) All India

	Maximum *t*	Critical Value	Verdict	Minimum *t*
First Order Dominance				
2002 Vs 1996	-13.56	4.13	ACCEPT	-271.7
2002 Vs 1988	-36.08	4.13	ACCEPT	-833.7
1996 Vs 1988	-27.07	4.13	ACCEPT	-327.4

Studentized Modulos Distribution with Degrees of freedom 276, ∞

1000-Violent Crime Rates: All India

	Maximum *t*	Critical Value	Verdict	Minimum *t*
First Order Dominance				
2002 Vs 1996	13.05	4.06	REJECT	-100.7
2002 Vs 1988	78.12	4.06	REJECT	-113.4
1996 Vs 1988	160.96	4.06	REJECT	-29.4
Second Order Dominance				
2002 Vs 1996	-23.82	4.06	ACCEPT	-107.74
2002 Vs 1988	-29.43	4.06	ACCEPT	-116.16
1996 Vs 1988	52.33	4.06	REJECT	-29.43

Studentized Modulos Distribution with Degrees of freedom 204, ∞

All Four Variables: All India Expenditure, All India District level Literacy, 1000-All India Under 5 Mortality, 1000-All India Violent Crime Rate.

	Maximum *t*	Critical Value	Verdict	Minimum *t*
First Order Dominance				
2002 Vs 1996	20.4	5.41	REJECT	-288.06
2002 Vs 1988	23.7	5.41	REJECT	-800.15
1996 Vs 1988	68.72	5.41	REJECT	-315.64
Second Order Dominance				
2002 Vs 1996	-0.32	5.41	ACCEPT	-296.78
2002 Vs 1988	-1.18	5.41	ACCEPT	-697.01
1996 Vs 1988	17.32	5.41	REJECT	-327.52

Studentized Modulos Distribution with Degrees of freedom 157052, ∞

Trivariate of All India Expenditure, All India District level Literacy, 1000-All India Under 5 Mortality

	Maximum *t*	Critical Value	Verdict	Minimum *t*
First Order Dominance				
2002 Vs 1996	8.70	5.43	REJECT	-290.3
2002 Vs 1988	4.21	5.43	ACCEPT	-846.5
1996 Vs 1988	2.26	5.43	REJECT	-330.4
Second Order Dominance				
2002 Vs 1996	-0.33	5.43	ACCEPT	-333.1
2002 Vs 1988	-1.26	5.43	ACCEPT	-866.5
1996 Vs 1988	-0.25	5.43	ACCEPT	-412.5

Studentized Modulos Distribution with Degrees of freedom 174537, ∞

Bivariate Distribution of Rural Literacy and Rural Expenditure

	Maximum *t*	Critical Value	Verdict	Minimum *t*
First Order Dominance				
2002 Vs 1996	8.30	5.01	REJECT	-273.3
2002 Vs 1988	4.52	5.01	ACCEPT	-625.9
1996 Vs 1988	1.42	5.01	ACCEPT	-275.8
Second Order Dominance				
2002 Vs 1996	-0.29	5.01	ACCEPT	-290.7
2002 Vs 1988	-1.76	5.01	ACCEPT	-731.4
1996 Vs 1988	-1.76	5.01	ACCEPT	-312.7

Studentized Modulos Distribution with Degrees of freedom 18615, ∞

Bivariate Distribution of Urban Literacy and Urban Expenditure

	Maximum *t*	Critical Value	Verdict	Minimum *t*
First Order Dominance				
2002 Vs 1996	5.11	5.04	REJECT	-159.6
2002 Vs 1988	5.00	5.04	ACCEPT	-520.01
1996 Vs 1988	1.59	5.04	ACCEPT	-286.1
Second Order Dominance				
2002 Vs 1996	2.86	5.04	ACCEPT	-290.7
2002 Vs 1988	-1.05	5.04	ACCEPT	-731.4
1996 Vs 1988	-0.27	5.04	ACCEPT	-312.7

Studentized Modulos Distribution with Degrees of freedom 21422, ∞

Bivariate Distribution of Literacy and Expenditure: Rural Vs Urban

	Maximum *t*	Critical Value	Verdict	Minimum *t*
First Order Dominance				
2002: Urban Vs Rural	0.97	4.98	ACCEPT	-21.0
1996: Urban Vs Rural	0.64	5.02	ACCEPT	-23.5
1988: Urban Vs Rural	0.03	5.07	ACCEPT	-22.7

Studentized Modulos Distribution with Degrees of freedom (16286, ∞),(19469, ∞) and (25694, ∞)

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