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Perfect Competition and the Keynesian Cross: Revisiting Tobin

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Abstract

I look at an exogenous decrease in the desire to save in a two-sector-two-period overlapping generations model, where the consumption good is capital-intensive and the elasticities of substitution in production are “small”. It is shown that there is a Keynesian-type multiplier at work, even though the model is a competitive one with full employment (and inelastic labour supply). It is reminiscent of Tobin (1975) who had shown thirty years ago that Keynesian results could be obtained with (short run) Marshallian dynamics (albeit in an ad-hoc model).

Keywords: Overlapping Generations, Two-sector Models, Multiplier, Keynesian Cross.
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1. Introduction

It is almost thirty years since James Tobin's paper "Keynesian Models of Recession and Depression" (Tobin (1975)) appeared. In that paper Tobin, following a suggestion by Milton Friedman, showed that Keynesian conclusions followed naturally if we appended a "Marshallian" (short-run) dynamic adjustment to a standard macro model. If, per contra, the dynamics was of the "Walrasian" variety, classical results followed. The framework that Tobin used was, of course, a standard work-horse of that era--an IS-LM-Phillips Curve model. Tobin's basic insight seems to have been forgotten since.

In the last two decades, various authors have sought to provide micro-foundations for macroeconomics. While macroeconomists of a classical persuasion tend to prefer a competitive framework, those who want to provide Keynesian economics with microeconomic underpinnings prefer a non-competitive (usually a monopolistically competitive) framework. In an imperfectly competitive framework (and/or in a model with increasing returns to scale) it is possible to obtain a multiplier-type relationship and policy intervention can (under certain circumstances) improve the welfare of all individuals[1].

There is, however, near-unanimity (the qualification "near" is probably redundant) that a multiplier cannot be obtained in a competitive model. For instance in the words of an influential survey (Matsuyama (1995)):"…the standard neoclassical paradigm, exemplified by Kenneth Arrow and Frank Hahn (1971), emphasizes the self-adjusting mechanisms of market forces with its efficient resource allocation. As different activities compete for scarce resources, expansion of one activity comes only at the expense of others, which tends to dampen any perturbation to the system." (p. 702). And, on the other hand: "The departure from perfect competition means that the firm, faced with downward sloping demands, sets prices above marginal cost…. Aggregate demand management could be effective in stimulating aggregate economic activity as well as raising the welfare of the economy." (p. 703).

In this paper, I revisit Tobin's insight but in an optimizing setting. I show that it is possible to obtain a multiplier—that is reminiscent of the text-book Keynesian-

[1] See, for instance, Blanchard and Kiyotaki (1987), Gali (1994), Mankiw (1988), Kiyotaki (1988), Startz (1984), (1989), Heijdra (1998) and Weitzman (1982); for surveys see Dixon and Rankin (1994), Matsuyama (1995) and Solow (1998).

cross diagram--in a perfectly competitive optimizing model. This is done in a two-sector overlapping generation model, where the consumption good is capital-intensive and the elasticities of substitution between inputs in both sectors are "low"—the latter assumption is equivalent to assuming that the short run dynamics is of the Marshallian variety[2]. The model satisfies the usual conditions for dynamic efficiency viz. the rate of interest exceeds the population growth rate[3]. The new equilibrium path does not Pareto-dominate the initial equilibrium, but may improve the welfare of all those individuals who "contribute" to the increase in aggregate demand (i.e., all generations barring the initially old).

The model is described in the next section and the aggregate demand experiment is introduced. In section 3, I provide an example with Leontief technologies, while section 4 looks at the case with non-zero elasticities of substitution in production. Finally, in section 5 there are some concluding comments.

2. The Model

The (closed) economy consists of overlapping generations of individuals or households with two-period lives. Each household supplies one unit of labour in the first period of its life and in the second period consumes the saving from the first period plus the return on these savings. No household is altruistically linked to any future generations i.e., there are no bequests or inheritances. The population is constant—the size of the population is assumed to be two, the size of each generation is unity. Agents have perfect foresight. We shall study the properties of the model by log-linearizing it around the initial steady state.

The representative household born in time period t maximises the following logarithmic utility function[4]

$$U_t = \log\ C_t^t + (1+\rho)^{-1} \log\ C_{t+1}^t \qquad t=\ldots.-2,-1,0,1,2\ldots. \qquad (1a)$$

[2] There is some empirical justification for both our capital intensity and low elasticities of substitution assumptions. On capital intensities see e.g., Takahashi, Mashiya and Sakagami (2004) who find that the Japanese investment goods sector is more labour-intensive (since 1975). The authors have calculated the capital-intensities for the two sectors for the UK, the US and Germany and found the investment good to be labour-intensive. Buffie (2001) summarizes time series studies as having obtained elasticities in production of around 0.5.

[3] See Abel et al. (1989), Buiter (1981), Lang (1996), Matsuyama (1991) and Shell (1971) for discussions on the possibility of overaccumulation in two-period overlapping generations models.

[4] Galor (1992)(section 6), Azariadis (1993) (example 13.5) and Cremers (2001) use the two-sector overlapping generations model with logarithmic utility.

where C_{t+i}^{t} is the consumption in period t+i of a household born in t and $\rho > 0$ is the rate of time preference.

Its lifetime budget constraint is

$$W_t = C_t^t + (1+r_{t+1})^{-1} \cdot C_{t+1}^{\ t} \qquad (1b)$$

where W_t is the wage rate in time period t and r_{t+1} the own interest rate on one period consumption loans between t and t+1.

This yields

$$C_t^t = [(1 + \rho)/(2 + \rho)]W_t \qquad (2a)$$

and $C_{t+1}^t = [(1 + r_{t+1}) / (2 + \rho)]W_t$ (2b)

The savings and indirect utility functions are given by (where m is a constant),

$$S_t \equiv W_t - C_t^t = (2 + \rho)^{-1} W_t \qquad (3)$$

$$V_t = V(W_t, 1 + r_{t+1}) = m + [(2 + \rho) \log W_t + \log(1 + r_{t+1})]/(1 + \rho) \qquad (4)$$

Note that savings in time period t do not depend on any variable in time period t+1 (e.g., the expected return on capital in t+1). This makes the model's dynamics scalar—i.e., it can be represented by a single difference equation (equation (7) below).

The production side of the economy is represented by two cost-equal-to-price equations. The consumption good (C) and the investment good (I) are produced under conditions of constant returns to scale using the two inputs, capital (K) and labour (L). All inputs are mobile between sectors instantaneously. Capital is assumed to depreciate completely in the process of production[5].

$$a_{LC} \cdot W_t + a_{KC} \cdot R_t = 1 \qquad (5a)$$

$$a_{LI} \cdot W_t + a_{KI} \cdot R_t = p_t \qquad (5b)$$

where a_{ij} is the requirement of the i^{th} input (i = K, L) in the production of the j^{th} good (j = C, I), p is the relative price of the investment good in terms of the numeraire good C, and R is the gross return on capital. Since we assume capital depreciates completely in the process of production, in equilibrium:

[5] This is for analytical convenience only. Dropping this requires capital gains on the sale of capital to be part of the return to holding of capital. That leaves the steady state, where there are no capital gains, unaffected.

$(1+r_{t+1}) = R_{t+1} / p_t$. The a_{ij}'s are functions of the relative factor prices unless the technologies are Leontief (as in section 3).

There are two goods markets (for C and I) and two factor markets (for K and L). By Walras' Law, if three of these are in equilibrium in any period, then so is the fourth one. We thus have (the aggregate demand shock is introduced below in equation (9))

$$a_{LC}.C_t + a_{LI}.I_t = 1 \quad (6a)$$

$$a_{KC}.C_t + a_{KI}.I_t = k_t \quad (6b)$$

$$p_t.I_t = (2+\rho)^{-1} W_t \quad (6c)$$

Equations (6a), (6b) and (6c) are the market-clearing conditions for the labour, capital and investment goods markets respectively. The variable C_t is the production of the consumption good, I_t is the output of the investment good and k_t is the capital stock (all are per worker magnitudes in time period t).

Finally, the dynamics of the economy is represented by the difference equation

$$k_{t+1} = I_t \quad (7)$$

Appendix 1 shows that we can solve equations (6a) and (6b) to get $I_t = I(p_t, k_t)$. Substituting this in equation (6c) we can solve $p_t = p(k_t)$[6]. Equation (7) can then be linearized around the steady state and written as

$$dk_{t+1} = \{(\partial I / \partial p)(\partial p / \partial k) + (\partial I / \partial k)\} dk_t \quad (8a)$$

Stability requires

$$-1 \le dk_{t+1} / dk_t = (\eta_{Ip}.\eta_{pk} + \eta_{Ik}) \le 1 \quad (8b)$$

where $dk_{t+i} \equiv k_{t+i} - k$ is the deviation of the t+i period capital per worker from its steady state value (a variable's steady state value is denoted without a time subscript). An η_{ij} is the elasticity of variable i with respect to j—these are given in Appendix 1.

The expression for η_{pk} will play a crucial role in obtaining the results that we do in this paper, so we turn to a detailed discussion of this expression. Equation (6c) can be differentiated logarithmically to obtain (Γ is a measure of increase in aggregate demand to be discussed below)

[6] This is explained at length in the paragraphs following equation (9).

$$\hat{W}_t = \hat{p}_t + \hat{I}_t + \Gamma$$

or (using (A1.2a), (A1.4a) and (A1.4c))

$$(\eta_{Wp} - 1 - \eta_{Ip})\hat{p}_t = \eta_{Ik}\hat{k}_t + \Gamma \tag{9}$$

If the consumption good is capital-intensive, $\eta_{Wp} > 1$ and $\eta_{Ik} < 0$ (from the Stolper-Samuelson Theorem and the Rybczinski Theorem, respectively). The term $\Psi \equiv (\eta_{Wp} - 1 - \eta_{Ip})$, measures the contribution of a change in price to excess demand for the investment good, ceteris paribus. The demand for capital goods is given by the elasticity of supply of savings η_{Wp}, while the elasticity of the supply responsiveness of investment goods (measured in units of consumption) is given by $1+\eta_{Ip}$. We assume that Ψ>0 i.e., that the (upward-sloping) demand curve for investment is more responsive to a price change than the supply curve (also upward-sloping)—i.e., the short-run dynamics is of the Marshallian-type (Ψ<0 is the Walrasian case)[7].

In figure 1, the II curve is drawn depicting p.I(p,k) and the SS curve for SηW(p)/[(2+ρ)]—ignore the broken lines for now. With p on the horizontal axis, when the II curve is flatter than the SS curve, we have Marshallian dynamics. Note that if we had output on the horizontal axis, we would get back the Keynesian-cross diagram (in its saving equals investment variant).

With our capital intensity and short run dynamics assumptions, equation (9) gives

$$\eta_{pk} \equiv \hat{p}/\hat{k} = \eta_{Ik}/\Psi < 0, \text{ and } \hat{p}/\Gamma > 0 \text{ (for } \Gamma>0).$$

3. An Exogenous Decrease in Desired Saving with Leontief Technologies

It would be enormously helpful for expository purposes to start off with the case where the elasticities of substitution in production are zero[8].

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[7] Note our interpretation of demand and supply for capital is at variance with what might be the natural interpretation of these terms in a one good dynamic model. There savings are taken to be the supply of capital and k_{t+1}, the next period's capital stock, constitutes the demand for it (see e.g., Diamond (1965), Galor and Polemarchakis (1986) and Persson (1985) for such a usage).

[8] Calvo (1978) was the first one, to the best of my knowledge, to use a two-sector overlapping generations model with Leontief technologies.

With Leontief technologies in both sectors, the stability requirement (see equation (8b)) is now (with $\eta_{Ip} = 0$), $-1 < \eta_{Ik} < 0$ which, in turn, requires $\lambda_{KC} > 2\lambda_{LC}$ (where λ_{ij} is the share of sector j in the total employment of input I, e.g., $\lambda_{KC} \equiv a_{KC} C / k$)[9].

Consider a decrease in desired saving e.g., an increase in the rate of time preference, ρ—we can call this an increase in "animal spirits"[10]. What does this do to national income, wages etc.? Is it true that in the new equilibrium the increase in income, if at all, comes at the expense of other activities? It is obvious that in the absence of unemployment of some resource (or, more generally, with elastic factor supplies), the scale of some activities would be reduced, as those of others are increased. The question is then whether this precludes the presence of a multiplier-type relationship?

If the economy were initially in a steady state, the economy jumps straight to a new steady state following a change in (the composition of) aggregate demand. To see this note that in equations (5a), (5b), (6a) and (6b), now the $a_{ij}'s$ are constants (i.e., do not depend on factor prices). From equation (6a) and (6b), I_t and C_t are determined in any period, given k_t. If we start from a steady state, then I = k (from equation (7)). So, from equations (6a), (6b) and (7) we determine the values of k, I and C. These are invariant to a change in desired saving.

Recalling, from (A1.2a), $W_t = W(p_t)$ and from (A1.4a) and (A1.4c) (with Leontief technologies), $I_t = I(k_t)$. With k_1 predetermined (and equal to the initial steady state value, if the system starts from a steady state), we have from equation (9) (with both $\eta_{Ip} = 0, \hat{k} = 0$)

$$\hat{W} = \hat{p} + dA / S \qquad (9')$$

where, $\Gamma \equiv$ dA/S (dA is the increase in aggregate demand in terms of the numeraire--note initially A=0, and $dA = [S/(2+\rho)]d\rho$).

[9] Note if technologies are Leontief, stability is ruled out if the consumption good is labour-intensive.

[10] Note this usage of the expression "animal spirits" is at variance with e.g., Weil (1989). Mine refers to an exogenous increase in the rate of time preference or an exogenous fall in savings, whereas Weil's model, being a one-sector model, uses the term to denote an increase in savings. See also Howitt and McAfee (1992) where the term refers to extraneous uncertainty.

Thus (below θ_{ij} is the share of the i^{th} input in the j^{th} sector price, e.g., $\theta_{LI} \equiv a_{LI}.W/p$, and $\Delta \equiv \theta_{LC}$ - θ_{LI} --see Appendix 1 for details),

$$\hat{p}/dA = -\Delta/(\theta_{KI}.S) \tag{10a}$$

$$\hat{W}/dA = \theta_{KC}/(\theta_{KI}.S) \tag{10b}$$

$$dW/dA = \theta_{KC}.(2+\rho)/\theta_{KI} > 2 \tag{10b'}$$

$$\hat{R}/dA = -\theta_{LC}/(\theta_{KI}.S) < 0 \tag{10c}$$

Now, the GDP (denoted by Q) is given by the identity

$$Q \equiv C + p.I$$

So, $dQ = pI.\hat{p}$ (by the envelope theorem)

$$dQ/dA = -\Delta/\theta_{KI} > 0 \tag{11}$$

In equation (11) above, dQ/dA is "likely" to be greater than unity since $(-\Delta/\theta_{KI})$ is "likely" to be greater than unity (the stability condition requires good C to be sufficiently capital intensive)—for the parameter values in Appendix 1, dQ/dA is 1.5.

Given these, the utility of generation 0 (the initially old)

$$dV_0 = kdR = -(R/p)(\theta_{LC}/\theta_{KI})dA < 0 \tag{12}$$

And those of the subsequent generations is given by (R/p>1 is the dynamic efficiency condition--see Appendix 2 for a derivation)

$$dV = [V_W((R/p)-1)\theta_{LC}/(p.\theta_{KI})].dA > 0 \tag{13}$$

where $V_W = (2+\rho)/((1+\rho).W)$.

We can sum this up as Proposition 1:

Proposition 1: If the technologies in both the sectors are Leontief and the economy is initially in a steady-state, then an exogenous decrease in saving takes the economy to a new steady instantaneously with a lower R/p, and higher W and Q. There is a multiplier effect on W, and possibly Q. As a consequence, the old (generation 0) lose, but everyone else is better off, if the economy is dynamically efficient.

The intuition for the multiplier process is that, given Leontief technologies and starting off from a steady state, k, and hence I, are fixed. Savings have to finance this given amount of I. This, in turn, fixes S/p. If now the rate of time preference increases, wages have to increase to generate the given amount of real savings (in terms of the I good). This requires p to increase, which raises Q. The rise in W increases welfare of generation 1 onwards, if R/p >1 (i.e., the interest rate was above the population growth rate). The increased W is accompanied by a fall in R that causes V_0 to fall. Note that all the action is coming from a change in relative prices—the real quantities, C, I and k, do not change. Also note that Rybczinski Theorem—a veritable Pandora's box in two sector models—plays no role in the results obtained.

In figure 2, the initial equilibrium is at E_0. An autonomous decline in desired saving moves the SS line down to S_1S_1. The new equilibrium is at E_1--the price of investment goods increases from p_0 to p_1.

A brief point about the perceived difference between Keynesian and classical models is in order here. It is often mentioned that classical models (including the Solow growth model) do not have an independent investment function. In this section, we have seen that in a completely neoclassical setting, we can have an independent investment function that is dictated by technology. Hence it is not surprising that we get results that one normally associates with Keynesian models[11].

4. The Multiplier with Non-Zero Elasticities of Substitution.

With non-zero elasticities of substitution in production, there are two changes: (i) the value of static multiplier changes because $\eta_{Ip} > 0$; and (ii) there is non-degenerate dynamics—one implication of this is that even with dynamic efficiency not all generation (barring generation 0, of course) may gain. I discuss what happens in periods 1, 2 and the steady state, leaving the details of the dynamics for Appendix 3.

Rewrite equation (9') as follows

$$\hat{W}_1 = \hat{p}_1 + \hat{I}_1 + (dA/S) \qquad (14)$$

As before, $W_t = W(p_t)$ but now $I_t = I(p_t, k_t)$. With k_1 predetermined, we have (remember $\Psi \equiv (\eta_{Wp} - 1 - \eta_{Ip})$)

$$\hat{p}_1 / dA = 1/(S\Psi) \qquad (15a)$$

[11] Robert Solow, in correspondence with the author, has queried whether the presence of a multiplier makes it a "Keynesian model". I would say that it is a competitive model with properties one expects in a Keynesian model.

$$\hat{W}_1 / dA = -\theta_{KC} /(\Delta S \Psi) \tag{15b}$$

$$dW_1 / dA = -\theta_{KC} (2+\rho)/(\Delta\Psi) > 2 \tag{15b'}$$

$$\hat{R}_1 / dA = \theta_{LC} /(\Delta S \Psi) < 0 \tag{15c}$$

The only change from the Leontief technologies case is that now the price change also elicits a supply response in the investment goods market.
The change in GNP ($Q \equiv C + p.I$) in period 1is given by

$$dQ_1 / dA = pI.\hat{p}_1 / dA = 1/\Psi > 0 \tag{16}$$

Note that both W_1 and Q_1 rise by more than in the Leontief case (for the assumed parameter values in Appendix 1, dQ_1 / dA is 2.5 approximately, compared to 1.5 in the Leontief case). This is because Ψ (which appears in the denominator) $\equiv (-\theta_{KI} / \Delta) - \eta_{Ip} < (-\theta_{KI} / \Delta)$ [12].

The changes in the next period (i.e., period 2) are given by

$$\hat{k}_2 = \hat{I}_1 = \eta_{Ip} \hat{p}_1 = [\eta_{Ip} /(S\Psi)]dA > 0 \tag{17a}$$

$$\hat{p}_2 = [(1+\eta_{Ip}\eta_{Ik}\Psi^{-1})/(S\Psi)]dA \tag{17b}$$

$$\hat{W}_2 = -[\theta_{KC}(1+\eta_{Ip}\eta_{Ik}\Psi^{-1})/(\Delta S\Psi)]dA \tag{17c}$$

$$\hat{R}_2 = [\theta_{LC}(1+\eta_{Ip}\eta_{Ik}\Psi^{-1})/(\Delta S\Psi)]dA \tag{17d}$$

Note, that if $\eta_{Wp} - 1 - \eta_{Ip}(1-\eta_{Ik}) < 0$ i.e., if $-\eta_{Ik}\eta_{Ip} > \Psi$, then $\hat{p}_2 < 0$ and $\hat{W}_2 < 0$. In this case the direct effect of the increased demand (i.e., dA/S) is outweighed by the decline in investment caused by increased capital accumulation (remember the investment good is labour-intensive).

If the initial equilibrium was at E_0 in figure 1, the change in the rate of time preference, shifts the SS curve down (the dashed line S_1S_1), with the period 1 equilibrium at E_1. In period 2, as capital is accumulated, the II curve shifts down to I_1I_1, with the new equilibrium E_2 to the left of E_1 (and possibly to the left of E_0).

The new steady state of the economy is obtained (by solving equations (6c) and (7) together).

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[12] Ψ>0 implies $(-\theta_{KI} / \Delta)(1-\varepsilon_I) - \lambda_{LC}\varepsilon_I /(\Delta\Omega) > 0$, while dQ_1 / dA requires $(-\theta_{KI} / \Delta)(1-\varepsilon_I) < 1 + \lambda_{LC}\varepsilon_I /(\Delta\Omega)$.

$$\Psi \hat{p} = \eta_{Ik}\hat{k} + (1/S).dA \quad (19)$$

$$\hat{k} = \eta_{Ip}\hat{p} + \eta_{Ik}\hat{k} \quad (20)$$

And, so

$$\hat{p}/dA = 1/[\Xi S] \quad (21a)$$

$$\hat{W}/dA = (-\theta_{KC})/[\Xi S\Delta] > 0 \quad (21b)$$

$$\hat{R}/dA = (\theta_{LC})/[\Xi S\Delta] < 0 \quad (21c)$$

$$\hat{k}/dA = \eta_{Ip}/\{(1-\eta_{Ik})\Xi S\} > 0 \quad (21d)$$

$$dQ/dA = 1/[\Xi] > 0 \quad (21e)$$

where $\Xi \equiv \eta_{Wp} - 1 - \eta_{Ip}(1-\eta_{Ik})^{-1} \geq \Psi$

Since $\Xi \geq \Psi$, the long run increases in p, W and Q are smaller than the corresponding ones in the short run--this is due to a crowding-in of capital, which lowers investment, ceteris paribus[13]. Startz (1989), in a monopolistically competitive model, had obtained a similar result and had attributed this to an entry of new firms and the consequent whittling away of profits. In Baxter and King (1993), however, the long run fiscal multiplier exceeds the short run one because of crowding-in of capital (see also Turnovsky and Sen (1991) for a crowding-in of capital in an open economy framework). This is because in an optimal growth model, in the steady state the rate of time preference has to equal the marginal product of capital. If labour supply increases (due to e.g., increased (lump sum) taxation), there is a crowding-in of capital to enable the capital-labour ratio to go back to its previous value.

Turning to the welfare effects

$$dV_0 = k_1.dR_1/dA = Rk.\hat{R}_1/dA = (R/p).\theta_{LC}.(\Delta\Psi)^{-1}dA < 0 \quad (22)$$

$$dV_1/dA = [(2+\rho).(\hat{W}_1/dA) + (\hat{R}_2/dA) - (\hat{p}_1/dA)]/(1+\rho)$$

$$= \{(2+\rho)(-\theta_{KC}\Delta^{-1}) - 1 + (1+\eta_{Ik}\eta_{Ip}\Psi^{-1})\theta_{LC}\Delta^{-1}\}/((1+\rho)\Psi S) > 0 \quad (23)$$

(this expression is positive because compared to the (steady state) welfare change with Leontief technologies, we have a positive term $(\eta_{Ik}\eta_{Ip}\theta_{LC})(\Psi\Delta)^{-1} > 0$).

[13] As mentioned above, even in period 2 the wage rate and output could fall if $(-\eta_{Ik}\eta_{Ip}) > \Psi$.

And the steady state utility change is given by

$$dV / dA = -V_W [((R / p) - 1) S(\hat{R} / dA)] > 0 \qquad (24)$$

The change in long run welfare is lower compared to the short run because as capital gets crowded in, some of the increased aggregate demand disappears—in figure 1, the II line shifts down--thereby checking the increase in wages and the fall in the interest rate. As long as the economy is dynamically efficient, the wage effect dominates, though (see Appendix 2).

The dynamics of prices is summarized in Appendix 3. Equation (A3.1) is the basic dynamic equation governing the evolution of $\hat{p}_t$. Equation (A3.2) is the solution to this difference equation. Equations (A3.3) and (A3.4) respectively give us the initial condition and the steady state values. Equation (A3.5) is the root of the equation. Equation (A3.6) gives the general expression for welfare.

The solution to the difference equation (A3.1) given in (A3.2) gives us some understanding about the possible reasons why the wage rate may fall in period 2 and with it the welfare of generation 2. Also note that the steady state welfare rises, so that the possible decline in welfare happens, if at all, only during the transition. To see this

$$\hat{p}_{t+1} / dA = ((\Psi S)^{-1} - (\Xi S)^{-1})\Theta^t + (S\Xi)^{-1} \qquad (25)$$

Since, the initial change in $\hat{p}_t$ is greater than its steady state value (since $\Psi<\Xi$), i.e., $\hat{p}_1$ overshoots its long-run value and in the odd periods following the shock (i.e., periods 2,4,6,….) this positive gap gets multiplied by a negative number Θ^i where i is the period since the shock (Θ is the root of the difference equation). In these periods there is a negative effect of capital accumulation on investment (and on prices and wages). There is a positive effect through $\hat{p}(= (S\Xi)^{-1})$ in all periods. In the initial stages the negative effect could dominate while as i becomes larger the negative effect dies out and the steady state effects of the fall in desired savings on price, wages, output and welfare is expansionary.

5. Conclusions

Revisiting Tobin's insight that in a model with Marshallian dynamics one can obtain Keynesian results, I looked for a multiplier-type expression in a two-sector overlapping generations model. Here in a perfectly competitive model with maximizing agents, I showed that Keynesian-type multipliers, reminiscent of the Keynesian-cross, can indeed be obtained. A decline in the saving propensity can cause wages and output to increase—since a given amount of investment, ceteris paribus, has to be financed, savings, and hence wages, increase. However, unlike macroeconomic models with monopolistic competition, the new equilibrium path is not Pareto-superior compared to the initial one—at least one generation loses along the new equilibrium path.

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APPENDIX 1

Equations (4) and (5) yield by logarithmic differentiation (see Atkinson and Stiglitz (1980), Chapter 6 for details)

$$\theta_{LC}\cdot\hat{W}_t + \theta_{KC}\cdot\hat{R}_t = 0 \qquad \text{(A1.1a)}$$

$$\theta_{LI}\cdot\hat{W}_t + \theta_{KI}\cdot\hat{R}_t = \hat{p}_t \qquad \text{(A1.1b)}$$

where θ_{ij} is the share of the i^{th} input in the j^{th} sector price (e.g., $\theta_{LI} \equiv a_{LI}.W/p$) and a hat over a variable denotes a percentage change.

From (A1.1a) and (A1.1b), we can solve for $\hat{W}_t$ and $\hat{R}_t$ in terms of $\hat{p}_t$. We thus have

$$\eta_{Wp} \equiv \hat{W}_t / \hat{p}_t = -\theta_{KC}/\Delta \qquad \text{(A1.2a)}$$

$$\eta_{Rp} \equiv \hat{R}_t/\hat{p}_t = \theta_{LC}/\Delta \qquad \text{(A1.2b)}$$

where $\Delta \equiv \theta_{LC} - \theta_{LI} = \theta_{KI} - \theta_{KC}$ and η_{ij} is the (partial) elasticity of variable i with respect to j. From equations (A1.2a) and (A1.2b) we see that η_{Wp} and η_{Rp} depend on capital intensities. Given our assumption that the consumption good is capital-intensive, $\Delta<0$. And hence by the Stolper-Samuelson Theorem, $\eta_{Wp} > 1, \eta_{Rp} < 0$.

Similarly by logarithmically differentiating (6a), (6b) and (6c) we have

$$\lambda_{LC}\cdot\hat{C}_t + \lambda_{LI}\cdot\hat{I}_t = [\hat{W}_t - \hat{R}_t][\lambda_{LC}.\theta_{KC}.\varepsilon_C + \lambda_{LI}.\theta_{KI}.\varepsilon_I] \qquad \text{(A1.3a)}$$

$$\lambda_{KC}\cdot\hat{C}_t + \lambda_{KI}\cdot\hat{I}_t = \hat{k}_t - [\hat{W}_t - \hat{R}_t][\lambda_{KC}.\theta_{LC}.\varepsilon_C + \lambda_{KI}.\theta_{LI}.\varepsilon_I] \qquad \text{(A1.3b)}$$

$$\hat{p}_t + \hat{I}_t = \hat{W}_t \qquad \text{(A1.3c)}$$

where λ_{ij} is the share of sector j in the total employment of input i and ε_j is the elasticity of substitution between inputs in the j^{th} industry.

From equations (A1.3a) and (A1.3b), we have the Rybczinski effects (which depend on assumed capital intensities)

$$\eta_{Ik} \equiv \hat{I}_t / \hat{k}_t = \lambda_{LC}/\Omega < 0 \qquad \text{(A1.4a)}$$

$$\eta_{Ck} \equiv \hat{C}_t / \hat{k}_t = -\lambda_{LI}/\Omega > 0 \qquad \text{(A1.4b)}$$

where $\Omega \equiv \lambda_{LC} - \lambda_{KC} < 0$ (by assumption).

From (A1.3a) and (A1.3b), we have the supply elasticities (which are independent of capital intensities)

$$\eta_{Ip} \equiv \hat{I}_t / \hat{p}_t = \{\lambda_{LC}\lambda_{KC}\varepsilon_C + (\lambda_{LC}\lambda_{KI}\theta_{LI} + \lambda_{KC}\lambda_{LI}\theta_{KI})\varepsilon_I\}/(\Delta\Omega) > 0 \qquad \text{(A1.4c)}$$

$$\eta_{Cp} \equiv \hat{C}_t / \hat{p}_t = -\{\lambda_{LI}\lambda_{KI}\varepsilon_I + (\lambda_{LC}\lambda_{KI}\theta_{KC} + \lambda_{KC}\lambda_{LI}\theta_{LC})\varepsilon_C\}/(\Delta\Omega) < 0 \qquad \text{(A1.4d)}$$

If we substitute for $\hat{W}_t$, from (A1.2a) into (A1.3c), we can solve (A1.3a), (A1.3b) and (A1.3c) for $\hat{C}_t$, $\hat{I}_t$ and $\hat{p}_t$ in terms of $\hat{k}_t$. In particular,

$$\eta_{pk} \equiv \hat{p}_t / \hat{k}_t = \eta_{Ik} / \Psi = (-\lambda_{LC}\Delta)[\theta_{KI}\Omega + \{\lambda_{LC}\lambda_{KC}\varepsilon_C + (\lambda_{LC}\lambda_{KI}\theta_{LI} + \lambda_{KC}\lambda_{LI}\theta_{KI})\varepsilon_I\}]^{-1}$$

(A1.5)

where $\Psi \equiv \eta_{Wp} - 1 - \eta_{Ip}$

The stability condition for equation (7) in the text requires

$$-1 \leq dk_{t+1} / dk_t = (\eta_{Ip}.\eta_{pk} + \eta_{Ik}) \equiv \Theta \leq 1 \qquad \text{(A1.6)}$$

Thus if the model is stable with $\eta_{pk} < 0$, the convergence is cyclical. The condition for stability turns out to be

$$[\theta_{KI}(\lambda_{KC} - 2\lambda_{LC}) - \{\lambda_{LC}\lambda_{KC}\varepsilon_C + (\lambda_{LC}\lambda_{KI}\theta_{LI} + \lambda_{KC}\lambda_{LI}\theta_{KI})\varepsilon_I\}]/(\Delta\Omega) > 0 \qquad \text{(A1.7)}$$

The condition in (A1.7) is more stringent than for $\hat{p}/\hat{k} < 0$ in (A1.5)--the term multiplying θ_{KI} is $\lambda_{KC} - 2\lambda_{LC}$ instead of $\lambda_{KC} - \lambda_{LC}$. Consider the following numerical example: $\lambda_{LC} = 0.15, \lambda_{KC} = 0.8, \theta_{LI} = 0.8, \theta_{LC} = 0.5, \varepsilon_I = \varepsilon_C = 0.4$. We have both $\hat{p}/\hat{k} < 0$ and $-1 < dk_{t+1} / dk_t < 0$.

APPENDIX 2

In the steady state if we have the general utility function $U(C^1.C^2)$, then the indirect utility function is

$V\left(W, {}^{R}\!/_{p}\right)$ (which is derived from $U(W-S, S.(R/p))$ where S is chosen optimally).

$dV = V_W dW + V_R d\,{}^{R}\!/_{p}$ Note $V_W = U_1 > 0, V_R = SU_2$

dV $= V_W\left(dW + S(R/p)^{-1} d(R/p)\right)$ because $U_1 = (R/p)U_2$

$= V_W\left(Idp - kdR + S(R/p)^{-1} d(R/p)\right)$ (from W+Rk=C+p.I and the envelope theorem implies dW+kdR=I.dp)

$$=V_W\left(kdp - kdR + S(R./p)^{-1}\left(\frac{dR}{p} - \frac{R}{p^2}dp\right)\right)=V_W\left(kdp - \frac{S}{{}^{R}\!/_{p}}\frac{R}{p^2}dp + \frac{S}{{}^{R}\!/_{p}}\frac{dR}{p} - kdR\right)$$

because the first two terms in the previous line cancel out (i.e., k=I=S/p), we have dV$= V_W.S.(1-(R/p))\hat{R}$ --this appears as equations (17) and (24) in the text.

APPENDIX 3

$$\hat{p}_{t+1} = \eta_{Ik}(\eta_{Wp} - 1)\Psi^{-1}\hat{p}_t + (1 - \eta_{Ik})(S\Psi)^{-1}dA \qquad \text{(A3.1)}$$

$$\hat{p}_{t+1} = ((S\Psi)^{-1} - \hat{p})\{\eta_{Ik}\Psi^{-1}(\eta_{Wp} - 1)\}^t + \hat{p} \quad \text{t=0,1,2,....} \qquad \text{(A3.2)}$$

$$\hat{p}_1 = (S\Psi)^{-1}dA \qquad \text{(A3.3)}$$

$$\hat{p} = (1 - \eta_{Ik})\{(\eta_{Wp} - 1)(1 - \eta_{Ik}) - \eta_{Ip}\}^{-1}S^{-1}dA > 0 \qquad \text{(A3.4)}$$

$$-1 < \Theta \equiv \eta_{Ik}(\eta_{Wp} - 1)\Psi^{-1} < 0 \qquad \text{(A3.5)}$$

$$dV_t = [S^{-1}\{(2 + \rho)\theta_{KC}\Delta^{-1} - 1\}(\hat{p}_t / dA) + \theta_{LC}\Delta^{-1}.(\hat{p}_{t+1} / dA)]dA$$

$$= [(-S\Delta)^{-1}\{(2 + \rho)\theta_{KC} + \Delta - \Theta\theta_{LC}\}\{(\hat{p}_1 / dA) - (\hat{p} / dA)\}\Theta^{t-1} +$$
$$\{(2 + \rho)\theta_{KC} + \Delta - \theta_{LC}\}\{(\hat{p} \ / dA)]dA$$

t=1,2,… (A3.6)

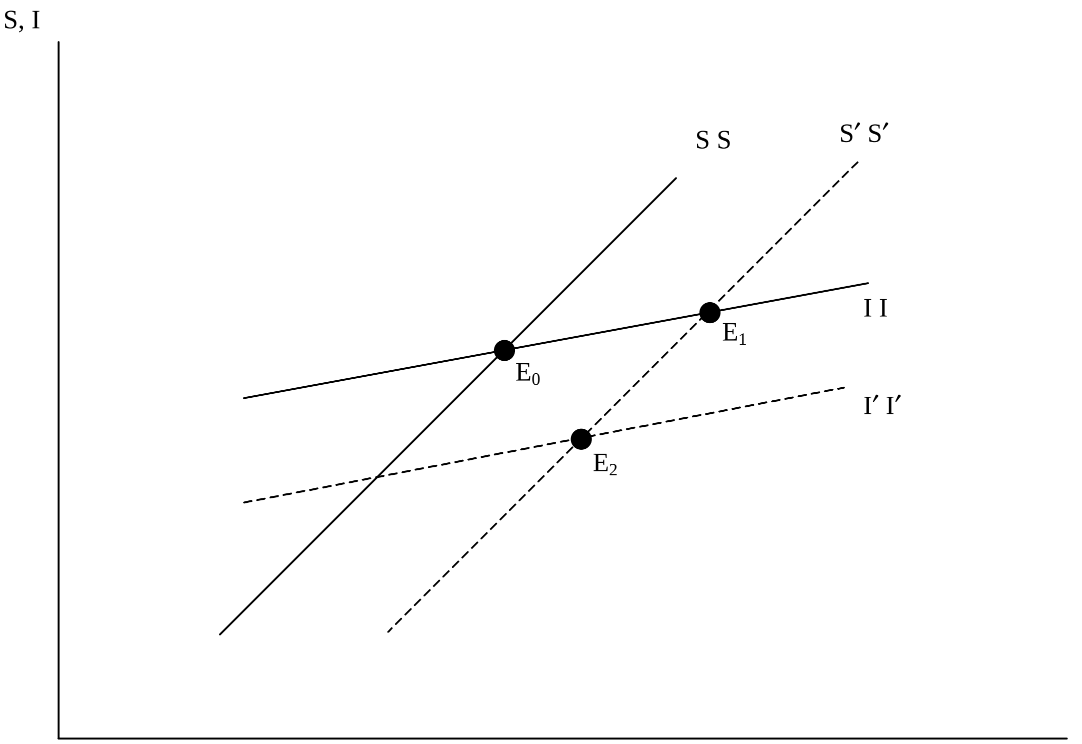

Figure 1

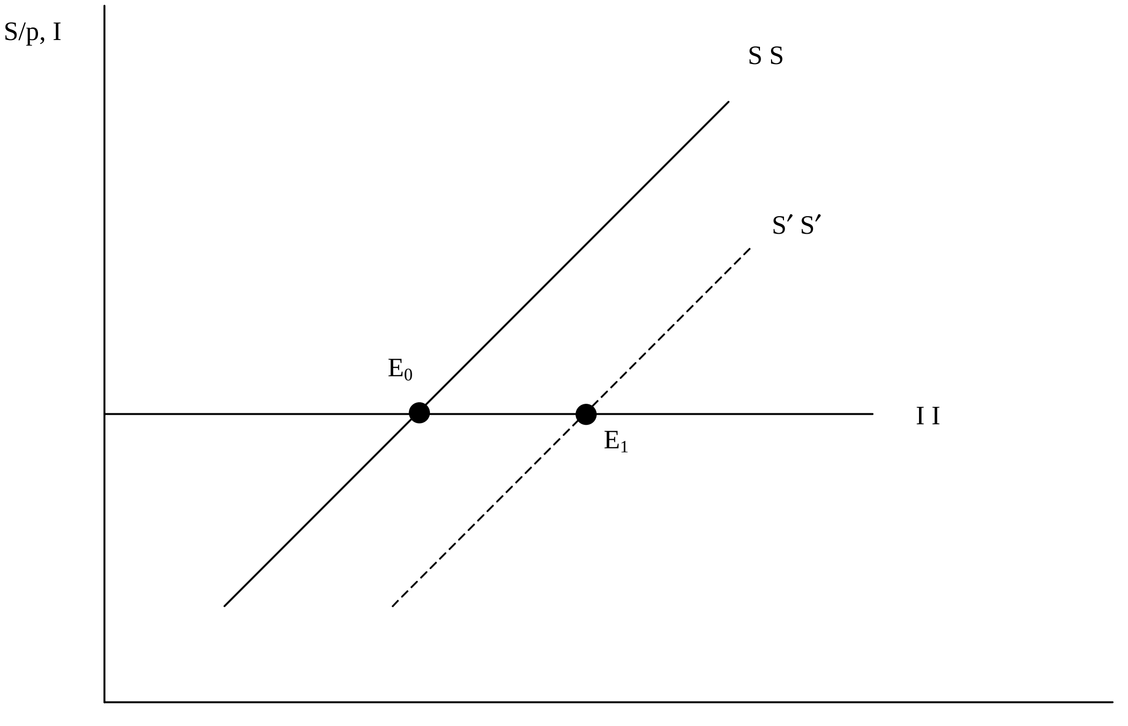

Figure 2

* Complete list of working papers is available at the CDE website:
http://www.cdedse.org/worklist.pdf