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Networks, Network Externalities and Market Segmentation

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Abstract

This paper models interaction between groups of agents by means of a graph where each node represents a group of agents and an arc represents bilateral interaction. It departs from the standard Katz-Shapiro framework by assuming that network benefits are restricted only amongst groups of linked agents. It shows that even if rival firms engage in Bertrand competition, this form of network externalities permits strong market segmentation in which firms divide up the market and earn positive profits.

Keywords: network structure, network externalities, price competition, market segmentation.

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1 Introduction

It has long been known that some goods and services (for example, telecommunications, computer software and hardware) generate network effects or externalities. The seminal paper by Katz and Shapiro (1985) defines a network effect to exist when the utility that a user derives from consuming a product depends on the number of other agents who consume either the same brand of the product, or another brand which is compatible. This way of modelling of the network effect is found throughout the large literature that has developed.[1] While this is reasonable in many contexts, we feel that in other instances it overlooks the fact that such positive externalities arise from the *specific* patterns of interaction between groups of users.

The examples we have in mind are primarily of software packages with specific functions such as word processing, accounting, data analysis and so on. The use of such packages have *local* network effects. Thus the utility to a user (say, a researcher in a University) of a word processing or data analysis package depends at least partly on the number of her research *collaborators* who use the same package, rather than on the total number of users of the package. A main advantage to two collaborators using the same package is sharing files. In many of these markets, there is a degree of incompatibility between brands. Two users using incompatible brands find it difficult if not impossible to share files; a program written on one software package cannot be read, or worked on, using a competing brand. Patterns of interaction and the generation of such local network effects can be observed in other kinds of

[1] There is by now a large literature analyzing important issues in markets subject to network effects. See, for instance Katz and Shapiro (1985, 1986), Farrell and Saloner (1985, 1986), Economides and Salop (1992), Farrell and Katz (2000), Matutes and Regibeau (1992), Choi (1994), Ellison and Fudenberg (2000), Waldman (1993). Economides (1996) provides an insightful overview.

economic activity as well. For example, in an environment where back office activities are outsourced, firms which manage outsourcing operations benefit from having systems and software compatible with clients' software.

We use the formal network structure proposed in the important recent paper of Jackson and Wolinsky (1996) to model the interaction between groups of users. In particular, the set of all consumers is partitioned into different groups or *nodes*, and two nodes are connected to each other if they "interact"[2]. Our main interest is in analysing whether the precise pattern of interactions - that is, the *specific* network structure- has any influence on market outcomes. For instance, suppose the overall "market" is the academic market for software. Does the fact that economists typically do not collaborate with physicists (that is, economists are not "linked" to physicists) matter in this market?

Since our model is motivated by examples such as software packages, capacity constraints are unimportant. It seems natural to assume that firms have unlimited capacity, and hence compete in prices. If firms produce competing, incompatible brands of the same *intrinsic* quality, and have the same constant marginal cost of production, existing models of network externalities would yield the Bertrand zero profit outcome. This is so for the Katz and Shapiro (1985) model as well, if it is modified to analyze price, rather than quantity competition. The main result in this paper is that if network effects are generated from patterns of interaction among users, then there exist outcomes in which firms do make *positive* profits, and there is *market segmentation* in the sense that rival firms divide or partition the overall mar-

[2]Although this kind of modelling has not been used so far in the literature on network externalities, the use of such network structures in other areas of economics is becoming increasingly popular. Dutta and Jackson (2003) contains several interesting papers in this genre.

ket into separate segments, with each firm selling to different segments. This accords well with casual observation, which suggests positive profit outcomes arise even when firms compete in prices and capacity is essentially unlimited.

Furthermore, we show that the graph (or interaction) structure graph (or interaction) structure matters; for some graphs, market segmentation can be ruled out in equilibrium. Thus, one way of interpreting our results is to say that there are interaction structures which convert the industry into a differentiated goods industry. However, there are other interaction structures - for instance, the complete graph where all users are linked to each other - where the goods remain homogeneous, and so firms do not earn positive profits. The discussion also shows that when positive profit equilibria exist, if firms could choose whether or not to make their brands mutually compatible, they would choose not to do so.

Recently, work on intermediation in two-sided markets (for example, markets for matchmaking services)has begun to analyze the consequences of the network benefit that one side of the market confers on the other.[3] The examples we cite show that patterns of interaction between groups of users can be more general. In this paper we make a beginning in attempting to understand how such interaction, and local network effects, affect market outcomes under oligopoly.

2 A Model of Network Externalities

Our model of network externalities in the context of a partial equilibrium duopoly has similarities to that of Katz and Shapiro (1985). A major difference is in the way in which we model network externalities. Another difference

[3] Armstrong (2002) provides a very interesting survey.

is that in our model firms compete in prices, in contrast to Katz and Shapiro (1985) who assumed that firms behaved a la Cournot.

Consumers

Consumers are partitioned into groups, and each group "interacts" with *some* but not necessarily all groups. For instance, consider the set of all faculty members in a university. Each department then constitutes a group. Economists may collaborate with political scientists and mathematicians, but perhaps not with physicists or other scientists. Similarly, members of the science departments may interact with each other, but not with sociologists. The pattern of such interactions is modeled as an *undirected graph* or *network* (I, g) where I is a set of n *nodes* and $g \subset I \times I$ is a set of *arcs.* Each group of consumers is located at a different node $i \in I$, and $ij \in g$ if consumers located at node i interact with consumers located at j. We assume that consumers within each group interact with each other and that if *some* consumers at node i interact with *some* consumers at j, then *all* consumers located at i interact with *all* consumers at j.[4]

We will say that the network (I, g) is *complete* if $g = \{ij | ij \in I \times I\}$. That is, all groups interact with *all* other groups in a complete network - this would correspond to the original Katz-Shapiro model of network externalities.

For each node i, let $L(i) = \{j \in I | ij \in g\}$. That is, $L(i)$ is the set of nodes that are linked to node i.

Let α_i denote the measure of consumers located at node i.[5] Each consumer wishes to consume at most one unit of a good. There are *two* brands of the

[4]This is without loss of generality since we can define the set of nodes appropriately in order to represent any pattern of interaction.

[5]Any single consumer has zero measure.

good - for example, different types of software. The two brands differ in inessential ways in the sense that each brand is functionally identical as far as consumers are concerned. Let r_i denote the *basic* willingness to pay for the good of a consumer who is located at node i. However, the total utility or surplus that a consumer gets from a particular brand of the good also depends on the number of other consumers with whom she interacts and who consume the same brand. Let p_j be the price of a unit of brand j, and α_{sj} be the measure of consumers at node s who consume brand j. Then the utility of a consumer at node i from buying a unit of brand j is

$$u_i(j, p_j) = r_i - p_j + \alpha_{ij} + \sum_{s \in L(i)} \alpha_{sj}.$$

So, by consuming brand j, a consumer at node i gets a gross benefit r_i and a *network benefit* of $\alpha_{ij} + \sum_{s \in L(i)} \alpha_{sj}$. We will refer to $p_j - \alpha_{ij} - \sum_{s \in L(i)} \alpha_{sj}$ as the *hedonic price* of brand j at node i.

Given any vector of prices, each consumer purchases the brand whose hedonic price is lower or abstains from buying either brand if both hedonic prices exceed her basic willingness to pay. Of course, each consumer has to have some *expectation* about other consumers' consumption decisions in order to estimate the network benefits. Following Katz and Shapiro(1985), we will assume that expectations are fufilled in equilibrium. We elaborate on this shortly.

Allocations

An allocation describes the pattern of consumption at each node corresponding to each vector of prices. More formally,

Definition 1 *An allocation a is a function $a : \Re^2_+ \rightarrow \Re^{2n}_+$, such that for all (p_1, p_2) and for all $i \in I$, $a_{i1}(p_1, p_2) + a_{i2}(p_1, p_2) \leq \alpha_i$.*

Here, $a_{ij}(p_1, p_2)$ is the amount of brand j consumed at node i corresponding to prices (p_1, p_2).

Consumers' decisions about which brand to purchase will determine which allocation is "observed" in the market. Since such allocations are the outcome of utility-maximising behaviour, it makes sense to impose some restrictions on "permissible" allocations.

Definition 2 *Let a be an allocation. Choose any non-negative prices* (p_1, p_2)*, and any node* $i \in I$*. Then, a satisfies*

(i) Individual Rationality if for $j = 1, 2$*,* $a_{ij}(p_1, p_2) > 0$ *implies that* $p_j - a_{ij}(p_1, p_2) - \sum_{s \in L(i)} a_{sj}(p_1, p_2) \leq r_i$

(ii) Incentive Compatibility if for $j = 1, 2$*,* $a_{ij}(p_1, p_2) > 0$ *implies that* $p_j - a_{ij}(p_1, p_2) - \sum_{s \in L(i)} a_{sj}(p_1, p_2) \leq p_k - a_{ik}(p_1, p_2) - \sum_{s \in L(i)} a_{sk}(p_1, p_2)$ *where* $k \neq j$*.*

Individual Rationality expresses the requirement that consumers will not purchase any commodity whose hedonic price exceeds their basic willingness to pay, while Incentive Compatibility incorporates the idea that consumers purchase the good with the lower hedonic price.

Throughout this paper, we assume that all allocations satisfy Incentive Compatibility and Individual Rationality. These are minimal requirements which arise straightaway from utility-maximising behaviour. Since the pattern of consumption also depends on consumers' expectations, it may be possible to justify or rationalise allocations which satisfy these restrictions, but are nevertheless non-intuitive simply because of the self-fulfilling nature of expectations. Suppose, for instance that "initial" prices of the two brands are p_1 and p_2. Now, let there be an increase in the price of brand 1, with p_2

remaining constant. If all consumers now expect everyone to switch to brand 1, then this may turn out to be self-fulfilling because the network externalities associated with brand 1 are now much larger and so the hedonic price of brand 1 is correspondingly lower at all nodes. The following assumption is imposed to bring about some regularity on how the pattern of consumption changes with changes in prices.

Assumption 1: An allocation a is non-perverse in prices if for all $i \in I$ and $j = 1, 2$, $a_{ij}(p_j, p_k)$ is non-increasing in p_j and non-decreasing in p_k.

By itself, Assumption 1 imposes a very weak restriction on how allocations change with respect to a change in prices. In particular, Assumption 1 still allows for allocations which seem somewhat counterintuitive. Consider, for example, a network structure in which nodes i and j are linked, and such that at prices (p_1, p_2), all consumers at node i are consuming say brand 1 because the hedonic price of brand 1 is smaller than the hedonic price of brand 2 by α_i. Suppose there is an arbitrarily small reduction in the price of p_2. Then, Assumption 1 allows for the possibility that *all* consumers at node i will switch brands and consume only brand 2. Of course, if all consumers expect this to happen, then the self-fulfilling nature of expectations guarantees that the allocation will satisfy Incentive Compatibility and Assumption 1. In order to rule out such changes, we impose the following assumption.

Assumption 2: For every $i \in I$, the component a_i of an allocation a is continuous except possibly at any (p_1, p_2) where the hedonic prices are equal.

Definition 3 *An allocation is admissible if it satisfies Assumptions 1 and 2.*

Since an individual's net utility depends on the actions of other consumers, the optimal decisions of consumers may depend on whether consumers can coordinate their actions. Consider, for example, a situation where node i is not linked to any other node, $p_1 - \alpha_i < r_i < p_1 < p_2$. Then, consumers at node i can derive some net utility if *all* consumers consume brand 1. On the other hand, *no* consumer on her own will want to consume either brand. In one subsequent result, we will assume that consumers at each node can coordinate their actions when this is mutually profitable.

Assumption C: At any node i and prices (p_1, p_2),if $\min_{j \in \{1,2\}}(p_j - \alpha_i - \sum\limits_{s \in L(i)} a_{sj}(p_1, p_2)) < r_i$, then $a_{i1}(p_1, p_2) + a_{i2}(p_1, p_2) = \alpha_i$.

Assumption C states that if consumers at any node can coordinate their consumption decisions and attain strictly positive utility, then no consumer will abstain from consumption.

Firms

There are two firms, each producing a different brand. For expositional purposes, let brand j refer to output produced by firm $j, j = 1, 2$. For simplicity, we assume that firms have zero cost of production.

Both firms anticipate the same allocation, and choose prices simultaneously to maximise profits. Given any allocation a, firm j's profit corresponding to prices (p_i, p_j) is

$$\pi_j(p_i, p_j; a) = p_j \sum_{i \in I} a_{ij}(p_i, p_j)$$

Equilibrium

An equilibrium will be a set of prices (p_1, p_2) and an admissible allocation such that each firm j maximises profit given the other firm's price and the allocation rule, while the allocation a satisfies individual rationality and incentive compatibility. Notice that the restrictions on a ensure that consumers' expectations are fulfilled in equilibrium.

Definition 4 *A vector* (p_1^*, p_2^*, a^*) *constitutes an equilibrium if*

(i) The allocation a^* *is admissible, and satisfies Individual Rationality and Incentive Compatibility*

(ii) For each $i = 1, 2$, $\pi_i(p_i^*, p_j^*; a^*) \geq \pi_i(p_i, p_j^*; a^*)$ *for all* p_i.

We first show that an equilibrium always exists.

Theorem 1 *For all graphs* g*, the vector* (p_1^*, p_2^*, a^*) *is an equilibrium where* $p_1^* = p_2^* = 0$ *and* a^* *is an admissible allocation with* $a_{i1}^*(p_1, p_2) = a_{i2}^*(p_1, p_2) = \frac{\alpha_i}{2}$ *for each* $i \in I$ *whenever* $p_1 = p_2$.

Proof: Consider any node i. Since $p_1^* = p_2^*$, and the allocation divides consumers equally between the two brands, the two hedonic prices must be equal at each node. Since the hedonic prices are also negative, the allocation satisfies incentive compatibility and individual rationality.

So, we only need to check that both firms are maximising profits. Notice that for each firm i, $\pi_i(p_1^*, p_2^*; a^*) = 0$. Clearly, neither firm has an incentive to lower price. Suppose firm i raises price to $p_i > 0$. Since a^* is admissible, $a_{si}^*(p_i, p_j^*) \leq a_{si}^*(p_i^*, p_j^*)$ at each node $s \in I$. But, this implies that the hedonic price of brand i is higher than that of brand j at each node. From incentive compatibility, $a_{si}^*(p_i, p_j^*) = 0$ at each node s. Hence, firm i does not gain by increasing price.

This completes the proof that (p_1^*, p_2^*, a^*) is an equilibrium. ■

Notice that in the equilibrium described in Theorem 1, the two hedonic prices are equal at each node. The pair of prices remain in equilibrium because neither firm wants to deviate by quoting a lower price since the "current" level is already zero. The lemma below shows that this is the only case when hedonic prices can be equal at any node. That is, if hedonic prices are equal at any node i, and brand j is consumed at this node, then the price of brand k ($k \neq j$) must be zero - the latter condition ensures that firm k has no incentive to lower price any further in order to capture a larger share of the market.

Lemma 1 *Suppose* (p_1, p_2, a) *is an equilibrium. Then, at all nodes* $i \in I$, *if* $p_1 - a_{i1}(p_1,p_2) - \sum_{s \in L(i)} a_{s1}(p_1,p_2) = p_2 - a_{i2}(p_1,p_2) - \sum_{s \in L(i)} a_{s2}(p_1,p_2)$, *then for* $j = 1, 2$ *and* $k \neq j$, *either* $a_{ij}(p_1,p_2) = 0$ *or* $p_k = 0$.

Proof. Suppose (p_1, p_2, a) is an equilibrium, and the two hedonic prices are equal at node i. Without loss of generality, let $a_{i1}(p_1,p_2) > 0$ and $p_2 > 0$. Suppose firm 2 lowers its price to $p_2' = p_2 - \epsilon$. Since a is admissible, $a_{i1}(p_1,p_2') \leq a_{i1}(p_1,p_2)$ and $a_{i2}(p_1,p_2') \geq a_{i2}(p_1,p_2)$. Since $p_2' < p_2$, the hedonic price of brand 2 is lower than that of brand 1 at node i for all permissible values of $a_{i1}(p_1,p_2')$. Since a satisfies incentive compatibility, it must be the case that $a_{i1}(p_1,p_2) = 0$ and $a_{i2}(p_1,p_2') = \alpha_i$.[6] So, firm 2 can capture the entire market at node i by lowering price. So, this increases profit by $a_{i1}(p_1,p_2)(p_2 - \epsilon)$. The loss of profit at other nodes can be made arbitrarily small by choosing an appropriately small ϵ.

Hence, firm 2 cannot be maximising profit at (p_1, p_2). This contradiction establishes the result. ■

3 Market Segmentation

Both firms had positive market share at each node in the equilibrium constructed in Theorem 1. However, this was not surprising since neither firm

[6] The latter follows because consumers at node i were purchasing at prices (p_1, p_2), and so had non-negative utility. Hence, they must be purchasing at price p_2' since the hedonic price of brand 2 is now lower.

had any incentive to cut into the other firm's market share as prices were driven down to zero. The purpose of this section is to show that some network structure(s) representing interactions between consumer groups may result in *segmented* markets with both firms earning *strictly positive profits although firms are competing in prices.* A formal definition of market segmentation follows.

Definition 5 *: An equilibrium (p_1, p_2, a) exhibits strong market segmentation if there are nodes i and j such that $a_{i1}(p_1, p_2) = \alpha_i$, $a_{j2}(p_1, p_2) = \alpha_j$ and $p_k > 0$ for $k = 1, 2$.*

Proposition 1 *: There exists a network with strong market segmentation.*

Proof: Consider a network (I, g) where $I = \{1, 2, 3\}$ and $g = \{12, 13\}$. That is, there are three consumer groups with group one connected to groups two and three. Note that nodes 2 and 3 are not connected. The description of the "market" is completed with the following specification and population shares and basic willingness to pay for each node.

(i) $r_1 = 10, \alpha_1 = 1$.

(ii) $r_2 = 60, \alpha_2 = 19$.

(iii) $r_3 = 5, \alpha_3 = 80$.

Consider an admissible allocation a^* which satisfies the following restrictions.

(i) $a^*_{11}(p_1, p_2) = \alpha_1$ if $p_1 - \alpha_1 - \alpha_3 \leq \min(p_2 - \alpha_2, r_1)$.

(ii) $a^*_{22}(p_1, p_2) = \alpha_2$ if $p_2 - \alpha_2 \leq \min(p_1 - \alpha_1, r_2)$.

(iii) $a^*_{31}(p_1, p_2) = \alpha_3$ if $p_1 - \alpha_1 - \alpha_3 \leq \min(p_2, r_3)$ or if $p_1 - \alpha_3 \leq \min(p_2 - \alpha_1, r_3)$.

Consider prices $p_1^* = 86, p_2^* = 79$. Then, we claim that (p_1^*, p_2^*, a^*) is an equilibrium with strong market segmentation.

To check our claim, we first observe that $a_{31}^*(p_1^*, p_2^*) = \alpha_3$, $a_{22}^*(p_1^*, p_2^*) = \alpha_2$, and $a_{11}^*(p_1^*, p_2^*) = \alpha_1$. It is easy to check that a^* satisfies individual rationality and incentive compatibility.

Now, we show that neither firm i has any incentive to deviate from p_i^*. First, if firm 1 raises its price, then consumers at node 3 will drop out of the market. So, firm 1 will not raise price. Next, in order to cut into firm 2's market share at node 2, firm 1 will have to reduce price to just below 61 from (ii) above. But, then firm 1's profit will be at most 6100, whereas its current profit is 6966. So, firm 1 has no incentive to change price if $p_2^* = 79$.

Finally, we check firm 2's incentives. It cannot raise price above 79 because consumers at node 2 will then drop out of the market. From (i) above, it would have to lower price to just below 24 in order to attract consumers at node 1. But, this gives it a profit of at most 480. It can capture the entire market if it charges a price less than 7. But, that would yield a profit of less than 700, whereas its current profit is 1501.

So, neither firm has any incentive to deviate from p_i^*. Hence, (p_1^*, p_2^*, a^*) is indeed an equilibrium. ■

How is it that both firms are earning positive profits despite being Bertrand duopolists? If prices are strictly positive, then lemma 1 implies that at each node, all consumers buy only one brand.[7] In particular, the incentive compatibility constraint cannot be binding. So, each firm i will have to lower its price by an amount ϵ_i strictly bounded away from zero in order to eat into its rival's market share. So, strong market segmentation can be sustained if ϵ_i is sufficiently large so as to make the revenue loss from its existing customers

[7] In general, consumers at some node may also refrain from buying either brand.

larger than the gain in revenue from new customers.

There is nothing pathological about the network structure used in the proof of the previous proposition. So, this suggests that market segmentation of this kind can arise quite generally, although we have not been able to derive any sufficient conditions.

In the remainder of this section, we show that there are types of network structures which *cannot* give rise to market segmentation. The first such structure is when all customers are linked to each other, while the second is when the network structure exhibits a specific type of *symmetry*: nodes can be ordered so that node i is connected only to nodes $i-1$ and $i+1$,[8] and all nodes have the same measure of consumers (say) α, while consumers at all nodes also derive the same gross benefit, say r.

Theorem 2 : *If (I, g) is a complete network, then there cannot be strong market segmentation.*

Proof: Suppose to the contrary that an equilibrium with strong market segmentation exists. Let (p_1, p_2) be the equilibrium prices. Since (I, g) is complete, the hedonic price of each brand is the same at all nodes. So, let (h_1, h_2) denote the hedonic prices corresponding to (p_1, p_2). Consider any node i where consumers buy brand 1. Incentive compatibility requires that $h_1 \leq h_2$. Similarly, by considering any node j where consumers buy only brand 2, we get $h_2 \leq h_1$.

Hence, $h_1 = h_2$. But,this contradicts Lemma 1. ■

The following lemma will be used in the proof of the next theorem.

[8] That is, g is a *circle*.

Lemma 2 *Suppose* (p_1, p_2, a) *is an equilibrium with strong market segmentation, and* a *satisfies Assumptions* 1 *and* 2*. Then for each brand* j*, there exists a node* i *such that* $a_{ij}(p_1, p_2) > 0$ *and* $u_i(j, p_j) = 0$*.*

Proof: Suppose that for every node i with $a_{ij}(p_1, p_2) > 0$, we have $u_i(j, p_j) > 0$. Since there is strong market segmentation, $p_j > 0$. Therefore, by Lemma 1,

$$p_j - a_{ij}(p_j, p_k) - \sum_{s \in L(i)} a_{sj}(p_j, p_k) < p_k - a_{ik}(p_j, p_k) - \sum_{s \in L(i)} a_{sk}(p_j, p_k)$$

By continuity of a, firm j can raise price p_j slightly - Incentive Compatibility and Individual Rationality of a ensure that consumption of brand j at each node remains as before. So, firm $j's$ profit is higher. This contradicts the assumption that (p_1, p_2, a) is an equilibrium. ■

Theorem 3 *Suppose* (I, g) *is a circle such that all consumers have the same basic willingness to pay* r*, and all nodes have the same measure of consumers* α*. Then there cannot be strong market segmentation if Assumption C is satisfied.*

Proof: Suppose to the contrary that (p_1, p_2, a) is an equilibrium with strong market segmentation. If $|I| \leq 3$, this is ruled out by Theorem 2. So let $|I| > 3$. We proceed in steps.

Step 1: At all nodes i, either $a_{i1}(p_1, p_2) = \alpha$ or $a_{i2}(p_1, p_2) = \alpha$. That is, all consumers at each node buy one of the two brands.

Proof of Step 1: We already know from lemma 1 that consumers at each node will completely specialise in one brand if they buy at all. So, we only need to prove that no consumer abstains from consumption.

Since there is market segmentation, there must be some node i where all consumers buy say brand 1. We want to show that no consumer at node $(i-1)$ abstains from consumption. Either consumers at node $(i+1)$ purchase brand 1 or they do not do so. In either case, from Individual Rationality,

$$p_1 - 2\alpha \leq r$$

Notice that if firm 1 lowers price slightly, then the hedonic price of brand 1 at node $(i-1)$ will be strictly lower if all consumers at node i buy 1. Assumption C ensures that no consumer abstains from consumption.

Step 2: If brand j is consumed at node i, then it is consumed at either node $(i-1)$ or node $(i+1)$.

Proof of Step 2: In view of Step 1, assume that brand k is consumed at nodes $(i-1)$ and $(i+1)$. By Incentive Compatibility at i, we have

$$p_j - \alpha < p_k - 2\alpha \tag{1}$$

The smallest possible hedonic price of brand k at node $(i-1)$ is $p_k - 2\alpha$ - this happens when consumers at $(i-2)$ consume k. The biggest possible hedonic price of brand j at $(i-1)$ is $p_j - \alpha$. Equation (1) shows that Incentive Compatibility is violated at node $(i-1)$.

Step 3: $p_1 = p_2 = r + 2\alpha$.

Proof of Step 3: Since g is a circle, Steps 1 and 2 imply that there exist nodes i and $(i+1)$ such that consumers at nodes i and $(i-1)$ consume brand j, while consumers at nodes $(i+1)$ and $(i+2)$ consume brand k. So, the hedonic prices of brands j and k at nodes i and $(i+1)$ respectively are $p_j - 2\alpha$ and $p_k - 2\alpha$. Also, if brand j is consumed at some node p, then its hedonic price at p cannot exceed $p_j - 2\alpha$.[9] Lemma 2 now completes the proof of Step 3.

[9] It could be $p_j - 3\alpha$ if j is consumed at both nodes $(p-1)$ and $(p+1)$.

Step 4: Suppose N_1 is the set of nodes where brand 1 is consumed. Without loss of generality, let $\# N_1 = n_1 \leq \frac{n}{2}$. Firm 1's profit is

$$\pi_1(p_1, p_2) = p_1 n_1 \alpha$$

Let firm 1 lower price to $p_1' = p_1 - \alpha - \epsilon$. It is easy to check that at (p_1', p_2), firm 1 captures the entire market. Its profit is now

$$\pi_1(p_1', p_2) = (p_1 - \alpha - \epsilon) n \alpha$$

Since $n_1 \leq \frac{n}{2}$, firm 1 can choose ϵ sufficiently small so that $\pi_1(p_1', p_2) > \pi_1(p_1, p_2)$.

Hence, (p_1, p_2, a) cannot be an equilibrium. This completes the proof of the theorem. ∎

Remark 1 *: Assumption C plays a crucial role in the theorem. If Assumption C does not hold, then even when the network is a symmetric circle, one can have market segmentation of the following kind :* $p_1 = p_2 = r + \alpha$, *and consumers at nodes* $i, i+2, i+4, \ldots$ *abstain from consumption, while consumers at nodes* $i+1, i+3, \ldots$ *consume either of the two brands.*

Remark 2 *Even if Assumption C holds, it is not in general true that strong market segmentation can occur at an equilibrium in all symmetric graphs.*

4 Discussion

We comment below on possible extensions of the basic model outlined in this paper.

We have assumed that the two firms produce *incompatible* brands. Suppose instead that the two brands are *fully* compatible. For a consumer at any node i, the network benefit from consuming either brand is then the same: it is the total measure of consumers of brands 1 and 2 at all adjacent nodes. Therefore, Incentive Compatibility implies that at any node i, consumers will simply buy the cheaper brand. The only equilibrium outcome possible then has zero prices and profits; strong market segmentation is ruled out. However, with *partial* compatibility, strong market segmentation can exist for exactly the same reason as in the basic model of this paper.

This has an obvious implication if the choice of compatibility is *endogenous*. Consider a network structure that permits equilibria with strong market segmentation when brands are incompatible. Suppose that before the firms compete in prices, they decide whether or not to make their brands compatible with each other, say, by providing a two way converter. Assume that if both play "Yes", then the brands are compatible, whereas if at least one plays "No", they are incompatible. Following this, there is price competition. If both play "Yes", price competition leads to zero profits. This is not an equilibrium, since if even a single firm plays "No", the firms can then coordinate on a positive profit, strong market segmentation equilibrium. This provides a justification for observing the existence of incompatible brands, even under price competition with unlimited capacities, and no differences in intrinsic product quality.

Addition of a link or edge to a graph increases the network effect at least on the nodes that are incident on the new edge. This can increase willingness to pay at these nodes, if they were consuming the same brand. However, whether profits increase in equilibrium depends crucially on the graph structure. For example, the equilibrium in Proposition 1 exhibits positive profits for the firms; however, if we add a link between nodes 2 and 3, we get a complete graph, which, by Theorem 2, implies a zero profit equilibrium. Similarly, one can show that if one starts with a graph for which strong market segmentation does not exist, and adds links/nodes to it, strong market segmentation can appear in the resulting graph. Thus it is not necessarily the case that if consumers are 'more connected', then the market tends to be less segmented - although the complete graph does not allow market segmentation and hence positive profits.

We note briefly that if consumers' expectations regarding network size, as captured in the allocation rule, are skewed enough, then a monopoly outcome is possible. In general, there can be multiple equilibria. This is a well known problem in network externalities models, arising from the possibility of multiple admissible network size expectations . A useful extension would be to consider reasonable restrictions on allocation rules to prune the number of equilibria. While Theorem 1 gives us the lowest profit equilibrium, it remains to characterize the highest profit equilibrium for a given network structure.

There can be various other context specific extensions to the model. One extension could be to study competition when brands may differ in intrinsic quality. Another could be to study models in which the different nodes have different physical interpretations. For instance, in several models of competition in two-sided markets (see Armstrong (2002)), agents on the two sides of a market have different roles, (e.g., consumers and retailers, with shopping malls as intermediaries) and it is reasonable to study the possibility that a firm such as a mall owner would charge different prices from consumers and retailers. The model in the present paper studies multilateral, rather than bilateral relationships; in applications where different nodes have distinct physical interpretations or roles, one can study the possibility of price discrimination by firms.

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