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Coalitional Power Structure in Stochastic Social Choice Functions with an Unrestricted Preference Domain*

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Abstract

When the individuals in Pattanaik and Peleg [16] are permitted to have weak preference orders, we show that: (i) as in their paper, there is a unique weight for each coalition; and (ii) for each feasible proper subset of the universal set and each preference profile, the society can be partitioned, so that, the weight of each coalition in this partition gives the probability of choosing *some* alternative which is best in the feasible set for at least one individual in the coalition. When the universal set is the feasible set, our result still holds provided certain additional conditions are satisfied. *Journal of Economic Literature* Classification Number: D71

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1 Introduction

Since the seminal work of Arrow [1] in social choice theory, many attempts have been made, without much success, to escape his impossibility theorem by modifying the conditions which he required the social choice rule to satisfy. The numerous impossibility results in the social choice literature bear testimony to the robustness of Arrow's impossibility result. These negative results are partly responsible for a line of research which considers probabilistic social choice rules to aggregate individual preferences on social alternatives. Probabilistic social choice rules are more general than their deterministic counterparts and increase the possibility for satisfactory aggregation of individual preferences.

Apart from opening up the possibility of escaping the Arrow-type impossibility results, an equally important and attractive aspect of the probabilistic framework is the scope it provides for incorporating certain notions of fairness and reasonable compromise into the collective decision-making process. For example, consider the situation of two seriously injured accident victims who each must have a pint of blood to survive, but there is only one pint of blood available and each individual wants to have it. In this situation of conflict of preferences, flipping a coin to determine the actual recipient of the single available pint of blood seems to incorporate a certain element of fairness and reasonable compromise which is lacking in deterministic social choice rules.

The Arrow conditions in the deterministic framework imply that the social choice procedure satisfies a neutrality property which plays a key role in the dictatorship theorem, namely, if any group of individuals is decisive over some pair of social alternatives, then that group must be decisive over every pair of social alternatives. A probabilistic analogue of neutrality is satisfied in the probabilistic framework when the Arrow conditions are appropriately translated into their probabilistic counterparts. However, in this framework neutrality is a more appealing principle because of the randomness present in probabilistic social choice rules. For example, a random dictatorship satisfies neutrality while avoiding many of the undesirable features of a deterministic dictatorship.

As in the deterministic case, the probabilistic social choice rules that are considered in the probabilistic social choice literature can be broadly classified into two categories, namely, those that map each social preference profile to a lottery over social preferences, and those that map each combination of a social preference profile and a feasible set (which is a subset of the universal set of social alternatives) to a social choice lottery over the feasible set. We refer to these kinds of rules as stochastic social welfare functions and stochastic social choice functions, respectively.

A major concern of probabilistic social choice theory is to characterize the properties of the power structures that can arise in this framework when the appropriate probabilistic analogues of the axiom systems used in the various impossibility theorems in the deterministic framework are adopted. Loosely speaking, by power structure we mean the distribution of the degree of influence in the social decision process that different groups of individuals may have. In the case of stochastic social welfare functions, this line of investigation was initiated by Barbera and Sonnenschein [6] and subsequently pursued by Bandyopadhyay, Deb and Pattanaik [2], Heiner and Pattanaik [12], and McLennan [14]. In this literature, the power of a coalition to determine the social choice probabilities in pairwise comparisons are induced by the probabilities assigned to the social preferences. However, unlike the deterministic framework, where the distribution of coalitional power in nonbinary choice situations is completely determined by the distribution of coalitional power for binary comparisions, it is not at all clear if the results in these papers can be used to derive restrictions on the distribution of coalitional power for nonbinary choice. This is a serious weakness if we maintain that from a social action perspective the significance of the probabilities assigned to the social preferences lies in the social choice probabilities they induce over each possible feasible set of social alternatives.

Among the literature which considers stochastic social choice functions, to our knowledge, Barbera and Valenciano [7], and Pattanaik and Peleg [16] are the only articles that systematically investigate the distribution of coalitional power.[1] However, as Barbera and Valenciano [7] implicitly consider only those feasible sets that contain exactly two social alternatives, their work also suffers from the same weakness mentioned in the previous paragraph. Pattanaik and Peleg [16], which is henceforth referred to as PP, consider feasible sets with arbitrary numbers of alternatives. For the axioms considered by them, they were able to characterize the distribution of coalitional power even when the feasible set contains more than two social alternatives.

PP considered stochastic social choice functions that satisfy three conditions they called *independence of irrelevant alternatives*, *ex-post Pareto optimality* and regularity.[2] Independence

[1]Most of the other works in this literature (Barbera [3,4], Fishburn [9], Fishburn and Gehrlein [10], and Intriligator [13] to mention a few) focus attention on the properties of particular stochastic social choice functions.

[2]Independence of irrelevant alternatives and ex-post Pareto optimality are respectively the probabilistic counterparts of Arrow's [1] Independence and Pareto conditions. Regularity is a natural probabilistic version of a rationality condition in the deterministic framework due to Chernoff [8] known as property α.

of irrelevant alternatives requires that the social choice lottery over a feasible set of alternatives must be the same for any two preference profiles which are identical when restricted to the feasible set. Ex-post Pareto optimality says that, if everyone prefers some alternative to another, then the social probability of choosing the later must be zero whenever both of them are feasible. Regularity requires that the social choice probability assigned to each alternative in a feasible set must not increase from its original value when the individual preferences remain the same but the feasible set is expanded by adding more alternatives. When the universal set of social alternatives contains at least four elements and individual preference orderings are strict, PP showed that a stochastic social choice function is essentially a random dictatorship if it satisfies their three conditions. More precisely, they first derived a unique weight for each individual such that the vector formed by these individual weights has the properties of a probability distribution over the set of all individuals. Then they showed that the probability of society's choosing an alternative from a feasible set, which is distinct from the universal set, is equal to the sum of the weights of those individuals who have that alternative as their best alternative in the feasible set. When the number of alternatives in the universal set exceeds the number of individuals in the society by at least two, their result extends to the case in which the feasible set is the universal set itself.

The assumption that individuals can only have strict preference orderings plays an important role in the result of PP. However, as the authors themselves pointed out, this is a rather restrictive assumption. Therefore, the next logical step is to permit individuals to have indifference between alternatives and ask whether there are any natural extensions of the results of PP with this expanded individual preference domain. The present paper pursues this particular line of investigation.

To understand the difficulty that arises when the individual preference domain is expanded to permit indifference between alternatives, consider a feasible set of alternatives B and a preference profile $\mathbf{R}$. If x and y are two alternatives in B and the individual preferences in $\mathbf{R}$ are strict, then, as there is only one best alternative in B for each individual, the intersection of the set of individuals who have x as their best alternative with the set of individuals who have y as their best alternative is empty. This fact is crucial for the weighted random dictatorship result of PP as it allows a specific partitioning of the society in which each member coalition consists of all those individuals who have the same best alternative in B. However, such a partition of the society may no longer exist if we permit the individual preferences in $\mathbf{R}$ to have indifference between alternatives. When the individual preferences in $\mathbf{R}$ are not necessarily strict, as each individual may have more than one best alternative in B, it is possible for two individuals to have best sets in B that intersect but are not equal to each other. So we must deal with the possibility that, given any two alternatives x and y in B, the two sets of individuals whose members respectively have x and y as a best alternative in B may intersect with each other.

Suppose there are at least four elements in the universal set of alternatives and the stochastic social choice function satisfies the three conditions of PP, namely, independence of irrelevant alternatives, ex-post Pareto optimality and regularity. Also, suppose B is a proper subset of the universal set of alternatives and $\mathbf{R}$ is a social profile of preference orderings. The society is then partitioned in such a way that each coalition S in the partition satisfies: (i) if individual i belongs to S and individual j does not belong to S, then their best sets in B do not have any common alternative; and (ii) if individual i belongs to S and there are at least two individuals in S, then at least one alternative which belongs to individual i's best set in B also belongs to the best set in B of some other individual in S. This way of partitioning the society is in some sense a generalization of the one described before for the case of strict individual preferences, because it yields the same partition of the society as before whenever the individual preferences in $\mathbf{R}$ are strict. We then show the following: (i) as in PP, there is a unique nonnegative weight for each individual in the society; and (ii) when B is the feasible set of alternatives and $\mathbf{R}$ is the social preference profile, the sum of the weights of all individuals belonging to the same coalition in the partition just described gives the probability that *some* alternative which is best in B for one of these individuals is chosen. If the social preference profile $\mathbf{R}$ is such that at least two alternatives do not belong to any of the best sets in the universal set, or the number of alternatives in the universal set exceeds the number of individuals in the society by at least two and the social preference profile $\mathbf{R}$ is such that at least one alternative does not belong to any of the best sets in the universal set, then the feasible set B does not have to be distinct from the universal set for our result to hold. These results reduce to the results of PP whenever everyone has unique best feasible alternatives. So our results are generalizations of those in PP.

In the next section, we introduce some prerequisite notation and definitions. The unique nonnegative weight of each coalition of individuals in the society is derived in section 3. The results of the paper, which we have briefly outlined above, are presented in section 4. We conclude in section 5.

2 Notation and Definitions

The universal set of alternatives, denoted by A, has m alternatives with $\infty > m \geq 3$. Let $\mathcal{A}$ be the set of all nonempty subsets of A (i.e. $\mathcal{A} = 2^A - \{\emptyset\}$).[3] An *ordering* on A is a reflexive, complete and transitive binary relation on A. We denote by $\mathcal{R}$ the set of all orderings on A.

Let $N = \{1, ..., n\}$ be the set of all individuals in the society, where $\infty > n \geq 2$. Also, let $\mathcal{N}$ be the set of all nonempty subsets of N (i.e. $\mathcal{N} = 2^N - \{\emptyset\}$). Then, as any subset of individuals in the society is a *coalition*, $\mathcal{N}$ is the set of all possible coalitions in the society. We denote the coalitions in $\mathcal{N}$ by S, $\hat{S}$, $\tilde{S}$, ...

Given any $S \in \mathcal{N}$, $\mathcal{R}^S$ denotes the $|S|$-fold Cartesian product[4] of $\mathcal{R}$. We use the term *preference profile* for the members of $\mathcal{R}^N$ and denote them by $\mathbf{R}$, $\hat{\mathbf{R}}$, $\tilde{\mathbf{R}}$, ... Given a preference profile $\mathbf{R} \in \mathcal{R}^N$, the ith coordinate of $\mathbf{R}$, which we denote by R_i, represents the preference relation on A of individual i in the preference profile $\mathbf{R}$. As usual, for each possible ordering $R_i \in \mathcal{R}$ of individual i, P_i and I_i denote the asymmetric and symmetric parts of R_i, respectively.

Definition 1: A *stochastic social choice function* (SSCF) is a function $F : A \times \mathcal{A} \times \mathcal{R}^N \rightarrow \Re_+$ which satisfies $\sum_{x \in B} F(x, B, \mathbf{R}) = \sum_{x \in A} F(x, B, \mathbf{R}) = 1 \quad \forall\, (B, \mathbf{R}) \in \mathcal{A} \times \mathcal{R}^N$.

Given a SSCF F, $(B, \mathbf{R}) \in \mathcal{A} \times \mathcal{R}^N$ and $x \in A$, we interpret $F(x, B, \mathbf{R})$ as the probability of x being chosen by the society when the feasible set is B and society's preference profile is $\mathbf{R}$. Thus, for each feasible set and each preference profile, a SSCF always assigns zero probability to any nonfeasible alternative..

Given any $(B, \mathbf{R}) \in \mathcal{A} \times \mathcal{R}^N$, we denote the set of all weakly Pareto optimal alternatives in B according to $\mathbf{R}$ by $WPAR(B, \mathbf{R})$; i.e.

$$WPAR(B, \mathbf{R}) \;=\; \{x \in B : \text{ there does not exist } y \in B \text{ such that } yP_ix \;\forall\, i \in N\}.$$

Given a SSCF F, let $POS(F, B, \mathbf{R})$ be the set of all alternatives that are assigned positive probabilities by F when the feasible set is $B \in \mathcal{A}$ and the preference profile is $\mathbf{R} \in \mathcal{R}^N$; i.e.

$$POS(F, B, \mathbf{R}) \;=\; \{x \in B : \; F(x, B, \mathbf{R}) > 0\}.$$

Definition 2: A SSCF F is *weakly Paretian ex-post* (WP) if and only if

$$POS(F, B, \mathbf{R}) \subseteq WPAR(B, \mathbf{R}) \quad \forall\, (B, \mathbf{R}) \in \mathcal{A} \times \mathcal{R}^N.$$

[3] Given any two sets D and E, we use the convention of letting $D - E = \{r \in D : r \notin E\}$.

[4] Given any set D, we follow the standard convention of denoting the number of elements in D by $|D|$.

Let $B \in \mathcal{A}$. For each $i \in N$ and each $R_i \in \mathcal{R}$, we denote the restriction of R_i to B by $R_i|B$. Similarly, for each $\mathbf{R} \in \mathcal{R}^N$, we denote the restriction of $\mathbf{R}$ to B by $\mathbf{R}|B = (R_1|B, ..., R_n|B)$. Also, we denote the set of all possible orderings on B by $\mathcal{R}|B$ and the n-fold Cartesian product of $\mathcal{R}|B$ by $\mathcal{R}^N|B$. Thus, for any $\mathbf{R} \in \mathcal{R}^N$, it is obvious that $R_i|B \in \mathcal{R}|B \ \forall\, i \in N$, and $\mathbf{R}|B \in \mathcal{R}^N|B$.

The following definition of *independence of irrelevant alternatives* (IIA), which is the appropriate counterpart of the one in the deterministic framework, is as given in PP.

Definition 3: A SSCF F satisfies IIA if and only if $\forall\, B \in \mathcal{A}$ and $\forall\, \mathbf{R}, \hat{\mathbf{R}} \in \mathcal{R}^N$

$$[\mathbf{R}|B = \hat{\mathbf{R}}|B] \implies [F(x, B, \mathbf{R}) = F(x, B, \hat{\mathbf{R}}) \ \forall\, x \in B].$$

The final condition we want to impose on a SSCF is *regularity*. This condition is a natural extension to the current framework of condition α of Chernoff [8], which is a minimum consistency condition for rationalizability of choice functions. For a more detailed discussion of this regularity condition the interested reader is referred to PP.

Definition 4: A SSCF F is *regular* if and only if $\forall\, B, \hat{B} \in \mathcal{A}$ and $\forall\, \mathbf{R} \in \mathcal{R}^N$

$$[x \in B \subseteq \hat{B}] \implies [F(x, B, \mathbf{R}) \geq F(x, \hat{B}, \mathbf{R})].$$

3 Coalitional Weights

In this section, corresponding to each SSCF F which satisfies certain conditions, we derive a unique nonnegative number for each coalition of individuals $S \in \mathcal{N}$ called the *weight* of coalition S according to the SSCF F. Given any partition of the society, the sum of these weights over all coalitions in this partition is equal to one. Although we carry out our analysis in a slightly different manner, these weights are essentially the same as those in PP.

Given a SSCF F which satisfies certain conditions, a coalition $S \in \mathcal{N}$ and any two pairs of alternatives (x, y) and (z, w), let $\mathbf{R}$ and $\hat{\mathbf{R}}$ be two preference profiles such that everyone in S prefers x to y and z to w according to their preferences in $\mathbf{R}$ and $\hat{\mathbf{R}}$ respectively, and everyone outside S prefers y to x and w to z according to their preferences in $\mathbf{R}$ and $\hat{\mathbf{R}}$ respectively. We then establish that the social probability of choosing x when $\{x, y\}$ is the feasible set and $\mathbf{R}$ is the preference profile must be the same as the social probability of choosing z when $\{z, w\}$ is the feasible set and $\hat{\mathbf{R}}$ is the preference profile. It is this social probability which is the weight of the coalition S according to the SSCF F. Thus, to some extent, the weight of a coalition S

according to a SSCF F indicates the power that coalition S has to influence the social choice probabilities. These weights play a key role in the characterization of the coalitional power structure.

We begin with a lemma which is the counterpart of Lemma 4.1 in PP. This lemma completely characterizes the condition under which the probability assigned to each alternative in a feasible set by a regular SSCF F remains unchanged when the feasible set is expanded.

Lemma 1 : *Suppose F is a regular* SSCF, *and $B, \hat{B} \in \mathcal{A}$ with $B \subseteq \hat{B}$. Then, for any $\mathbf{R} \in \mathcal{R}^N$, $POS(F, \hat{B}, \mathbf{R}) \subseteq B$ if and only if $F(x, B, \mathbf{R}) = F(x, \hat{B}, \mathbf{R}) \ \forall \ x \in B$.*

Proof: Let $B, \hat{B} \in \mathcal{A}$ with $B \subseteq \hat{B}$ and $\mathbf{R} \in \mathcal{R}^N$.

(*Necessity*): Suppose $POS(F, \hat{B}, \mathbf{R}) \subseteq B$. Then we have

$$1 = \sum_{x \in \hat{B}} F(x, \hat{B}, \mathbf{R}) = \sum_{x \in POS(F, \hat{B}, \mathbf{R})} F(x, \hat{B}, \mathbf{R}) \leq \sum_{x \in B} F(x, \hat{B}, \mathbf{R}) \leq 1.$$

Thus, $\sum_{x \in B} F(x, \hat{B}, \mathbf{R}) = 1 = \sum_{x \in B} F(x, B, \mathbf{R})$. But by regularity, $F(x, B, \mathbf{R}) \geq F(x, \hat{B}, \mathbf{R}) \ \forall \ x \in B$. Hence, $F(x, B, \mathbf{R}) = F(x, \hat{B}, \mathbf{R}) \ \forall \ x \in B$.

(*Sufficiency*): Suppose $F(x, B, \mathbf{R}) = F(x, \hat{B}, \mathbf{R}) \ \forall \ x \in B$. Then $1 = \sum_{x \in B} F(x, B, \mathbf{R}) = \sum_{x \in B} F(x, \hat{B}, \mathbf{R})$, which implies that $POS(F, \hat{B}, \mathbf{R}) \subseteq B$. ||

The next two lemmas consider the neutrality features of the stochastic social choice function.

Lemma 2 : *Let F be a regular* SSCF *that satisfies* WP *and* IIA, *and let $x, y, z \in A$ be three distinct alternatives. If $S \subseteq N$ and $\mathbf{R}, \hat{\mathbf{R}} \in \mathcal{R}^N$ are such that xP_iy and $x\hat{P}_iz \ \forall \ i \in S$, and yP_ix and $z\hat{P}_ix \ \forall \ i \in N - S$, then $F(x, \{x, y\}, \mathbf{R}) = F(x, \{x, z\}, \hat{\mathbf{R}})$.*

Proof: Let $x, y, z \in A$ be distinct. Suppose $S \subseteq N$ and $\mathbf{R}, \hat{\mathbf{R}} \in \mathcal{R}^N$ are such that xP_iy and $x\hat{P}_iz \ \forall \ i \in S$, and yP_ix and $z\hat{P}_ix \ \forall \ i \in N - S$. Consider $\tilde{\mathbf{R}} \in \mathcal{R}^N$ such that $x\tilde{P}_iy\tilde{P}_iz \ \forall \ i \in S$ and $y\tilde{P}_iz\tilde{P}_ix \ \forall \ i \in N - S$. WP implies that $F(z, \{x, y, z\}, \tilde{\mathbf{R}}) = 0$. Then we get

$$\begin{aligned}
F(x, \{x, z\}, \hat{\mathbf{R}}) &= F(x, \{x, z\}, \tilde{\mathbf{R}}) && \text{[by IIA]} \\
&\geq F(x, \{x, y, z\}, \tilde{\mathbf{R}}) && \text{[by regularity]} \\
&= F(x, \{x, y\}, \tilde{\mathbf{R}}) && \text{[by Lemma 1 and } F(z, \{x, y, z\}, \tilde{\mathbf{R}}) = 0] \\
&= F(x, \{x, y\}, \mathbf{R}) && \text{[by IIA]}\,.
\end{aligned}$$

By a similar argument, we can also conclude that $F(x, \{x, y\}, \mathbf{R}) \geq F(x, \{x, z\}, \hat{\mathbf{R}})$. Hence, $F(x, \{x, y\}, \mathbf{R}) = F(x, \{x, z\}, \hat{\mathbf{R}})$. ||

Lemma 3 *: Let F be a regular* SSCF *that satisfies* WP *and* IIA, *and let $x, y, z \in A$ be three distinct alternatives. If $S \subseteq N$ and $\mathbf{R}, \hat{\mathbf{R}} \in \mathcal{R}^N$ are such that xP_iy and $z\hat{P}_iy$ $\forall\ i \in S$, and yP_ix and $y\hat{P}_iz$ $\forall\ i \in N - S$, then $F(x, \{x, y\}, \mathbf{R}) = F(z, \{y, z\}, \hat{\mathbf{R}})$.*

Proof: Let $x, y, z \in A$ be distinct. Suppose $S \subseteq N$ and $\mathbf{R}, \hat{\mathbf{R}} \in \mathcal{R}^N$ are such that xP_iy and $z\hat{P}_iy$ $\forall\ i \in S$, and yP_ix and $y\hat{P}_iz$ $\forall\ i \in N - S$.

Using Lemma 2, we get $F(y, \{x, y\}, \mathbf{R}) = F(y, \{y, z\}, \hat{\mathbf{R}})$, which implies that

$$1 - F(y, \{x, y\}, \mathbf{R}) \;=\; 1 - F(y, \{y, z\}, \hat{\mathbf{R}}).$$

Hence, $F(x, \{x, y\}, \mathbf{R}) = F(z, \{y, z\}, \hat{\mathbf{R}})$. $\|$

Given a SSCF F, define the correspondence $\alpha_F : 2^N \rightarrow\rightarrow \Re_+$ as follows: for any $S \subseteq N$,

$$\alpha_F(S) = \left\{ \alpha \in \Re_+ : \begin{array}{l} \alpha = F(x, \{x, y\}, \mathbf{R}) \text{ for some distinct } x, y \in A \text{ and } \mathbf{R} \in \mathcal{R}^N \\ \text{such that } xP_iy\ \forall\ i \in S \text{ and } yP_ix\ \forall\ i \in N - S \end{array} \right\}.$$

If a SSCF F satisfies WP, then it is easy to check that

(1) $\quad \alpha_F(N) \;=\; \{1\}$, and $\alpha_F(\emptyset) \;=\; \{0\}$.

Also, given a SSCF F, it follows from the definition of α_F that

(2) $\quad$ for any $S \subseteq N$: $\quad [\alpha \in \alpha_F(S)] \iff [(1 - \alpha) \in \alpha_F(N - S)]$.

Now, given a SSCF F and a coalition $S \in \mathcal{N}$, consider any pair of distinct alternatives $x, y \in A$ and a preference profile $\mathbf{R} \in \mathcal{R}^N$ such that xP_iy $\forall\ i \in S$ and yP_ix $\forall\ i \in N - S$. As long as $F(x, \{x, y\}, \mathbf{R}) = F(x, \{x, y\}, \hat{\mathbf{R}})$ for every $\hat{\mathbf{R}} \in \mathcal{R}^N$ that is identical to $\mathbf{R}$ when restricted to $\{x, y\}$, which is the case if F satisfies IIA, the social probability of choosing x from the feasible set $\{x, y\}$ when $\mathbf{R}$ is the preference profile, $F(x, \{x, y\}, \mathbf{R})$, can be interpreted as the probability for coalition S to be almost decisive for x over y in the SSCF F. As a consequence, if $\alpha_F(S)$ contains a single nonnegative number, then the probability for coalition S to be almost decisive for some alternative over another alternative is invariant to the pair of distinct alternatives considered. Further, this probability is equal to the single number in $\alpha_F(S)$. Proposition 1 demonstrates that $\alpha_F(S)$ is a singleton under our assumptions.

Proposition 1 *: Suppose F is a regular* SSCF *that satisfies* WP *and* IIA. *Then $|\alpha_F(S)| = 1$ for each $S \subseteq N$.*

Proof: Clearly, $|\alpha_F(\emptyset)| = |\alpha_F(N)| = 1$.

Suppose $S \in \mathcal{N}$ such that $S \neq N$. Let $\hat{\mathbf{R}}, \tilde{\mathbf{R}} \in \mathcal{R}^N$ and $x, y, z, w \in A$ be such that $x\hat{P}_iy$ and $z\tilde{P}_iw$ $\forall\, i \in S$, and $y\hat{P}_ix$ and $w\tilde{P}_iz$ $\forall\, i \in N - S$. Then it is sufficient to show that $F(x, \{x, y\}, \hat{\mathbf{R}}) = F(z, \{z, w\}, \tilde{\mathbf{R}})$.

If $x = z$, then, by Lemma 2, $F(x, \{x, y\}, \hat{\mathbf{R}}) = F(z, \{z, w\}, \tilde{\mathbf{R}})$. So we suppose that $x \neq z$. As $m \geq 3$, let $v \in A$ be distinct from x and z. Now, consider $\mathbf{R}', \mathbf{R}'' \in \mathcal{R}^N$ such that $xP_i'z$ and $vP_i''zP_i''x$ $\forall\, i \in S$, and $zP_i'x$ and $xP_i''zP_i''v$ $\forall\, i \in N - S$. Then we get the following sequence of equalities: $F(x, \{x, y\}, \hat{\mathbf{R}}) = F(x, \{x, z\}, \mathbf{R}') = F(v, \{v, z\}, \mathbf{R}'') = F(v, \{v, x\}, \mathbf{R}'') = F(z, \{x, z\}, \mathbf{R}'') = F(z, \{z, w\}, \tilde{\mathbf{R}})$, where the first, third and last equalities are due to Lemma 2, and the second and fourth equalities are due to Lemma 3. Hence, we have $F(x, \{x, y\}, \hat{\mathbf{R}}) = F(z, \{z, w\}, \tilde{\mathbf{R}})$ as desired. ||

The significance of Proposition 1 is that, for every coalition $S \in \mathcal{N}$, the single number in $\alpha_F(S)$ is a good indicator of the power of coalition S, and hence, there is a possibility of using α_F to derive restrictions on the structure of coalitional power. It is worth noting that Proposition 1 can be viewed as a probabilistic version of the neutrality feature implied by the Arrow conditions in the deterministic framework.

Henceforth, we shall treat α_F as a function; i.e. for each $S \subseteq N$, $\alpha_F(S)$ is a nonnegative real number rather than a set containing a single nonnegative real number. We have the following straightforward corollary to Proposition 1.

Corollary 1 : *If a regular* SSCF F *satisfies* WP *and* IIA, *then* $\alpha_F(S) + \alpha_F(N - S) = 1$ *for each* $S \subseteq N$.

Proof: Follows from (2) and Proposition 1. ||

To characterize the coalitional power structure in terms of the function α_F it is essential that α_F be additive, so that, given any partition of the society, the sum of $\alpha_F(S)$ over all coalitions S in the partition is equal to one, the power of the grand coalition N. Before we show this additivity property of α_F, we need to introduce some notation and prove a preliminary proposition.

Given $B \in \mathcal{A}$, $i \in N$ and $R_i \in \mathcal{R}$, let

$$G_1(R_i|B) \;=\; \{x \in B : \; xR_iy \quad \forall\, y \in B\}; \quad \text{and}$$

$$G_p(R_i|B) = \left\{ x \in B - \bigcup_{q=1}^{p-1} G_q(R_i|B) : \; xR_iy \quad \forall\, y \in B - \bigcup_{q=1}^{p-1} G_q(R_i|B) \right\} \quad \forall\, p \geq 2.$$

Thus, $G_1(R_i|B)$ is the best set in B according to R_i, $G_2(R_i|B)$ is the second best set in B according to R_i and so on.

Given $x \in B \in \mathcal{A}$ and $\mathbf{R} \in \mathcal{R}^N$, let

$$\begin{aligned} \beta(\emptyset, \mathbf{R}|B) &= \emptyset, \\ \beta(S, \mathbf{R}|B) &= \bigcup_{i \in S} G_1(R_i|B) \quad \forall\, S \in \mathcal{N}, \quad \text{and} \\ L(x, B, \mathbf{R}) &= \{i \in N : \; x \in G_1(R_i|B)\}. \end{aligned}$$

So $\beta(S, \mathbf{R}|B)$ is the set of all alternatives that belong to the best set in B according to the preference in the profile $\mathbf{R}$ of at least one individual in the coalition S, and $L(x, B, \mathbf{R})$ is the set of all individuals who have x in their best sets in B according to their preferences in $\mathbf{R}$.

Consider a SSCF F that satisfies regularity, WP and IIA, and any feasible set B distinct from A. Suppose $x \in B$ and $\mathbf{R}$ is a preference profile such that $x \in G_1(R_j|B)$ for some individual j, and either x is the unique member of $G_1(R_i|B)$ or $x \notin G_1(R_i|B)$ for each individual i. Then the following proposition, which is in the spirit of Claim 4.7 of PP, shows that we can find some feasible set $\{z, w\}$ and a preference profile $\hat{\mathbf{R}}$ that satisfies $z\hat{P}_iw$ for each $i \in L(x, B, \mathbf{R})$ and $w\hat{P}_iz$ for each $i \in N - L(x, B, \mathbf{R})$ such that the social probability of choosing x from B, $F(x, B, \mathbf{R})$, is at least as large as that of choosing z from $\{z, w\}$, $F(z, \{z, w\}, \hat{\mathbf{R}})$.

Proposition 2 : *Suppose F is a regular* SSCF *that satisfies* WP *and* IIA. *If $x \in B \in \mathcal{A}$ and $\mathbf{R} \in \mathcal{R}^N$ are such that $B \neq A$ and $\beta(L(x, B, \mathbf{R}), \mathbf{R}|B) = \{x\}$, then $F(x, B, \mathbf{R}) \geq \alpha_F(L(x, B, \mathbf{R}))$.*

Proof: Suppose $x \in B \in \mathcal{A}$ and $\mathbf{R} \in \mathcal{R}^N$ are such that $B \neq A$ and $\beta(L(x, B, \mathbf{R}), \mathbf{R}|B) = \{x\}$. Let $y \in A - B$ and $\hat{B} = B \cup \{y\}$. Also, let $\hat{\mathbf{R}} \in \mathcal{R}^N$ be such that $\hat{\mathbf{R}}|B = \mathbf{R}|B$, $G_2(\hat{R}_i|\hat{B}) = \{y\}$ $\forall\, i \in L(x, B, \mathbf{R})$ and $G_1(\hat{R}_i|\hat{B}) = \{y\}$ $\forall\, i \in N - L(x, B, \mathbf{R})$. Then WP implies $F(z, \hat{B}, \hat{\mathbf{R}}) = 0$ $\forall\, z \in \hat{B} - \{x, y\}$. So, by Lemma 1, $F(x, \hat{B}, \hat{\mathbf{R}}) = F(x, \{x, y\}, \hat{\mathbf{R}}) = \alpha_F(L(x, B, \mathbf{R}))$. Because of regularity and IIA, we also have $F(x, \hat{B}, \hat{\mathbf{R}}) \leq F(x, B, \hat{\mathbf{R}}) = F(x, B, \mathbf{R})$. Hence, we have $F(x, B, \mathbf{R}) \geq \alpha_F(L(x, B, \mathbf{R}))$ as desired. ||

We are now ready to formally state our desired result in the form of Proposition 3, which is similar to Claim 4.8 of PP. This proposition shows that, if F is a regular SSCF that satisfies WP and IIA, then α_F is a subadditive function, which is additive whenever there are four or more social alternatives.

Proposition 3 : *Let F be a regular* SSCF *that satisfies* WP *and* IIA,*and suppose $S, T \in \mathcal{N}$ are disjoint. Then $\alpha_F(S) + \alpha_F(T) \geq \alpha_F(S \cup T)$. If $m \geq 4$, then $\alpha_F(S) + \alpha_F(T) = \alpha_F(S \cup T)$.*

Proof: Let $S, T \in \mathcal{N}$ be disjoint. The proposition is trivially true if $S \cup T = N$. So we suppose $S \cup T \neq N$. Let $x, y, z \in A$ be distinct. Also, consider $\mathbf{R} \in \mathcal{R}^N$ such that $xP_iyP_iz \; \forall \; i \in S$, $yP_izP_ix \; \forall \; i \in T$, and $zP_ixP_iy \; \forall \; i \in N - (S \cup T)$. Then we have the following:

$$
\begin{aligned}
\alpha_F(S) + \alpha_F(T) &= F(x, \{x, z\}, \mathbf{R}) + F(y, \{x, y\}, \mathbf{R}) \\
&\geq F(x, \{x, y, z\}, \mathbf{R}) + F(y, \{x, y, z\}, \mathbf{R}) && \text{[by regularity]} \\
&= 1 - F(z, \{x, y, z\}, \mathbf{R}) \\
&\geq 1 - F(z, \{y, z\}, \mathbf{R}) && \text{[by regularity]} \\
&= 1 - \alpha_F(N - (S \cup T)) \\
&= \alpha_F(S \cup T) && \text{[by Corollary 1].}
\end{aligned}
$$

Now, suppose $m \geq 4$. Then we have $\{x, y, z\} \neq A$, $L(x, \{x, y, z\}, \mathbf{R}) = S$, $L(y, \{x, y, z\}, \mathbf{R}) = T$, $L(z, \{x, y, z\}, \mathbf{R}) = N - (S \cup T)$, $\beta(S, \mathbf{R}|\{x, y, z\}) = \{x\}$, $\beta(T, \mathbf{R}|\{x, y, z\}) = \{y\}$ and $\beta(N - (S \cup T), \mathbf{R}|\{x, y, z\}) = \{z\}$. So, using Proposition 2 and regularity, we get

$$F(x, \{x, y, z\}, \mathbf{R}) \;\geq\; \alpha_F(S) \;=\; F(x, \{x, z\}, \mathbf{R}) \;\geq\; F(x, \{x, y, z\}, \mathbf{R}).$$

Hence, $F(x, \{x, y, z\}, \mathbf{R}) = \alpha_F(S)$. Then, using a similar argument, we can also conclude that $F(y, \{x, y, z\}, \mathbf{R}) = \alpha_F(T)$ and $F(z, \{x, y, z\}, \mathbf{R}) = \alpha_F(N - (S \cup T))$. Therefore,

$$
\begin{aligned}
\alpha_F(S) + \alpha_F(T) &= F(x, \{x, y, z\}, \mathbf{R}) + F(y, \{x, y, z\}, \mathbf{R}) \\
&= 1 - F(z, \{x, y, z\}, \mathbf{R}) \\
&= 1 - \alpha_F(N - (S \cup T)) \\
&= \alpha_F(S \cup T) && \text{[by Corollary 1].} \quad \|
\end{aligned}
$$

Thus, given an universal set with at least four social alternatives and any regular SSCF F that satisfies WP and IIA, it follows from Proposition 3 that $\alpha_F(S) = \sum_{i \in S} \alpha_F(\{i\}) \; \forall \; S \in \mathcal{N}$. Further, if $\{S^q\}_{q=1}^{Q}$ is a partition of the society,[5] then the sum of $\alpha_F(S^q)$ over all coalitions S^q in the partition $\{S^q\}_{q=1}^{Q}$ is equal to one. So, for each coalition of individuals $S \in \mathcal{N}$, we refer to $\alpha_F(S)$ as the *weight* of the coalition S according to F.

We conclude this section with the following corollary, which implies the weighted random dictatorship result of PP (Theorem 4.11).

[5] So $\{S^q\}_{q=1}^{Q}$ is such that $S^q \subseteq N \; \forall \; q = 1, ..., Q$, $S^1 \cup ... \cup S^Q = N$ and $S^{q'} \cap S^{q''} = \emptyset$ for $1 \leq q' < q'' \leq Q$.

Corollary 2 : *Suppose $m \geq 4$. Let F be a regular* SSCF *that satisfies* WP *and* IIA. *If $B \in \mathcal{A}$ and $\mathbf{R} \in \mathcal{R}^N$ are such that $B \neq A$ and $|G_1(R_i|B)| = 1 \; \forall \; i \in N$, then $F(x, B, \mathbf{R}) = \alpha_F(L(x, B, \mathbf{R})) \; \forall \; x \in \beta(N, \mathbf{R}|B)$ and $F(x, B, \mathbf{R}) = 0 \; \forall \; x \in B - \beta(N, \mathbf{R}|B)$.*

Proof: Suppose $B \in \mathcal{A}$ and $\mathbf{R} \in \mathcal{R}^N$ are such that $B \neq A$ and $|G_1(R_i|B)| = 1 \; \forall \; i \in N$. Then $\beta(L(x, B, \mathbf{R}), \mathbf{R}|B) = \{x\} \; \forall \; x \in \beta(N, \mathbf{R}|B)$. So, using Proposition 2, we get

$$(3) \qquad F(x, B, \mathbf{R}) \;\geq\; \alpha_F(L(x, B, \mathbf{R})) \quad \forall \; x \in \beta(N, \mathbf{R}|B).$$

Clearly, $\{L(x, B, \mathbf{R})\}_{x \in \beta(N, \mathbf{R}|B)}$ is a partition of N. Therefore, Proposition 3 implies that

$$\sum_{x \in \beta(N, \mathbf{R}|B)} \alpha_F(L(x, B, \mathbf{R})) \;=\; \alpha_F(N) \;=\; 1.$$

Now, because of (3) and the last equation, we get

$$(4) \qquad 1 \;\geq\; \sum_{x \in \beta(N, \mathbf{R}|B)} F(x, B, \mathbf{R}) \;\geq\; \sum_{x \in \beta(N, \mathbf{R}|B)} \alpha_F(L(x, B, \mathbf{R})) \;=\; 1.$$

Then (3) in conjunction with (4) imply $F(x, B, \mathbf{R}) = \alpha_F(L(x, B, \mathbf{R})) \; \forall \; x \in \beta(N, \mathbf{R}|B)$, and $F(x, B, \mathbf{R}) = 0 \; \forall \; x \in B - \beta(N, \mathbf{R}|B)$. $\quad ||$

4 The Structure of SSCFs

Suppose there are at least four alternatives in the universal set A. Let B be a feasible proper subset of A, and let $\mathbf{R}$ be a preference profile. If the individual preferences in $\mathbf{R}$ are strict, then $\{L(x, B, \mathbf{R})\}_{x \in \beta(N, \mathbf{R}|B)}$ defines a partition of the society that satisfies $\beta(L(x, B, \mathbf{R}), \mathbf{R}|B) = \{x\}$ for each $x \in \beta(N, \mathbf{R}|B)$. Using this partition, one can derive the weighted random dictatorship result in PP from the structure of coalitional power for two-element feasible sets. But such a partition of the society may no longer exist once the individual preferences in $\mathbf{R}$ are not restricted to be strict. However, using two key properties of the partition $\{L(x, B, \mathbf{R})\}_{x \in \beta(N, \mathbf{R}|B)}$ when individual preferences in $\mathbf{R}$ are strict, we can generate a partition of the society which can be used to derive the structure of coalitional power for nonbinary choice from the restrictions for two-element feasible sets derived in the previous section. These two key properties of the partition $\{L(x, B, \mathbf{R})\}_{x \in \beta(N, \mathbf{R}|B)}$ when individual preferences in $\mathbf{R}$ are strict are: (i) $\beta(L(x, B, \mathbf{R}), \mathbf{R}|B) \cap \beta(N - L(x, B, \mathbf{R}), \mathbf{R}|B) = \emptyset$ for every $x \in \beta(N, \mathbf{R}|B)$; and (ii) for each $x \in \beta(N, \mathbf{R}|B)$, if $L(x, B, \mathbf{R})$ has more than one member and $i \in L(x, B, \mathbf{R})$, then there is some other member $j \in L(x, B, \mathbf{R})$ such that $G_1(R_i|B) \cap G_1(R_j|B) \neq \emptyset$.

Thus, given A, B and $\mathbf{R}$ as above, let $S \in \mathcal{N}$ be a coalition in the partition of the society which satisfies the above mentioned two properties. So there is nothing in common between the best sets in B of any two individuals according to their preferences in the profile $\mathbf{R}$ if only one of them belongs to the coalition S. Also, if the coalition S has more than one member, then every individual in S has some best feasible alternative according to her preference in $\mathbf{R}$ which is also a best feasible alternative according to the preference in $\mathbf{R}$ of some other individual in S. Our main result shows that, for S satisfying the above properties, the social probability of choosing one of the alternatives from the set $\beta(S, \mathbf{R}|B)$ when B is the feasible set and $\mathbf{R}$ is the social preference profile is equal to the weight of the coalition S.

The steps we use to prove our result are as follows: (i) we first establish our result when $m = 4$; (ii) the next crucial step shows that, for any feasible proper subset of A and any preference profile, the social probability of choosing an alternative which is a best feasible alternative for no one in the society is zero; and (iii) finally, using an induction argument, we prove our result for the more general $m \geq 4$ case.

Given any $B \in \mathcal{A}$ and any $\mathbf{R} \in \mathcal{R}^N$, let $\mathcal{P}(B, \mathbf{R})$ be the unique partition of N such that:[6]

(P1) if $S, T \in \mathcal{P}(B, \mathbf{R})$ and $S \neq T$, then $\beta(S, \mathbf{R}|B) \cap \beta(T, \mathbf{R}|B) = \emptyset$; and

(P2) if $i \in S \in \mathcal{P}(B, \mathbf{R})$ and $|S| \geq 2$, then $G_1(R_i|B) \cap G_1(R_j|B) \neq \emptyset$ for some $j \in S - \{i\}$.

Thus, $\mathcal{P}(B, \mathbf{R})$ is the partition of the society which satisfies the two important properties discussed above.

The first proposition in this section is our result on the structure of coalitional power for the case $m = 4$. As with most impossibility results, the proof exploits the fact that, given any feasible proper subset B of the universal set A and a preference profile $\mathbf{R}$, there are sufficient degrees of freedom to choose another preference profile $\hat{\mathbf{R}}$ such that both $\mathbf{R}$ and $\hat{\mathbf{R}}$ are the same on B and $\hat{\mathbf{R}}$ also has certain desirable properties on some other subsets of A. The new profile $\hat{\mathbf{R}}$ is then used to derive some restrictions on the social choice probabilities corresponding to some feasible sets distinct from B. Then, by invoking IIA and regularity, these restictions are shown to imply the desired restictions when B is the feasible set and $\mathbf{R}$ is the preference profile.

Proposition 4 : *Suppose* $m = 4$. *Let* F *be a regular* SSCF *that satisfies* WP *and* IIA. *If* $B \in \mathcal{A}$ *but* $B \neq A$, *and* $\mathbf{R} \in \mathcal{R}^N$, *then* $\sum_{x \in \beta(S, \mathbf{R}|B)} F(x, B, \mathbf{R}) = \alpha_F(S)\ \forall\ S \in \mathcal{P}(B, \mathbf{R})$.

[6] It can be easily checked that $\mathcal{P}(B, \mathbf{R})$ is uniquely determined by (P1) and (P2).

Proof: Let $B \in \mathcal{A}$ but $B \neq A$, and $\mathbf{R} \in \mathcal{R}^N$. Then we have the following exhaustive list of possibilities: **(a)** $|G_1(R_i|B)| = 1 \ \forall\ i \in N$; **(b)** $|B| = 2$ and $|G_1(R_j|B)| = 2$ for some $j \in N$; **(c)** $|B| = 3$, $|\beta(N, \mathbf{R}|B)| = 2$ and $|G_1(R_k|B)| = 2$ for some $k \in N$; and **(d)** $|B| = 3$ and $\beta(N, \mathbf{R}|B) = B$.

(a) $|G_1(R_i|B)| = 1 \ \forall\ i \in N$: Corollary 2 implies the proposition in this case.

(b) $|B| = 2$ and $|G_1(R_j|B)| = 2$ for some $j \in N$: Then $\mathcal{P}(B, \mathbf{R}) = \{N\}$, and the proposition holds.

(c) $|B| = 3$, $|\beta(N, \mathbf{R}|B)| = 2$ and $|G_1(R_k|B)| = 2$ for some $k \in N$: Then $\mathcal{P}(B, \mathbf{R}) = \{N\}$. Let $\beta(N, \mathbf{R}|B) = \{x, y\}$, $B - \beta(N, \mathbf{R}|B) = \{z\}$ and $A - B = \{w\}$. Also, let $\{N_1, N_2, N_3\}$ be the partition of N such that $G_1(R_i|B) = \{x, y\} \ \forall\ i \in N_1$, $G_1(R_i|B) = \{x\} \ \forall\ i \in N_2$ and $G_1(R_i|B) = \{y\} \ \forall\ i \in N_3$. Now, consider $\hat{\mathbf{R}} \in \mathcal{R}^N$ such that $\hat{\mathbf{R}}|B = \mathbf{R}|B$, $G_2(\hat{R}_i|A) = \{w\} \ \forall\ i \in N_1 \cup N_2$, and $a\hat{P}_i w \ \forall\ a \in B$ if $i \in N_3$. Clearly, as $x\hat{P}_i w \ \forall\ i \in N$, $w \notin WPAR(A, \hat{\mathbf{R}})$. So WP implies $F(w, A, \hat{\mathbf{R}}) = 0$. Let $\hat{B} = \{y, z, w\}$. Then it can be verified that $G_1(\hat{R}_i|\hat{B}) = \{y\} \ \forall\ i \in N_1 \cup N_3$, $G_1(\hat{R}_i|\hat{B}) = \{w\} \ \forall\ i \in N_2$, and $\mathcal{P}(\hat{B}, \hat{\mathbf{R}}) = \{N_1 \cup N_3, N_2\}$. Hence, using Corollaries 1 and 2, we get $F(y, \hat{B}, \hat{\mathbf{R}}) + F(w, \hat{B}, \hat{\mathbf{R}}) = \alpha_F(N_1 \cup N_3) + \alpha_F(N_2) = 1$, which implies $F(z, \hat{B}, \hat{\mathbf{R}}) = 0$. Then, because of regularity, $F(z, A, \hat{\mathbf{R}}) = 0$. But we also know that $F(w, A, \hat{\mathbf{R}}) = 0$. So we have $F(x, A, \hat{\mathbf{R}}) + F(y, A, \hat{\mathbf{R}}) = 1$. Then, using regularity once more, we get $F(x, B, \hat{\mathbf{R}}) + F(y, B, \hat{\mathbf{R}}) = 1$. Therefore, as $\mathbf{R}|B = \hat{\mathbf{R}}|B$, IIA implies $F(x, B, \mathbf{R}) + F(y, B, \mathbf{R}) = 1$. Hence, $\sum_{a \in \beta(N, \mathbf{R}|B)} F(a, B, \mathbf{R}) = 1 = \alpha_F(N)$.

(d) $|B| = 3$ and $\beta(N, \mathbf{R}|B) = B$: If $\mathcal{P}(B, \mathbf{R}) = \{N\}$, then the proposition is obvious. So we suppose $\mathcal{P}(B, \mathbf{R}) \neq \{N\}$. As the proposition is true in case of possibility **(a)**, we only need to consider the case where $|\beta(S, \mathbf{R}|B)| = 2$ for some $S \in \mathcal{P}(B, \mathbf{R})$. Then it can be easily checked that $\mathcal{P}(B, \mathbf{R}) = \{S, N - S\}$, $|\beta(S, \mathbf{R}|B)| = 2$ and $|\beta(N - S, \mathbf{R}|B)| = 1$. So let $\beta(S, \mathbf{R}|B) = \{x, y\}$ and $\beta(N - S, \mathbf{R}|B) = \{z\}$. Then, because of Corollary 1, it is sufficient to show that $F(z, B, \mathbf{R}) = \alpha_F(N - S)$. However, as Proposition 2 implies $F(z, B, \mathbf{R}) \geq \alpha_F(N - S)$, the proof is complete if we show that $F(z, B, \mathbf{R}) \leq \alpha_F(N - S)$. Now, as before, let $\{w\} = A - B$. Also, let $\tilde{\mathbf{R}} \in \mathcal{R}^N$ be such that $\tilde{\mathbf{R}}|B = \mathbf{R}|B$, $G_2(\tilde{R}_i|A) = \{w\} \ \forall\ i \in N$ such that $G_1(R_i|B) \cap \{x\} \neq \emptyset$, and $x\tilde{P}_i w$ and $y\tilde{P}_i w \ \forall\ i \in N$ such that $G_1(R_i|B) \cap \{x\} = \emptyset$. Clearly, as $x\tilde{P}_i w \ \forall\ i \in N$, $w \notin WPAR(A, \tilde{\mathbf{R}})$. So WP implies $F(w, A, \tilde{\mathbf{R}}) = 0$. Next, let $\tilde{B} = \{y, z, w\}$. Then it can be verified that $|G_1(\tilde{R}_i|\tilde{B})| = 1 \ \forall\ i \in N$ and $L(z, \tilde{B}, \tilde{\mathbf{R}}) = N - S$. So, using regularity and Corollary 2, we get $F(z, A, \tilde{\mathbf{R}}) \leq F(z, \tilde{B}, \tilde{\mathbf{R}}) = \alpha_F(N - S)$. Therefore, as we already know that $F(w, A, \tilde{\mathbf{R}}) = 0$, Corollary 1 implies that $F(x, A, \tilde{\mathbf{R}}) + F(y, A, \tilde{\mathbf{R}}) = 1 - F(z, A, \tilde{\mathbf{R}}) \geq 1 - \alpha_F(N - S) = \alpha_F(S)$. Then, because of regularity, we get $F(x,B,\tilde{\mathbf{R}}) + F(y,B,\tilde{\mathbf{R}}) \geq \alpha_F(S)$. So, as $\tilde{\mathbf{R}}|B = \mathbf{R}|B$, IIA implies $F(x,B,\mathbf{R}) + F(y,B,\mathbf{R}) \geq \alpha_F(S)$. Hence, using Corollary 1 once more, we get $F(z,B,\mathbf{R}) \leq 1 - \alpha_F(S) = \alpha_F(N-S)$.

This completes the proof of Proposition 4. $\|$

Given a SSCF F that satisfies IIA and any $B \in \mathcal{A}$, let $F|_B : B \times (2^B - \{\emptyset\}) \times \mathcal{R}^N|B \to \Re_+$ be the restriction of F to nonempty subsets of B; i.e. for any $\hat{B} \in 2^B - \{\emptyset\}$ and any $\hat{\mathbf{R}}|B \in \mathcal{R}^N|B$:

$$F|_B(x,\hat{B},\hat{\mathbf{R}}|B) = F(x,\hat{B},\mathbf{R}) \quad \forall\, x \in B \text{ and any } \mathbf{R} \in \mathcal{R}^N \text{ such that } \hat{\mathbf{R}}|B = \mathbf{R}|B.$$

It can be easily checked that, if F satisfies regularity, WP and IIA, then $F|_B$ also satisfies regularity, WP and IIA for any $B \in \mathcal{A}$. This observation is important for the proof of our next proposition.

The final result required for the proof of the main result is Proposition 5. The proof uses a simple induction logic. Depending on whether the feasible set B has two or more fewer alternatives, or exactly one fewer alternative than the universal set, the proof has two parts. In the first case, we consider the restriction of the SSCF to a superset of B with exactly one alternative less than in the universal set and exploit the observation made above about the properties it inherits from the original SSCF. In the later case, as in the proof of Proposition 4, we rely on the amount of freedom available to choose a new preference profile with certain desirable properties, and then appeal to IIA and regularity.

Proposition 5 *: Suppose $m \geq 4$. Let F be a regular* SSCF *that satisfies* WP *and* IIA. *If $B \in \mathcal{A}$ but $B \neq A$, and $\mathbf{R} \in \mathcal{R}^N$, then $F(x,B,\mathbf{R}) = 0\ \forall\, x \in B - \beta(N,\mathbf{R}|B)$.*

Proof: Because of Proposition 4, the proposition is true for $m = 4$. Then, using an induction argument, the proof is complete if we show that the proposition holds for $m = t+1$ whenever it holds for $m = t \geq 4$ ($t < \infty$). So let us suppose that the proposition holds for $m = t \geq 4$ ($t < \infty$). Consider A such that $|A| = m = t+1$. Let $B \in \mathcal{A}$ but $B \neq A$, and $\mathbf{R} \in \mathcal{R}^N$. Then there are two possibilities: **(a)** $|B| \leq t-1$; and **(b)** $|B| = t$.

(a) $|B| \leq t-1$: Clearly, there exists $B^o \in \mathcal{A}$ such that $|B^o| = t$ and $B \subset B^o$. Then, as the proposition is true for $m = t$, it must be the case that $F|_{B^o}(a,B,\mathbf{R}|B^o) = 0\ \forall\, a \in B - \beta(N,\mathbf{R}|B)$. Hence, $F(a,B,\mathbf{R}) = 0\ \forall\, a \in B - \beta(N,\mathbf{R}|B)$.

(b) $|B| = t$: Let $x^* \in B - \beta(N,\mathbf{R}|B)$. Also let $\{w\} = A - B$. Then we must show that $F(x^*,B,\mathbf{R}) = 0$. If $|G_1(R_i|B)| = 1\ \forall\, i \in N$, then, because of Corollary 2, the proposition is true. So we suppose $|G_1(R_j|B)| \geq 2$ for some $j \in N$. Let $\hat{x}, \hat{y} \in G_1(R_j|B)$ be distinct. Then $B - \{\hat{x}, \hat{y}, x^*\} \neq \emptyset$ as $|B| = t \geq 4$. So let $\hat{z} \in B - \{\hat{x}, \hat{y}, x^*\}$, and define $\hat{B} = (B - \{x^*, \hat{z}\}) \cup \{w\}$ and $\hat{\hat{B}} = A - \{\hat{x}, \hat{y}\}$. Now, consider $\hat{\mathbf{R}} \in \mathcal{R}^N$ such that $\hat{\mathbf{R}}|B = \mathbf{R}|B$, $G_2(\hat{R}_i|A) = \{w\}$ if $i \in N$ is such that $\{\hat{x}, \hat{y}\} \cap G_1(R_i|B) \neq \emptyset$, and $a\hat{P}_i w \; \forall \; a \in B$ if $i \in N$ is such that $\{\hat{x}, \hat{y}\} \cap G_1(R_i|B) = \emptyset$. Then it is easily verified that $|\hat{B}| = |\hat{\hat{B}}| = t - 1$, $w \in \hat{B} - \beta(N, \hat{\mathbf{R}}|\hat{B})$ and $x^* \in \hat{\hat{B}} - \beta(N, \hat{\mathbf{R}}|\hat{\hat{B}})$. So, using the conclusion drawn in possibility (a) above, we get $F(w, \hat{B}, \hat{\mathbf{R}}) = 0$ and $F(x^*, \hat{\hat{B}}, \hat{\mathbf{R}}) = 0$. Then regularity implies $F(w, A, \hat{\mathbf{R}}) = 0$ and $F(x^*, A, \hat{\mathbf{R}}) = 0$. Therefore, $\sum_{a \in A - \{x^*, w\}} F(a, A, \hat{\mathbf{R}}) = 1$. But, clearly, $A - \{x^*, w\} = B - \{x^*\} \subset B \subset A$. So, using regularity, we get $\sum_{a \in B - \{x^*\}} F(a, B, \hat{\mathbf{R}})$ $= 1$. Then $\hat{\mathbf{R}}|B = \mathbf{R}|B$ and IIA implies $\sum_{a \in B - \{x^*\}} F(a, B, \mathbf{R}) = 1$. Hence, $F(x^*, B, \mathbf{R}) = 0$.

This completes the proof of Proposition 5. ||

We are now ready to present our characterization of the structure of coalitional power when the universal set has four or more alternatives. This theorem states that the distribution of power under a SSCF F, which satisfies regularity, WP and IIA, is such that, given a feasible proper subset B of the universal set and a preference profile $\mathbf{R}$, for each coalition S in the partition $\mathcal{P}(B, \mathbf{R})$, the social probability of choosing an alternative that is a best feasible alternative for *some* member of S is equal to the weight of S.

Theorem: *Suppose $m \geq 4$. Let F be a regular* SSCF *that satisfies* WP *and* IIA. *If $B \in \mathcal{A}$ but $B \neq A$, and $\mathbf{R} \in \mathcal{R}^N$, then $\sum_{x \in \beta(S, \mathbf{R}|B)} F(x, B, \mathbf{R}) = \alpha_F(S) \; \forall \; S \in \mathcal{P}(B, \mathbf{R})$.*

Proof: We have already proved the theorem for $m = 4$ in Proposition 4. So, if we prove the theorem for $m = t + 1$ whenever it holds for $m = t \geq 4$ $(t < \infty)$, an induction argument completes the proof of the theorem for any finite $m \geq 4$.

Suppose the theorem holds for $m = t \geq 4$ $(t < \infty)$. Consider A such that $|A| = m = t + 1$. Let $B \in \mathcal{A}$ but $B \neq A$, and $\mathbf{R} \in \mathcal{R}^N$. By Proposition 5, $\sum_{a \in \beta(N, \mathbf{R}|B)} F(a, B, \mathbf{R}) = 1 = \alpha_F(N)$. So we suppose $\mathcal{P}(B, \mathbf{R}) \neq \{N\}$. Let $w \in A - B$. Now pick any $\hat{S} \in \mathcal{P}(B, \mathbf{R})$, and define $\bar{B}$ $= B \cup \{w\}$, $\hat{B} = \bar{B} - \beta(\hat{S}, \mathbf{R}|B)$ and $\tilde{B} = \bar{B} - \beta(N - \hat{S}, \mathbf{R}|B)$. Consider $\hat{\mathbf{R}} \in \mathcal{R}^N$ such that $\hat{\mathbf{R}}|B = \mathbf{R}|B$, $G_2(\hat{R}_i|\bar{B}) = \{w\}$ if $i \in N - \hat{S}$, and $a\hat{P}_i w \; \forall \; a \in B$ if $i \in \hat{S}$. Then it is obvious that $\hat{B} \neq A$ and $w \in \hat{B} - \beta(N, \hat{\mathbf{R}}|\hat{B})$. So Proposition 5 implies that $F(w, \hat{B}, \hat{\mathbf{R}}) = 0$. Therefore, using regularity, we get

$$(5) \quad F(w, \bar{B}, \hat{\mathbf{R}}) = 0.$$

It can also be verified that $\tilde{B} \neq A$, $L(w, \bar{B}, \hat{\mathbf{R}}) = N - \hat{S}$, and $\beta(N - \hat{S}, \hat{\mathbf{R}}|\tilde{B}) = \{w\}$. Then

Proposition 2 implies that $F(w, \bar{B}, \hat{\mathbf{R}}) \geq \alpha_F(N - \hat{S})$. So, because of Corollary 1 and Proposition 5, $\sum_{a\in\beta(N,\hat{\mathbf{R}}|\bar{B})-\{w\}} F(a, \bar{B}, \hat{\mathbf{R}}) \leq \alpha_F(\hat{S})$ must hold. Then $\sum_{a\in\beta(N,\hat{\mathbf{R}}|\bar{B})-\{w\}} F(a, \bar{B}, \hat{\mathbf{R}}) \leq \alpha_F(\hat{S})$ follows from regularity, which together with (5) gives

$$(6) \qquad \sum_{a\in\beta(N,\hat{\mathbf{R}}|\bar{B})} F(a, \bar{B}, \hat{\mathbf{R}}) \;\leq\; \alpha_F(\hat{S}).$$

Then, as $\bar{B} - \beta(N, \hat{\mathbf{R}}|\bar{B}) = B - \beta(\hat{S}, \mathbf{R}|B)$, (6), Corollary 1 and regularity imply that

$$(7) \qquad \sum_{a\in B-\beta(\hat{S},\mathbf{R}|B)} F(a, B, \hat{\mathbf{R}}) \;\geq\; \sum_{a\in B-\beta(\hat{S},\mathbf{R}|B)} F(a, \bar{B}, \hat{\mathbf{R}}) \;\geq\; \alpha_F(N - \hat{S}).$$

Using $\hat{\mathbf{R}}|B = \mathbf{R}|B$ and IIA in (7), we then get $\sum_{a\in B-\beta(\hat{S},\mathbf{R}|B)} F(a, B, \mathbf{R}) \geq \alpha_F(N - \hat{S})$, which in conjunction with Corollary 1 gives $\sum_{a\in\beta(\hat{S},\mathbf{R}|B)} F(a, B, \mathbf{R}) \leq \alpha_F(\hat{S})$. But $\hat{S}$ was arbitrarily chosen from $\mathcal{P}(B, \mathbf{R})$. Therefore, we can conclude that

$$(8) \qquad \sum_{a\in\beta(S,\mathbf{R}|B)} F(a, B, \mathbf{R}) \;\leq\; \alpha_F(S) \qquad \forall\, S \in \mathcal{P}(B, \mathbf{R}).$$

Then, as $\mathcal{P}(B, \mathbf{R})$ is a partition of N, it follows from (8) and Propositions 3 and 5 that

$$1 \;=\; \sum_{a\in\beta(N,\mathbf{R}|B)} F(a, B, \mathbf{R}) \;=\; \sum_{S\in\mathcal{P}(B,\mathbf{R})} \sum_{a\in\beta(S,\mathbf{R}|B)} F(a, B, \mathbf{R}) \;\leq\; \sum_{S\in\mathcal{P}(B,\mathbf{R})} \alpha_F(S) \;=\; 1.$$

Hence, (8) must hold as an equality for every $S \in \mathcal{P}(B, \mathbf{R})$. This completes the proof of the theorem. ||

Given any $S \in \mathcal{N}$, let

$$\mathcal{F}_S \;=\; \left\{F:\; \textstyle\sum_{x\in\beta(S,\mathbf{R}|B)} F(x, B, \mathbf{R}) = 1 \quad \forall\, (B, \mathbf{R}) \in \mathcal{A} \times \mathcal{R}^N\right\}.$$

$\mathcal{F}_S$ can be interpreted as the class of all SSCFs that give oligarchic power to the coalition S in the following sense – given any feasible set B and any preference profile $\mathbf{R}$, an alternative which is not best in B for anyone in the coalition S according to their preferences in $\mathbf{R}$ has zero probability of being the socially chosen feasible alternative. Clearly, for each $i \in N$, $\mathcal{F}_{\{i\}}$ is the appropriate stochastic counterpart to the class of deterministic social choice functions in which individual i is a weak dictator.[7]

The next two straightforward corollaries of our main result essentially show that, under the conditions of the Theorem, a SSCF can be interpreted as a weighted random oligarchy or as a weighted random dictatorship. The weighted random dictatorship result of PP (Theorem 4.11) can also be viewed as a special case of Corollary 4 given below.

[7]When we restrict individual preference relations to the set of all linear orderings on A, $\mathcal{F}_{\{i\}}$ is equivalent to the class of decision schemes in which individual i is a dictator as defined in PP.

Corollary 3 : *Suppose $m \geq 4$, and F is a regular* SSCF *that satisfies* WP *and* IIA. *Let $B \in \mathcal{A}$ but $B \neq A$, and $\mathbf{R} \in \mathcal{R}^N$. If $\{d_T\}_{T \in \mathcal{P}(B,\mathbf{R})}$ is a set of* SSCFs *such that $d_T \in \mathcal{F}_T$ for each $T \in \mathcal{P}(B,\mathbf{R})$, then*

$$\sum_{x \in \beta(S,\mathbf{R}|B)} F(x,B,\mathbf{R}) = \sum_{x \in \beta(S,\mathbf{R}|B)} \sum_{T \in \mathcal{P}(B,\mathbf{R})} \alpha_F(T) d_T(x,B,\mathbf{R}) \quad \forall\, S \in \mathcal{P}(B,\mathbf{R}).$$

Proof: Suppose $B \in \mathcal{A}$ but $B \neq A$, and $\mathbf{R} \in \mathcal{R}^N$. Let $\{d_T\}_{T \in \mathcal{P}(B,\mathbf{R})}$ be such that $d_T \in \mathcal{F}_T$ for each $T \in \mathcal{P}(B,\mathbf{R})$. Also, let $S \in \mathcal{P}(B,\mathbf{R})$. Then we have

$$(9) \quad \sum_{x \in \beta(S,\mathbf{R}|B)} \sum_{T \in \mathcal{P}(B,\mathbf{R})} \alpha_F(T) d_T(x,B,\mathbf{R}) = \sum_{T \in \mathcal{P}(B,\mathbf{R})} \left[\alpha_F(T) \sum_{x \in \beta(S,\mathbf{R}|B)} d_T(x,B,\mathbf{R}) \right].$$

By definition, $\beta(S,\mathbf{R}|B) \cap \beta(\hat{S},\mathbf{R}|B) = \emptyset$ and $\sum_{x \in \beta(\hat{S},\mathbf{R}|B)} d_{\hat{S}}(x,B,\mathbf{R}) = 1 \;\forall\, \hat{S} \in \mathcal{P}(B,\mathbf{R}) - \{S\}$. So $\sum_{x \in \beta(S,\mathbf{R}|B)} d_{\hat{S}}(x,B,\mathbf{R}) = 0 \;\forall\, \hat{S} \in \mathcal{P}(B,\mathbf{R}) - \{S\}$. Hence, (9) can be rewritten as

$$(10) \quad \sum_{x \in \beta(S,\mathbf{R}|B)} \sum_{T \in \mathcal{P}(B,\mathbf{R})} \alpha_F(T) d_T(x,B,\mathbf{R}) = \alpha_F(S) \sum_{x \in \beta(S,\mathbf{R}|B)} d_S(x,B,\mathbf{R}) = \alpha_F(S).$$

However, because of the Theorem, (10) is all that we need to show. ||

Corollary 4 : *Suppose $m \geq 4$, and F is a regular* SSCF *that satisfies* WP *and* IIA. *Let $B \in \mathcal{A}$ but $B \neq A$, and $\mathbf{R} \in \mathcal{R}^N$. If $\{d_i\}_{i \in N}$ is a set of* SSCFs *such that $d_i \in \mathcal{F}_{\{i\}}$ for each $i \in N$, then*

$$\sum_{x \in \beta(S,\mathbf{R}|B)} F(x,B,\mathbf{R}) = \sum_{x \in \beta(S,\mathbf{R}|B)} \sum_{i \in N} \alpha_F(\{i\}) d_i(x,B,\mathbf{R}) \quad \forall\, S \in \mathcal{P}(B,\mathbf{R}).$$

Proof: The proof is omitted as it is similar to that of Corollary 3. ||

Suppose one of the following two conditions are satisfied in addition to those specified in our Theorem: (i) the preference profile $\mathbf{R}$ is such that there are at least two alternatives that are not best in the universal set for anyone according to their preferences in $\mathbf{R}$; or (ii) the number of alternatives in the universal set exceeds the number of individuals in the society by at least two, and the preference profile $\mathbf{R}$ is such that there is at least one alternative which is not best in the universal set for anyone according to their preferences in $\mathbf{R}$. Then a simple consequence of regularity is that the conclusion of Proposition 5 remains valid even when the universal set is the feasible set. Therefore, Lemma 1 allows us to consider the universal set as the feasible set in our Theorem. This extended result is formally stated as our last proposition.

Proposition 6 : *Suppose $m \geq 4$. Let F be a regular* SSCF *that satisfies* WP *and* IIA, *and let $\mathbf{R} \in \mathcal{R}^N$. If* (i) $|A - \beta(N,\mathbf{R}|A)| \geq 2$, *or* (ii) $m \geq n + 2$ *and* $|A - \beta(N,\mathbf{R}|A)| \geq 1$, *then* $\sum_{x \in \beta(S,\mathbf{R}|A)} F(x,A,\mathbf{R}) = \alpha_F(S) \;\forall\, S \in \mathcal{P}(A,\mathbf{R})$.

Proof: Suppose m, n and $\mathbf{R} \in \mathcal{R}^N$ are such that: (i) $|A - \beta(N,\mathbf{R}|A)| \geq 2$, or (ii) $m \geq n+2$ and $|A - \beta(N,\mathbf{R}|A)| \geq 1$. We first show that in both cases $F(x,A,\mathbf{R}) = 0 \; \forall \; x \in A - \beta(N,\mathbf{R}|A)$.

(i) $|A - \beta(N,\mathbf{R}|A)| \geq 2$: Let $z \in A - \beta(N,\mathbf{R}|A)$ and $B = \beta(N,\mathbf{R}|A) \cup \{z\}$. Clearly, $B \neq A$ and $\{z\} = B - \beta(N,\mathbf{R}|B)$. By Proposition 5, we then have $F(z,B,\mathbf{R}) = 0$. Therefore, regularity implies $F(z,A,\mathbf{R}) = 0$.

(ii) $m \geq n+2$ and $|A - \beta(N,\mathbf{R}|A)| \geq 1$: As $|A - \beta(N,\mathbf{R}|A)| \geq 2$ has already been considered, we only look at $|A - \beta(N,\mathbf{R}|A)| = 1$. Suppose, for each $x \in \beta(N,\mathbf{R}|A)$, there exists $i \in N$ such that $G_1(R_i|A) = \{x\}$. Then $n \geq |\beta(N,\mathbf{R}|A)|$, which implies $m \geq n + 2 > n + 1 \geq |\beta(N,\mathbf{R}|A)| + 1 = m$, an impossibility. Hence, there is an alternative in $\beta(N,\mathbf{R}|A)$, say w, such that $\{w\} \neq G_1(R_i|A)$ for every $i \in N$. Let $A - \beta(N,\mathbf{R}|A) = \{y\}$ and $\hat{B} = A - \{w\}$. Then $y \in \hat{B} - \beta(N,\mathbf{R}|\hat{B})$. So $F(y,\hat{B},\mathbf{R}) = 0$ follows from Proposition 5. Therefore, by regularity, $F(y,A,\mathbf{R}) = 0$.

Thus, $POS(F,A,\mathbf{R}) \subseteq \beta(N,\mathbf{R}|A) \subset A$ holds in any case. Then, because of Lemma 1, we have

$$(11) \quad F(x,\beta(N,\mathbf{R}|A),\mathbf{R}) \;=\; F(x,A,\mathbf{R}) \quad \forall \; x \in \beta(N,\mathbf{R}|A).$$

Obviously, $\mathcal{P}(\beta(N,\mathbf{R}|A),\mathbf{R}) = \mathcal{P}(A,\mathbf{R})$ and $\beta(S,\mathbf{R}|\beta(N,\mathbf{R}|A)) = \beta(S,\mathbf{R}|A) \; \forall \; S \in \mathcal{P}(A,\mathbf{R})$. Hence, (11) in conjunction with the Theorem imply

$$\sum_{x \in \beta(S,\mathbf{R}|A)} F(x,A,\mathbf{R}) = \sum_{x \in \beta(S,\mathbf{R}|\beta(N,\mathbf{R}|A))} F(x,\beta(N,\mathbf{R}|A),\mathbf{R}) = \alpha_F(S) \; \forall \; S \in \mathcal{P}(A,\mathbf{R}). \quad \|$$

In Proposition 6 the two additional conditions are independent of each other. However, when preferences are restricted to be strict, as $m \geq n+2$ implies $|A - \beta(N,\mathbf{R}|A)| \geq 2$, the second condition is stronger than the first. Thus, although PP's additional condition for their extended result (Theorem 4.14) is $m \geq n+2$, our Proposition 6 shows that their result is actually a consequence of a weaker condition on the preference profile, namely, $|A - \beta(N,\mathbf{R}|A)| \geq 2$.

Needless to say, using Proposition 6, it is easy to show that Corollaries 3 and 4 also hold when we consider the universal set as the feasible set provided at least one of the pair of additional conditions specified in Proposition 6 is satisfied.

5 Conclusion

We provided a natural extension of the results in PP to the case in which individuals might have indifference between alternatives. When there are at least four elements in the universal set of alternatives and the stochastic social choice function is regular, weakly Paretian ex-post and satisfies independence of irrelevant alternatives, as in PP, there is a unique nonnegative weight associated with each coalition of individuals in the society. For each social preference profile, when the above mentioned conditions hold and the feasible set is a proper subset of the universal set of social alternatives, we showed that the society can be partitioned into coalitions of individuals in such a way that the sum of the social probabilities of all the alternatives in the union of the best sets of the members of each coalition is equal to the weight of the coalition. When the universal set of social alternatives itself is the feasible set, we showed that our result still holds provided the social preference profile is such that there are at least two alternatives that are not best in the universal set for anyone, or the number of alternatives in the universal set exceeds the number of individuals in the society by at least two and the preference profile is such that there is at least one alternative which is not best in the universal set for anyone.

Although this paper derived restrictions on the distribution of coalitional power for SSCFs, it is easy to construct examples to show that the results presented here do not fully characterize the structure of coalitional power under SSCFs. So how close to a complete characterization are our results? We offer the following simple answer to this question. The examples in PP can be extended consistently to our expanded preference domain in a straightforward manner to show that our Theorem may no longer hold when any one of its conditions is violated, and also, Proposition 6 may not be true when both additional conditions specified in it are dropped.[8] Thus, in a way, the results of the paper can be conceived as representing an almost complete characterization of the structure of SSCFs in our framework.

[8]These extended examples can be provided to the interested reader on request.

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