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A. BANERJI
Email: a.banerji@econdse.org
J. V. MEENAKSHI
Email: meena@econdse.org
Delhi School of Economics

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Department of Economics, Delhi School of Economics

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A. BANERJI & J. V. MEENAKSHI

Abstract

This paper undertakes structural estimation of asymmetric auction models in a market for basmati, and detects the presence of a cartel consisting of a large (in market share) local miller and commission agents purchasing for large distant millers. The contracts between the distant millers and their commission agents help to explain the specific form that collusion takes. Simulations indicate that (i) the cartel gains considerably by colluding, over the competitive outcome; (ii) however, sellers (farmers) do not lose significantly under collusion when the commission agents bid; (iii) a knowledgeable auctioneer would choose much higher starting prices for auctions when commission agents bid, compared with the observed starting prices. The paper also shows that *efficient collusion*, the form of collusion commonly assumed in the literature, does not explain the data well.

Keywords: Auctions, Cartels, Agricultural Markets.
JEL Classification: L1, L4, Q13.

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1. Introduction

Collusion amongst bidders in auctions has attracted the attention of antitrust cells the world over, and, increasingly, spurred academic research. In an important early paper, Hendricks and Porter (1989) stress the need to recognize that the possibility of collusion, and the form that it takes, is likely to depend on specific characteristics of the market in question. In the present paper, we undertake the detection of collusion amongst buyers of basmati paddy in a wholesale market in Panipat[1], using a dataset from a primary survey that we conducted. In this market, sales take place through open ascending auctions. We find evidence of a simple yet novel form of collusion, which is related to the fact that some of the bidders have nonstandard payoffs, specific to the context of this market. In particular, payoffs for a subset of bidders, conditional on winning, are increasing in the win price. We also argue that the characteristics of this market work against the possibility of 'efficient collusion', (the predominant form of collusion discussed in the literature)[2], and show that the data do not support it. The paper thus illustrates the general point about market characteristics and forms of collusion made by Hendricks and Porter. It also contributes to the relatively small literature analyzing the structure of grain markets in developing countries. Many such markets in India and elsewhere use auctions to transact grain, but surprisingly, very few studies use auction theory to characterize these markets and study their features.

The buyers of basmati paddy in Panipat market are mill owners who process basmati paddy into various grades of basmati rice. Their mills and size of operations displays great variation. At one end, there are small millers who sell the processed rice in the domestic market under generic names; at the other, there are large millers with recognized brand names and a significant presence in international markets.

Apart from buyer asymmetry, a more important distinction in this market is between local and distant mills. Many millers have mills located relatively close to the Panipat market; typically, such millers themselves participate in the bidding. On the other hand, a few large millers are located at considerably larger distances from the

[1] Panipat is a district in the Northern Indian state of Haryana.

[2] In efficient collusion, if a bidding ring wins the auction, the good goes to that member of the ring who has the highest valuation for it.

market, and employ the services of commission agents (called *pakka arhtias* in local parlance) to bid at auctions and buy grain on their behalf. As payment for this service, a commission agent gets 3% of the sale price of any lot of grain (i.e., the win price) that he purchases. Ordinarily, bidders' payoffs in auctions decrease in the win price, and one incentive to collude, ordinarily, is that collusion tends to depress win prices. While a fuller description of the commission agents and their relationship to the millers they buy for is in Section 2.3, we note here that their payoff from winning a grain lot *increases* in the win price. Despite this, we find that they are part of a cartel.

As stated earlier, the form of collusion that appears to be in operation is not 'efficient collusion'. In such collusion, the cartel members have a mechanism by which their values for the object to be auctioned are revealed; the member with the highest value is designated to bid for the object. The open and stringently time bound nature of ascending auctions in Panipat (as described later) militates against the kind of revelation of values that is necessary for efficient collusion. More importantly, conditional upon winning, a commission agent prefers to win at a higher price. So, it is not obvious that excluding all but one cartel members from bidding, as in efficient collusion, is desirable for this kind of player.

We find that the collusive model which best describes the data (Section 4) takes place within a four player cartel, three of whose members are commission agents bidding on behalf of distant millers, the remaining member being a large local miller who is present at the auctions. The cartel divides up the lots to be auctioned, and, for a given lot, it is *either* the local miller who is designated to bid, *or* the *group* comprising the 3 commission agents. In such collusion, each commission agent's probability of winning increases by excluding the large local miller from bidding, whereas the win price does not drop significantly, since all 3 commission agent cartel members bid; the net effect of collusion is favorable for all the cartel players (Section 5.1).

The auction models that we use are independent (conditional on observed characteristics of a lot such as quality) private values (IPV) models that incorporate buyer asymmetry. The methodology is to estimate structural parametric models. In particular, the estimation here is supported by a recent identification result (Athey and Haile (2002), Meilijson (1981)) which demonstrates that for private values models of ascending auctions, one can invert the joint distribution of win price and winner identity to uncover the latent distributions of all bidders.

Structural estimation of auction models, which began with Paarsch (1992), has expanded fast into a large literature. The papers by Laffont, Ossard and Vuong (1995), Donald and Paarsch (1996), Guerre, Perrigne and Vuong (2000), Campo (2002), Hendricks, Pinkse and Porter (2003), and Hong and Shum (2003), form a small and incomplete list. Sareen (2002) is a recent survey. Bajari and Hortacsu (2003) display some experimental evidence in support of this kind of structural estimation, by way of answering criticism about the strict rationality postulates that govern the behavior of players in these models. The empirical literature on collusion and its detection is also growing fast. Besides Hendricks and Porter (1989) mentioned above, Baldwin, Marshall and Richard (1997), Porter and Zona (1999), Bajari and Ye (2004) are but a few interesting examples.

The rest of the paper is arranged as follows. Section 2 contains a description of Panipat market, the auction process, the players, and our dataset. Section 3 describes the models to be estimated. Section 4 discusses the estimation results, which suggest that the model incorporating the collusive scheme briefly discussed above describes the data best. Section 5 discusses results from several simulations that use the parameter estimates of the collusive model. First, it is shown that the payoffs to cartel players from the above collusive scheme are much higher than from competition, given our parameter estimates (Section 5.1). Since the collusion is not ‘efficient’, this result is not obvious, and must be demonstrated to hold, for given parameter estimates. Next, we compare win prices under collusion and noncooperation (Section 5.2). We show that the unconventional collusion in this market has, on average, a much smaller downward impact on win prices than it would be otherwise. Finally, in Section 5.3, though we eschew computing optimal reserve prices, we show that an auctioneer who is aware of the collusive scheme would use starting prices that differ vastly between lots on which the large local miller is designated to bid, and those on which the group of commission agents bid. Section 6 concludes.

2. The Basmati Paddy Market in Panipat

Basmati is an aromatic, long-grain rice variety, grown only in Northern India and Pakistan. A large proportion of Indian basmati is exported. Premium basmati on the Indian domestic market retails at prices ten times greater than those for cheap rice varieties. The supply chain begins with farmers, who harvest basmati paddy from the middle of October to late December, and bring the grain to regulated[3] markets to be auctioned[4]. The paddy is bought by private millers, who process it into rice. Processing includes de-husking, cleaning, polishing, and separating the rice into different quality grades. The rice then finds it way into retail domestic and international markets in various ways -- as generic basmati rice, under the miller's brand name, or under the brand names of different retailers or trading houses.

Panipat is a small (in terms of volume of transactions) regulated market situated in the state of Haryana. Our data set corresponds to the 1999 harvest season when basmati arrivals totaled approximately 127,000 quintals.

2.1. The Auction Process

In order to sell their paddy, farmers must contract with market agents known as *katcha arhtias*, whose job it is to weigh the grain, display the grain in lots in the market yard, and provide overnight storage, in return for 2 percent of the price at which the grain sells. The lots are sold one by one through an ascending auction. The auctioning of a lot begins with the auctioneer drawing out a handful of grain from it and visually inspecting it for quality. Following this, he announces a (per quintal) starting price. Interested buyers also pick up handfuls of grain and visually assess its quality. The auctioneer then raises the price incrementally and rapidly, while potential buyers may drop out by throwing down the grain they had picked up. The win price is the price at which the last but one buyer drops out. The auctioneer receives 0.8 percent of the win price.

[3] Regulated markets for foodgrains were set up under the Market Regulation Act. Such a market is run by a market committee whose functions include maintaining the physical infrastructure of the market place, supervise weighment, settle disputes, appoint auctioneers, maintain market records on grain arrivals and sales, and ensure that no unauthorized costs are passed on to the farmer.

[4] In some large markets, sales seem to take place primarily through bilateral deals between buyers and sellers, apparently because the number of auctioneers is too small to handle the large volume of grain arrivals.

2.2. The Data

The data set was constructed using a primary survey that we conducted in October-November 1999. This information was supplemented using market committee records, and personal interviews with millers and farmers. The market committee records provide us data on each lot that was auctioned from the middle of October through December. These data, for each lot, include the identities of the farmer (whose lot was auctioned) and the buyer, the amount of paddy sold (in quintals), and the win price (Rs. per quintal). These records cannot be used as a basis for analyzing the auctions as they do not record the quality of the lot sold. Basmati paddy is heterogeneous in several quality characteristics, variations in which affect the sale price. Second, the market committee data do not contain information on the auctioneer's starting price. Third, there is no record of either the number of potential bidders for a lot, or the number of bidders that are active post the announcement of the starting price. Finally, there is no record of the prices, during an auction, at which various players dropped out, and the identities of these players.

In order to redress these shortcomings, we tracked a random sample of 495 basmati auctions spread over four weeks in the peak marketing season. This forms our core data set. For each lot in this data set, we were able to record different quality characteristics of the paddy, the auctioneer's starting price, the number of active bidders (the number observed after the starting price was announced), the win price, and the identity of the winner. Due to the rapidity with which auctions take place, and the large number of farmers present, however, it was not possible to observe and record the identities of all active bidders and the prices at which they dropped out of the bidding. Thus we have no record of participation rates of players, although in some instances one can make reasonable guesses.

In order to record quality characteristics, we talked to agricultural scientists, market committee officials, agricultural experts working with the government, and bidders at the auctions to find out what characteristics affected paddy prices. From these conversations, seven quality characteristics emerged as being potentially important. These are: moisture content, uniformity of grain size, grain luster, the percentages of chaff, green and immature grain and broken grains, and a category of 'other' variables (encapsulating evidence of disease or pest infestation). If the moisture content is too low, the grain is brittle and prone to breaking while milling; but beyond a point, more moisture simply means there is less rice per kilogram of paddy. The ideal pre-milling moisture content is around 14 percent. If the grain is dull, broken, or immature, this translates after milling into low grade rice. The buyers at the auction perform simple visual and other tests to evaluate quality (such as breaking the grain and looking at the cross section for visual evidence of brittleness). For each lot in our sample, we performed similar tests while the auction was on, and evaluated each characteristic on a scale of either 1 to 3 (worst to best) or 1 to 2 (poor and good)[5]. Table 1 displays summary statistics for our sample.

Table 1: Summary Statistics of the Data

	Mean	Standard Deviation	Minimum	Maximum
Win price (Rs./quintal)	955	99	619	1196
Starting price (Rs./quintal)	862	85	500	1100
Number of active bidders	3.52	1.06	1	7
Moisture content (3=ideal)	2.69	0.49	1	3
Uniformity in grain size	2.22	0.57	1	3
Absence of chaff	2.21	0.61	1	3
Absence of brokens	1.60	0.49	1	2
Grain luster	1.81	0.39	1	2
Green and immature grain	1.21	0.41	1	2
Other	1.39	0.49	1	2

[5] Lab testing a sample from each lot for quality was not practicable.

Note the rather wide range in the start and win prices, in itself one indication of the substantial heterogeneity in the quality of basmati lots sold. The modal number of active bidders (those we observed participating in the bidding after the announcement of the start price) ranged between 3 and 4.

Apart from the above variables, we constructed a variable capturing the number of *potential* bidders on a given day, from the market committee records. After consulting market committee officials and auctioneers, we determined that this number would correspond to the number of players who won more than one lot on auction on a given day[6]. The number of potential bidders is thus a day-specific variable and varies between 5 and 10.

2.3. The Buyers and Bidders at the Basmati Auctions

As mentioned above, basmati paddy is bought by private millers, for processing and sale in domestic and international markets. A first feature about the buyers at Panipat market is asymmetry. Anecdotal evidence suggests that the size of mills differs greatly across players, and that millers with larger, more modern mills have the benefit of economies of scale. Some of these 'large' millers also cater to international markets that are perhaps not accessible to small players but are lucrative. Differential size of mills may translate, in the auction, to differential valuations for grain, and therefore, different rates of winning the auctions[7]. There were 4 large buyers (by market share) in Panipat. Their market shares in our sample are (Player G: 18%); (Player A: 13%); Player B: 12%; Player D: 11%.[8] Other buyers had market shares well below 5%; we refer to these as 'small' buyers.

Other than buyer asymmetry, the important distinction to be made, in the context of the auctions at Panipat, is between *local* millers, whose mills are within a radius of about ten kilometers from the market, and *distant* millers. Most of the millers are local (including all the small buyers), but there are important distant millers with mills which

[6] Cases of potential buyers who end up not winning anything on a given day appear to be few, and are compensated for by cases of buyers who win a few lots and drop out of the market. By and large, most buyers stay the course and bid for lots for an entire day of auctions.
[7] Since we do not observe participation rates, this is a conjecture but a reasonable one.
[8] The names of buyers are withheld to protect their identities.

are far off from the market. Mills are family owned. Local mills are represented at the bidding most often by a family member and sometimes by a close associate of the family owning the mill. As opposed to this, 'commission agents' called *pakka arhtias* bid for, and purchase grain on behalf of, the distant millers. Their contracts to purchase grain stretch over the entire season. Each distant miller has one such commission agent (who is licensed to operate in regulated markets such as Panipat) to purchase grain on his behalf.

During the bidding for a lot, the commission agent is in contact with his employer by mobile phone. This contact enables the agent to communicate the quality of the grain to the miller, and the miller to inform the agent of his valuation or willingness to pay for the lot. If this agent wins the auction, he gets 3% of the win price. This form of contract (with a 'commission' equal to a percentage of the sale price) has a long history. The commission agent is responsible for delivering the grain to the miller, who laboratory-tests a sample before accepting it. Because of this, and because the market committee records the sale price of each lot of grain, it is hard for a commission agent to miscommunicate the quality of grain during the bidding, or to misreport the win price.

Nevertheless, the form of the contract implies that a commission agent's payoff, and incentives, are not standard. As a receiver of a percentage of the win price, it is in his interest to win the lot at a higher, rather than a lower price [9].

In the Panipat market, Player G is a large local miller. Players A, B and D are commission agents representing distant mills. The distant millers have larger operations than Player G. Player G sells several grades of rice both in the domestic market and the Middle East. Some of the rice that Player G sells domestically, and all the rice that he sells outside India, sells under the brand names of trading houses and retailers. In contrast, the distant millers sell domestically, and to a lesser extent internationally, under their own, well-recognized brand names.

The estimation in Section 4 works with models of (conditionally) independent private values. The large market shares of 4 players and the small shares of the others suggests that the larger players' values are drawn from stochastically dominant

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[9] Indeed, if the miller is willing to pay up to v, the agent would prefer the price to go all the way up to v. Of course, winning at this price in an ascending auction ought to warn the miller about his agent's probity.

distributions. In order to probe the form that collusion among the four large players might take, we first summarize our sample data using ordinary least squares. The dependent variable is log win price, and the explanatory variables are: (i) the seven quality variables for paddy; (ii) dummies for each of the weeks of our survey; (iii) a dummy for Player G that takes the value 1 if he won the particular lot, and 0 if he did not. (iv) A similar dummy for the group of *pakka arhtias* comprising Players A, B and D. Table 2 presents the results.

Table 2: OLS Regression of log win price on quality characteristics

Dependent Variable: Log Win Price	Estimate	t-ratio
Moisture content	0.04	5.09
Uniformity in grain size	0.05	8.89
Absence of chaff	0.04	6.51
Absence of broken grain	0.03	5.15
Grain luster	0.02	2.50
Green & immature grain	0.03	4.42
Other factors	0.05	6.91
Week 2 dummy	0.001	0.22
Week 3 dummy	0.04	6.14
Player G dummy	-0.001	-0.13
Pakka arhtia dummy	0.01	1.99
Number of active bidders	0.02	7.88
Constant	6.26	265.95
R-squared = 0.62		

In an IPV model of an ascending auction, the win price of a given lot of grain is the second highest valuation (if this is greater than the starting price) amongst the set of values that the bidders have for the lot. The positive signs on the quality variables therefore indicate that players' valuations are positively related to quality characteristics. The dummy variables associated with the large players show that Player G, the large local miller, had win prices corresponding to the market average (controlling for the right hand variables), whereas the win prices for the commission agents were higher. If the value distributions of the large players dominate those of small players, this is consistent with the kind of collusive model described in Section 3.2: a cartel comprising Players G, A, B and D collude, assigning bidding rights on a lot to either Player G (with some probability π) or to the *group* including players A, B and D (with probability (1 - π)). Therefore, when Player G bids and wins, there are fewer competing potential bidders, and all of them are small players. The win price is a value drawn from a small player's distribution. When a commission agent, say Player A wins, there are more competing potential bidders, including players B and D; the win price is therefore likely to be higher on average.

3. The Models

We estimate simple asymmetric independent (conditional on observed quality parameters) private values auction models. The main assumptions common to these models are (i) Each potential bidder *i* has a valuation (or willingness to pay) v_i for a given lot, which is privately known to the bidder. (ii) v_i is a realization of a random variable V_i whose distribution F_i is common knowledge. (iii) The random variables $V_1,...,V_n$ are independent. The distributions F_i, $i = 1,...,n$, are lot specific and depend, for instance, on the (observed) quality characteristics of the lot. The assumed independence is conditional on these lot specific characteristics (we suppress the conditioning variables for convenience).

We believe that the asymmetric IPV assumption is a reasonable approximation for this market. For the millers who participate in the bidding, the valuation for a lot of grain is interpreted as the difference between the revenue from selling the processed rice and the costs of processing, transport and storage. The commission agents who bid on behalf of millers are in constant mobile phone contact with the millers during the bidding. Therefore, it is reasonable to assume that these agents have valuations communicated to them by the millers. Processing costs are known to vary across mills, but are privately known to millers[10]. Other firm characteristics that may be privately known and contribute to differences in valuations are the markets (domestic or foreign) that a firm sells in, and the quality grades of rice that it deals in[11].

The assumption of asymmetry (that the valuations of different millers spring from different latent distributions) is better than an *a priori* restriction of symmetry, particularly because market shares of buyers (Section 2) are skewed. Although the data do not record participation rates of bidders, there is good reason to believe that the large players listed in Section 2 have systematically higher valuations than the smaller players. This results in their winning auctions more frequently, which is reflected in their market shares.[12] This is despite the fact that collusion (which we establish later) amongst them restricts each of them to bid on fewer lots than would have been the case in the absence of collusion. From the observed market shares (as recorded by the market committee) and obtaining descriptions of the mills, in terms of their size and turnover, over the course of our interviews with various market players, we assume that there are four latent value distributions: G_i, $i = 1,2,3$ (1 = Player G, 2 = Player A, 3 = Players B and D) and *F* (distribution for the small players)[13].

The different models imply different joint distributions for the win price and the observed winner. The use of this joint distribution for estimation arises out of a recent identification result (Athey and Haile (2002) (Theorems 2(a), 3(i)) and Meilijson (1981)). The result implies that under the (conditional) IPV assumption, the joint distribution of the second highest order statistic of the random sample $v_1,...,v_n$ of valuations and the identity of the winner uniquely identifies the latent distributions from which the valuations are drawn.

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[10] One key determinant of processing cost is the *conversion factor* or ratio of unbroken rice per kilo of paddy. This depends on mill-specific machinery in a processing unit, along with quality characteristics of the paddy such as moisture content, and whether the processed output is parboiled or raw rice. Conversion factors (conditional on paddy quality) vary significantly across mills, and are privately known.

[11] Our data lists the win price for each lot, but information on other bids is absent. Thus simple tests of the IPV specification, such as a regression of the highest bid on the second highest bid and quality variables, cannot be performed to check for presence of correlation. The absence of resale reduces the appropriateness of an alternative, common values framework.

[12] From the market committee records, we find that Player G exited the market on November 19, 1999, with more than a month left in the peak marketing season. Our sample ends soon after. Therefore, G's market share in our sample is higher than it is in the population (determined by the market committee records).

[13] This specification of distributions turns out to be robust to changes.

3.1. The Noncooperative Model

It is well known that in this standard model, it is a dominant strategy for a player with valuation *v* to stay with the bidding until the price ascends to equal this valuation, and to drop out thereafter. Therefore, the winner is the bidder with the highest valuation, and the win price is the second highest valuation, amongst the values $v_1,...,v_n$, provided that both valuations exceed the auctioneer's starting price *r*. The win price is therefore a realization of the second highest order statistic. On the other hand, the win price is *r* if exactly one valuation exceeds the starting price, and the lot is unsold if all valuations are less than the starting price. In our sample 7 percent of the lots sold at the starting price, and we did not observe any unsold lots.

Our data includes, for each auctioned lot, a number *p* of potential bidders, of which *m* are small players, the auctioneer's starting price *r*, a number *n* of active bidders (whose values exceed *r*), the win price *w*, and the identity of the winner. Under the noncooperative model, there are therefore *m* + 4 potential bidders (i.e. 4 large players). So for example, if Player G (whose values are drawn from the distribution G_1) wins at a price *w* greater than *r*, the joint distribution of the second highest order statistic, the winner being equal to Player G, and *n* out of *m* + 4 potential bidders drawing values higher than *r* is given by Equation 1 below.

$$H_G(w) = G_2(r)(G_3(r))^2 F(r)^{m-n+1} C(m,n-1)\left[\int_r^w (F(t)-F(r))^{n-1} dG_1(t) + (F(w)-F(r))^{n-1}(1-G_1(w))\right] +$$

$$(G_3(r))^2 C(m,n-2)F(r)^{m-n+2}\left[\begin{array}{l}\int_r^w (F(t)-F(r))^{n-2}(G_2(t)-G_2(r))dG_1(t) + (F(w)-F(r)).^{n-2} \\ . \qquad .(G_2(w)-G_2(r))(1-G_1(w))\end{array}\right] +$$

$$2G_2(r)G_3(r)C(m,n-2)F(r)^{m-n+2}\left[\begin{array}{l}\int_r^w (F(t)-F(r))^{n-2}(G_3(t)-G_3(r))dG_1(t) + (F(w)-F(r))^{n-2}. \\ \qquad .(G_3(w)-G_3(r))(1-G_1(w))\end{array}\right] +$$

$$G_2(r)C(m,n-3)F(r)^{m-n+3}\left[\begin{array}{l}\int_r^w (F(t)-F(r))^{n-3}(G_3(t)-G_3(r))^2 dG_1(t) + (F(w)-F(r))^{n-3}. \\ \qquad .(G_3(w)-G_3(r))(1-G_1(w))\end{array}\right] +$$

$$2G_3(r)C(m,n-3)F(r)^{m-n+3}\left[\begin{array}{l}\int_r^w (F(t)-F(r))^{n-3}(G_2(t)-G_2(r))(G_3(t)-G_3(r))dG_1(t) + \\ (F(w)-F(r))^{n-3}(G_2(w)-G_2(r))(G_3(w)-G_3(r))(1-G_1(w))\end{array}\right] +$$

$$C(m,n-4)F(r)^{m-n+4}\left[\begin{array}{l}\int_r^w (F(t)-F(r))^{n-4}(G_2(t)-G_2(r))(G_3(t)-G_3(r))^2 dG_1(t) + \\ (F(w)-F(r))^{n-4}.(G_2(w)-G_2(r))(G_3(w)-G_3(r))^2(1-G_1(w))\end{array}\right] \qquad (1)$$

The first of the six terms in Eq(1) corresponds to events in which Players A, B and D have values less than the starting price *r*, so that the *n* active bidders consist of Player G (the winner) and *n*-1 small players. As there are *m* small potential bidders, there are *C*(*m*, *n*-1) such events. Note that the probability $(1-F(r))^{n-1}$ that *n*-1 small players have values exceeding *r* is not shown, as it gets cancelled with the same term that crops up in the denominator in the lower truncated terms appearing in the square brackets. In the square brackets, the integral covers events in which all *n* bidders' values are less than or equal to the win price, and the following term relates to the event that the winner Player G's value strictly exceeds the win price, while the other values are less than or equal to the win price. In similar fashion to the entire first term, the second to sixth terms capture events in which one or more of Players A, B and D have values exceeding *r*. Finally, note that Eq(1) is valid if the number *n* of active bidders is at least 4. If *n* = 3, the last term drops out; if *n* = 2, the last three terms drop out. The density $h_G(w)$ corresponding to Eq(1) is obtained by differentiating it with respect to *w* (see Appendix A). The maximum likelihood estimation in Section 4 uses this density for data points in which the winner is Player G and the win price is greater than the starting price. Similar expressions $H_i(w), i = A, B, D, s,$ hold when the winners are, respectively, Player A,B,D or a specific small buyer; the corresponding densities $h_i(w)$ are used in the maximum likelihood estimation of the model.

On the other hand, Player G wins a lot *at* the starting price *r* if only his value for the lot exceeds *r*. In this case, the joint density of the win price (which equals *r*) and the winner being Player G is given by equation 2.

$$\Pr(winprice = r, winner = G) = F(r)^m G_2(r)(G_3(r))^2[1 - G_1(r)] \quad (2)$$

Similar expressions are used when other players win at the starting price for a lot.

3.2. Bid Rotation between Player G and the Group of Commission Agents

In this alternative model, the four large players form a cartel. They collude by allocating bidding rights on each lot to either Player G, or the *group* comprising Players A, B and D. This nonstandard form of bid rotation is driven by the fact that the commission agents buy *on behalf* of millers, and their incentives are not the same as those of the millers they represent. Player G is a local miller. His chances of winning a given lot in the absence of bidding by the other large players is higher than in the noncooperative model. Moreover, the win prices are also likely to be lower, as there are now fewer potential bidders, and in particular bidders likely to have relatively high values are absent. Therefore, if Player G is assigned to bid on a sufficiently high number of lots, his expected payoff is greater than under noncooperation. On the other hand, a large commission agent's payoff from winning is a fraction of the win price. This payoff is higher the higher is the win price. Therefore, a bid rotation scheme in which all 3 commission agents bid for a lot has two opposing effects on the expected payoff of a commission agent, relative to noncooperation. Excluding Player G improves a commission agent's chances of winning, but also reduces the expected win price. If the net effect on his expected payoff is positive, such a collusive scheme may work, for instance, by a threat to revert to noncooperation in future auctions if a player deviates from colluding.[14]

Under the hypothesis that collusion takes this form, the joint distribution of the win price, a specific player being the winner, and *n* out of *p* bidders being active is somewhat different. For example, for a lot which is won by Player G (at a price greater than the starting price), we know that the cartel has assigned bidding rights for this lot to Player G. Therefore, the joint distribution is given by Eq(3) below.

$$H_G^C(w) = F(r)^{m-n+1} C(m, n-1) \left[\int_r^w (F(t) - F(r))^{n-1} dG_1(t) + (F(w) - F(r))^{n-1} (1 - G_1(w)) \right] \quad (3)$$

The major difference between this and the corresponding noncooperative expression (Eq(1)) is that the large commission agents are absent, as Player G is bidding. The density $h_G^C(w)$ is the derivative of Eq(3) with respect to *w* (see Appendix A). For lots won by one of the large commission agents, we infer that the cartel has assigned the group comprising Players A, B and D (but not Player G) to bid for the lot. Joint densities $h_i^C(w), i = A, B, D$ for the large commission agents are therefore derived by excluding Player G from the set of potential bidders. For lots won by Player G at the starting price, the joint density analogous to Eq(2) is given by Eq(4) below.

$$h_G^C(r) = F(r)^m [1 - G_1(r)] \quad (4)$$

Specifying a joint distribution and density when a specific small player wins a lot is more complicated, since we do not observe whether Player G, or the group of commission agents, was bidding for this lot. Suppose the win price is higher than *r,* a specific small player wins, and *n* bidders are active (their values exceed *r*). We first work out the joint density $h_{s,G}^C(w)$ that would obtain if Player G were the large player bidding, and the joint density $h_{s,CA}^C(w)$ that would obtain if the large commission agents were bidding. For such lots, we then use the joint density $a h_{s,G}^C(w) + (1-a) h_{s,CA}^C(w)$. The number *a* may be interpreted as the probability, conditional on a specific small player winning

[14] Of course, such a threat, and the suggested ordering of collusive payoffs being higher, is not necessary to sustain collusion. More sophisticated strategies using minmax threats and 'reconciliation states' also work. Recent work (Skrzypacz and Hopenhayn (2004)) analyzes how efficient collusion can be sustained in a repeated English auction setting with independent private values.

the lot, that Player G is a bidder. Similarly, for lots selling to specific small players at starting prices, we use the density $ah^{C}_{s,G}(r) + (1-a)h^{C}_{s,CA}(r)$.

3.3. Other Models

In addition to the above, we have estimated several other models. Two of these are worth reporting, as they can provide alternative benchmarks. We report estimates of these models in Appendix B. The first of these is the *efficient collusion* model. In such collusion, the cartel of colluding players may set up a pre-auction 'knockout' in which players reveal their values, the player with the highest (revealed) value gets to bid on the object, and transfer payments are worked out across players. In such a mechanism, players have an incentive to reveal their true values (see Graham and Marshall (1987) for an early discussion).

In the market that we study, efficient collusion amongst the four cartel members is not a promising prospect for two reasons. One, auctions are conducted in the open, and take place rapidly; both these factors probably preclude opportunities for revealing valuations and hence efficient collusion. Two, the incentives of the commission agents are nonstandard, so that it is not obvious that excluding all but one cartel member from bidding is best for such an agent (as this may lead to too low a win price). Nevertheless, we estimate an efficient collusion model and test it against the bid rotation described in Section 3.2, as it is one of the more commonly discussed models of collusion in the empirical auctions literature.

The joint density of the win price and a specific cartel member being the winner, and *n* bidders being active, now involves an integral. For example, suppose a cartel player, say Player G, is the winner, and the win price exceeds the starting price. This implies that the other cartel members have values less than Player G's value, and Player G's value exceeds the win price. These members don't bid. Thus the win price must be the value of one of the small players, and the rest of the small players must have values less than the win price. Note however that cartel members who do not bid can also have values exceeding the win price (hence the integral). The joint density is therefore given by Eq(5) below.

$$h_G^E(w) = C(m, n-1)F(r)^{m-n+1}(n-1)(F(w) - F(r))^{n-2} f(w).\int_w^\infty G_2(t)(G_3(t))^2 dG_1(t) \quad (5)$$

The expression takes into account that any of the (*n*-1) small players with values exceeding the starting price could have the value which equals the win price; so this can happen in (*n*-1) ways. The integral describes the event that Player G's value exceeds *w*, and is the highest value amongst the cartel members. On the other hand, if Player G wins *at* the starting price, none of the small players' values exceed the starting price. Hence the joint density is

$$h_G^E(r) = F(r)^m \int_r^\infty G_2(t)(G_3(t))^2 dG_1(t) \quad (6)$$

Joint densities when the winner is some other cartel member are similar. If the winner of a given lot is a specific small player outside the cartel, the joint density we work with uses the information that all other players have values less than or equal to the win price, while the small player who wins has a higher value. Therefore, the expressions are closer in form to those used for the other models.

Finally, we report results for a *standard bid rotation* model. In this model, the collusive scheme is that for any given lot of grain, exactly one of the four cartel players gets the right to bid. Who gets to bid for a lot depends on the outcome of a randomization; revelation of values is not involved. For a large commission agent, the fact that other large players are absent, while increasing the probability of winning, decreases the expected win price. Again, since such an agent's payoff increases in the win price, it is not clear that such exclusion of other large players is necessarily beneficial. We note here that the regression in Section 2 provides some evidence against standard bid rotation, and in favor of the collusive scheme described in Section 3.2. Under standard bid rotation, we would not expect that the large commission agents would have significantly higher win prices compared to Player G's win prices; but this is a distinct possibility under bid rotation between Player G and the *group* of large commission agents as a whole.

4. Estimation and Results

In order to estimate the non-cooperative and the various models of collusion, we specify that the latent distributions of valuations F and $G_i, i = 1,2,3$ for a given lot *t* are lognormal with means $x_t\beta$ and $x_t\beta + \mu_i, i = 1,2,3$, respectively. Here, x_t is a vector of characteristics for lot *t*, including the seven quality characteristics, dummies for two weeks, and a constant term; β is the corresponding vector of parameters. The μ_i are mean shifters for the distributions of the large players. We make the simplifying assumption that the three distributions G_i have the same variance. We estimate all models using the method of maximum likelihood, with densities specified as indicated in the previous section.

We find that the collusive scheme described in Section 3.2, viz. bid rotation between Player G, (the large local miller) and the *group* of large commission agents (Players A, B and D) best describes the data. Parameter estimates of this model are set out below in Table 3. Estimates for the non-cooperative model, along with the efficient collusion and the standard bid rotation models, are given in Appendix B.

Table 3: Parameter Estimates for Model with Bid Rotation Between Player G and group of Commission Agents

	Estimate	t-ratio
Moisture content	0.04	6.25
Uniformity in grain size	0.05	9.52
Absence of chaff	0.03	5.70
Absence of brokens	0.04	5.62
Grain luster	0.03	2.93
Green & immature grain	0.04	5.29
Other factors	0.06	9.30
Week 2 dummy	-0.03	-3.31
Week 3 dummy	0.03	4.40
Difference in mean for Player G (μ_1)	0.16	17.37
Difference in mean for Player A (μ_2)	0.24	14.87
Difference in mean, Players B and D (μ_3)	0.24	19.99
Variance *F*	0.21	33.96
Variance G_i	0.07	19.79
Constant	6.10	250.44
Log Likelihood value	-574.2	

The log likelihood value for this model is much higher than that for the other models (Appendix B). We also test between the above model and the benchmark non-cooperative model using Vuong's (1989) test. The test statistic evaluates to 2.02 and favours this form of the collusive model over the noncooperative model.

The signs on the quality variables are positive, indicating that players' values are positively affected by higher quality. Further, the large local miller, Player G, and the three commission agents (A, B and D) draw values from distributions (conditional on quality) which have higher means than the mean of the small players' distribution. This fits well with the fact that the four cartel players have large market shares while the others do not. Notice also that the mean for Player G's distribution is significantly lower than for the group of commission agents. One expects, therefore, that Player G benefits greatly if the group of commission agents does not bid when he does.

5. Simulations

5.1. A Comparison of Payoffs of the Large Players in the Presence and Absence of Collusion

In order to get a sense of how profitable it is for the cartel to collude per the model in Section 3.2, we conduct simulations to evaluate the expected payoffs under collusion and noncooperation. We fix the distributions of the various players, using the parameter estimates $\hat{\beta}, \hat{\mu}_i, i = 1,2,3$, of the collusive model reported in Table 3. The vector x_t is evaluated at the average quality of the lots that sold in the third week of our sample. We then simulate 10,000 auctions with the players drawing values from their respective distributions. To simulate the noncooperative model, we have all players draw valuations and bid at every auction; thus there is one value drawn randomly from the distribution G_1, one from G_2, two from G_3, and several (varying from 5 to 10) from F. To simulate collusion, we evaluate one scenario where Player G is the only large player, and another scenario in which Players A, B and D (but not Player G) are the large players who participate. In either case, Player G's payoff, when he wins, is the difference between his valuation and the win price. His average payoff per lot is the sum of these differences for lots that he wins, divided by 10,000. For a commission agent (Player A or B or D), the average payoff per lot is 3 percent of the sum of win prices (for lots won by the agent), divided by 10,000 [15]. We use a starting price of Rs. 893 per quintal, the average for that week.

Table 4 presents these payoffs for the large players, under collusive and non-cooperative behavior[16]. We vary the number of small potential bidders between 5 and 10. Columns 2 and 3 give Player G's payoffs per quintal under collusion and

[15] In most of our simulations comparing collusive versus noncooperative payoffs, very few lots are unsold (i.e., at least one valuation exceeds the starting price). This is consistent with what we observed in our sample; we recorded no lot that remained unsold once a starting price had been announced.

[16] Simulations with other quality vectors indicate similar results.

noncooperation. Columns 4 and 5 give average win prices per quintal for Player B or D (win prices for Player A are very similar as well, and hence not reported), under collusion and noncooperation. Player B's payoff for a lot is 3% of this times the number of quintals in the lot.

Table 4: Expected Payoffs (Rs./quintal) of Large Players under Collusion and Noncooperation

# small potential bidders	For G under collusion	For G under noncooperation	For B/D under collusion	For B/D under noncooperation
m = 5	17.45	1.10	224.23	210.60
m = 6	14.42	1.05	203.10	202.70
m = 7	11.66	0.97	192.86	178.81
m = 8	9.46	0.87	175.06	165.48
m = 9	7.64	0.68	162.99	155.82
m = 10	6.53	0.62	155.33	150.43

As indicated above, for each of the large players, the expected payoffs under collusion exceed that of noncooperative play. Since the average lot of grain is about 50 quintals, the difference in expected payoffs per lot is fifty times the difference between columns 1 and 2 (for Player G) and 3% of 50 times the difference between columns 3 and 4 (for Players B and D). Given the larger expected payoffs under collusion, the cartel can be sustained, in a repeated auctions setting, by deterring deviations using a threat to revert to noncooperation[17]. This reiterates the key point that given our parameter estimates, the nonstandard collusive scheme described in Section 3.2 can work, owing to the different incentives of commission agents, and serves as a further validation of the collusive model.

5.2. The Effect of Collusion on Win Price

[17] We do not model this formally. Such formalization would require attention to some key details, such as the capacity constraints on the mills owned by cartel players, and their current inventories of paddy. Data on these are unavailable.

We also use simulations to compute average win prices under collusive and noncooperative behavior, fixing the distributions in the manner described above. As before, under bid rotation between Player G and the group comprising Players A, B and D, we consider two scenarios: one in which Player G is the only large bidder, and the other where the commission agents are the large bidders. The simulation of non-cooperation assumes all four large bidders participate in the bidding.

Given that this is a nonstandard form of collusion, it is interesting to assess the extent to which market prices are thus depressed. For comparisons, we also simulate win prices under standard bid rotation amongst the 4 large players (Section 3.3) although this form of collusion does not characterize this market. However, the standard bid rotation model might characterize an alternative market structure, one where the 4 large players are all local millers. In that case, close associates would dictate their bidding, and their payoffs (unlike those of the commission agents) would be standard. If efficient collusion is hard to carry out due to the open and rapid nature of the auctions, standard bid rotation may be a viable option. Table 5 provides the results.

Table 5: Average win prices under two forms of collusion and under non-cooperation (Rs./quintal)

Number of small players *m* drawing valuations from *F*	Collusion between G & group of *pakka arhtias*		Non-cooperation	Standard bid rotation	
	G bids	*Pakka arhtias* bid		G bids	A bids
5	960.91	1048.42	1049.38	959.12	983.80
6	970.53	1052.53	1055.44	970.24	995.27
7	983.07	1058.75	1060.22	983.17	1007.75
8	991.94	1061.77	1064.14	992.27	1016.77
9	1003.58	1066.86	1069.52	1002.85	1025.94
10	1012.49	1071.27	1073.73	1012.47	1034.47

Note first that collusion between Player G and the commission agents results in significantly lower win prices (Rs.60 - 90 per quintal, compared to the noncooperative case), when Player G is the large player assigned to bid for a lot. However, when the commission agents bid, the difference declines sharply, to between Rs. 1-3 per quintal. This is because on these lots, there is only one less large bidder; when Player G bids, there are 3 fewer large bidders. Thus farmers lose substantially only on lots where G bids.

Note that the higher average prices when the commission agents *bid* is similar to the OLS regression result (Table 2) that showed that they pay higher prices (than Player G) when they *win*.

Had this market been characterized by standard bid rotation instead, the prices received by farmers would have been substantially lower; by about Rs. 60-90 per quintal on lots that Player G bids for, to approximately Rs. 40-60 lower on lots that Player A bid on. So, we may infer that the form of collusion which describes the data best, namely, bid rotation between Player G and the *group* of commission agents, lowers prices much less than other forms of collusion.

5.3. The Effect of Alternative Starting Prices on Win Prices and Sales Percentage

It is natural to ask whether, given the estimated latent distributions, the starting prices for lots maximize the expected revenue of the sellers (farmers). However, in the grain market setting of this paper, the auctioneer is an intermediary and not the owner of the grain. This leads to interesting possibilities and questions (about the optimality of the reserve prices, for example) that are otherwise absent. These questions are the subject of ongoing research and beyond the scope of this paper. Nevertheless, it is useful to explore in a simulation, (given the form of collusion that we detect), the expected revenue that alternative starting prices result in, for a lot of grain of given quality. In Table 6, we present an extract of the kind of results that we get as an answer to this question[18]. In this extract, we again take the case of a lot of average quality in week 3 of the sample and simulate 10,000 auctions each with Player G, and the group of commission agents, as the large bidders.

[18] If bidders are symmetric, the optimal reserve price does not depend on the number of bidders. Since our setting has asymmetry, the simulation reports results varying the number of bidders.

Table 6: The Effect of Alternative Starting Prices on Win Price and Sale Percentage

	Player G bids		*Pakka arhtias* bid	
Starting Price (*m* : # small potential bidders)	% lots sold	Average win price of lots sold (Rs./quintal)	% lots sold	Average win prices of lots sold (Rs./quintal)
m = 5				
800	99.9	947.75	100	1048.41
850	99.4	951.50	100	1048.41
875	97.9	956.03	100	1048.41
900	94.9	963.30	100	1048.42
925	89.9	973.50	100	1048.50
950	82.5	986.94	100	1049.00
1000	62.8	1021.69	100	1053.25
m = 8				
800	100	987.80	100	1061.77
850	99.9	988.53	100	1061.77
875	99.6	989.86	100	1061.77
900	98.4	993.12	100	1061.77
925	95.8	998.99	100	1061.80
950	91.3	1008.09	100	1062.08
1000	76.2	1035.68	100	1064.88
m = 10				
800	100	1010.38	100	1071.27
850	99.9	1010.78	100	1071.27
875	99.7	1011.49	100	1071.27
900	99.2	1013.16	100	1071.27
925	97.7	1016.83	100	1071.28
950	94.8	1023.20	100	1071.34
1000	83.1	1045.44	100	1076.40

If we assume that an unsold lot yields a payoff of 0, then multiplying the average win price with the percentage of lots sold gives a measure of expected revenue. For Player G, we observe that higher starting prices reduce the percentage of lots sold and increase the average win price, and that the net effect is to reduce expected revenue. Of the starting prices reported, the lowest one, Rs.800, corresponds to the highest expected revenue, irrespective of the number of bidders. As opposed to this, when the group of commission agents bid, the highest starting price reported, Rs.1000, corresponds to the highest expected revenue. So, if the auctioneer can detect the pattern of bidding corresponding to our collusive scheme, one instrument available to him is to tailor the starting price quite differently, and substantially higher, when the commission agents bid. In our data set, the average starting price when the commission agents A, B or D win is higher than when G wins, by about Rs.50. Our simulations indicate that the difference ought to be much larger, in order to maximize expected revenue.

Conclusions

This paper examines basmati paddy auctions in a grain market in North India, using new data from a primary survey, through the structural estimation of several simple auction models. It introduces a key feature that characterizes this market -- that some distant millers buy grain through commission agents, whose contracts make their payoffs nonstandard. In particular, their payoffs conditional on winning increase in the win price. This feature may help to explain the form of collusion that best explains the data: bid rotation between a large local miller and the *group* of commission agents purchasing on behalf of large distant millers. The paper also shows that efficient collusion within this cartel does not explain the data well.

Simulations then indicate the following points. (i) The payoffs to cartel members under collusion of the form described are higher than under noncooperation. (ii) This form of collusion has little downward impact on win prices on lots on which the group of commission agents bid. (iii) An auctioneer concerned about expected revenue, and aware of the form of collusion, would choose much higher starting prices for lots on which the commission agents bid, versus those on which the local large miller bids.

Like almost all empirical work on structural estimation of auctions, this paper treats each auction in the data set as a separate single unit auction. But grain auctions across the harvest and marketing months can alternatively be viewed as a multiunit, sequential auction. This view has the potential to considerably enrich the work in this paper, especially as the kind of collusion that seems to be taking place is better justified in a framework of repeated auctions. There is some recent, interesting empirical work in dynamic frameworks (Jofre-Bonet and Pesendorfer (2003); Donald, Paarsch and Robert (2002)), and an attempt can be made to adapt such methods. However, data on at least two additional fronts are needed before one can explicitly model a dynamic auction and estimate it. The first is a full set of bids (or prices at which bidders drop out)[19]; the second is data on mills' inventories of paddy across the season.

[19] This data will considerably enrich estimation (and testing) of the static theory as well, by enlarging the variety of models that can be used considerably beyond the conditionally independent private values model.

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Appendix A.

1. The Density Corresponding to $H_G(w)$ (Eq.(1))

$$
\begin{aligned}
&h_G(w) = \left(1 - G_1(w)\right) \Big\{ G_2(r)(G_3(r))^2 F(r)^{m-n+1} C(m,n-1)\big[(n-1)(F(w)-F(r)^{n-2} f(w)\big] + \\
&(G_3(r))^2 C(m,n-2)F(r)^{m-n+2}\big[(F(w)-F(r))^{n-2} g_2(w) + (n-2)(F(w)-F(r))^{n-3} f(w)(G_2(w)-G_2(r))\big] + \\
&2G_2(r)G_3(r)C(m,n-2)F(r)^{m-n+2}\big[(F(w)-F(r))^{n-2} g_3(w) + (n-2)(F(w)-F(r))^{n-3} f(w)(G_3(w)-G_3(r))\big] + \\
&G_2(r)C(m,n-3)F(r)^{m-n+3}\big[(F(w)-F(r))^{n-3} g_3(w) + (n-3)(F(w)-F(r))^{n-4} f(w)(G_3(w)-G_3(r))\big] + \\
&2G_3(r)C(m,n-3)F(r)^{m-n+3}\begin{bmatrix} (F(w)-F(r))^{n-3}\{(G_2(w)-G_2(r))g_3(w) + (G_3(w)-G_3(r))g_2(w)\} + \\ (n-3)(F(w)-F(r))^{n-4} f(w)(G_2(w)-G_2(r))(G_3(w)-G_3(r)) \end{bmatrix} + \\
&C(m,n-4)F(r)^{m-n+4}\begin{bmatrix} (F(w)-F(r))^{n-4}\{2(G_2(w)-G_2(r))(G_3(w)-G_3(r))g_3(w) + \\ (G_3(w)-G_3(r))^2 g_2(w)\} + \\ (n-4)(F(w)-F(r))^{n-5} f(w)(G_2(w)-G_2(r))(G_3(w)-G_3(r))^2 \end{bmatrix} \Big\} \qquad (A1)
\end{aligned}
$$

2. The Density Corresponding to $H_G^C(w)$ (Eq.(3))

$$
h_G^C(w) = F(r)^{m-n+1} C(m,n-1)(1-G_1(w))(n-1)(F(w)-F(r))^{n-2} f(w) \qquad (A2)
$$

Appendix B.

1. Maximum Likelihood Estimates of the Noncooperative Model

	Estimated Coefficient	t-ratio
Moisture content (3=ideal)	0.04	7.86
Uniformity in grain size (3=best)	0.06	17.06
Presence of chaff (3=best)	0.04	12.12
Presence of brokens (2=least broken)	0.03	7.13
Grain lustre (2=best)	0.03	4.75
Green & immature grain (2=best)	0.06	12.26
Other factors (2=best)	0.05	8.67
Week 2 dummy	0.01	1.07
Week 3 dummy	0.05	12.26
Difference in mean for Player G	0.76	25.92
Difference in mean for Player A	0.74	25.10
Difference in mean for Players B and D	0.74	25.77
Constant	5.48	176..46
Log Likelihood value	-675.43	

2. Maximum Likelihood Estimates of the Efficient Collusion Model

	Estimated Coefficient	t-ratio
Moisture content (3=ideal)	0.04	5.99
Uniformity in grain size (3=best)	0.06	12.68
Presence of chaff (3=best)	0.03	6.92
Presence of brokens (2=least broken)	0.04	5.75
Grain lustre (2=best)	0.03	3.05
Green & immature grain (2=best)	0.07	9.99
Other factors (2=best)	0.04	5.51

Week 2 dummy	-0.03	-2.87
Week 3 dummy	0.04	5.88
Difference in mean for Player G	0.09	5.83
Difference in mean for Player A	0.06	3.68
Difference in mean for Players B and D	0.04	3.17
Constant	6.11	297.70
Log Likelihood value	-866.23	

3. Maximum Likelihood Estimates of the Individual Bid Rotation Model

	Estimated Coefficient	t-ratio
Moisture content (3=ideal)	0.05	6.47
Uniformity in grain size (3=best)	0.05	11.40
Presence of chaff (3=best)	0.04	7.62
Presence of brokens (2=least broken)	0.03	5.62
Grain lustre (2=best)	0.03	0.03
Green & immature grain (2=best)	0.06	9.46
Other factors (2=best)	0.05	6.27
Week 2 dummy	-0.03	-3.55
Week 3 dummy	0.02	3.59
Difference in mean for Player G	0.80	Neg
Difference in mean for Player A	0.15	14.39
Difference in mean for Players B and D	0.80	Neg
Constant	6.10	288.55
Log Likelihood value	-601.81	

* Complete list of working papers is available at the CDE website: **http://www.cdedse.org/worklist.pdf**