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Abstract

This paper models the univariate dynamics of seasonally unadjusted quarterly macroeconomic time series for the Indian economy including industrial production, money supply (broad and narrow measures) and consumer price index. The seasonal integration-cointegration and the periodic models are employed. The 'best' model is selected on the basis of a battery of econometric tests including comparison of out-of-sample forecast performance.

The results suggest that a periodically integrated process with one unit root best captures the movements in industrial production. The other variables do not exhibit periodically varying dynamics, though narrow money and consumer price index exhibit nonstationary seasonality. For the index of industrial production, the periodic model yields the best out-of-sample forecasts, while for broad money, the model in first differences performs best. In narrow money and the consumer price index, incorporating nonstationary seasonality does not lead to significant gains in forecast accuracy. Finally, we find significant Periodic conditional heteroskedasticity in industrial production, with error variance in the first two quarters (highest and lowest economic activity quarters, respectively) almost three times that in the other two quarters.

Keywords: Seasonality, Integration, Periodic Integration, Forecast Performance

JEL Classification: C22, C53

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1 Introduction

Seasonality is an important component of most macroeconomic time series, sometimes swamping the other movements in them. Therefore for a long time the practice in econometric literature was to adjust away this component, prior to using the series. However, in the past three decades there has been an increasing inclination towards modelling seasonality instead of adjusting it away. This is due to three reasons. One, it was realised that seasonal adjustment distorts inference in dynamic models. An example of this distortion is in the tests for integration and cointegration (seasonal as well as non-seasonal). Two, seasonal cycles are found to have important information, which would be lost if one were to work with seasonally adjusted data[1]. Finally, it has been found that in some cases seasonal and other components are not separable from each other. In such cases seasonal adjustment gives rise to seasonal and non-seasonal components which are not orthogonal to each other. This defeats the very purpose of seasonal adjustment, and the only way to study such series is without seasonal adjustment. Given these, it is clear that working with seasonally adjusted data would not only give misleading results for policy modelling but this throwing away of information might also result in loss of forecast accuracy.

Due to the non-trivial characteristics of seasonality, the models required to capture movements in seasonally unadjusted sub-annual time series are different from those required for annual time series and seasonally adjusted time series. Two models which have been found to describe most of the macroeconomic time series well are the seasonal integration model and the periodic model, possibly with periodic integration. Though the latter embeds the former, the two models exhibit different characteristics, both univariate and multivariate.

This paper attempts to model the movements in four macroeconomic variables for the Indian economy, covering real output, money supply and prices. Both narrow and broad money have been taken. This has been done with the objective of selecting the model which gives the best out-of-sample forecasts. This also helps us choose the appropriate modelling strategy for relations among these series, since the appropriate modelling strategy for a set of time series depends on the univariate properties of the individual time series.

We find that the movements in the index of industrial production are best captured by

[1] This information may appear in three ways, mainly: one, gradual changes in amount and pattern of seasonal variation over time; two, the relations between amount and pattern of seasonal variation in different series and three, in periodically varying dynamics, univariate as well as multivariate.

a periodically integrated process with one unit root. Neither of the other three variables exhibits periodically varying dynamics. A model in first difference and seasonally varying intercepts captures the dynamics of broad money well, while narrow money and consumer price index exhibit non-stationary seasonality generated by seasonal unit roots. For out-of-sample forecasts, we find that for index of industrial production the periodic model gives the best forecasts, while for broad money the model in first differences performs best. In narrow money and consumer price index, incorporating nonstationary seasonality does not lead to significant gains in forecast accuracy. For macroeconomic modelling, the finding of periodic integration has an important implication. The relation of a series with periodic integration with any variable that is non-periodically integrated is essentially periodically varying. Finally, one important finding of this study is that of periodic variation in conditional variance in the index of industrial production. It is found that the error variance in the first two quarters of the year (respectively, the highest and the lowest economic activity quarters for the Indian economy) is almost three times as much as that in the other two quarters.

The rest of the paper is organised as follows. The following section contains a brief overview of the models found to be most useful for modelling seasonality in macroeconomic time series. Section 3 builds the motivation for this study by reviewing the results of studies for other countries and India. The econometric methodology of this paper is discussed in section 4, along with the details of the data used, followed by a discussion of results in section 5. Section 6 contains the methodology and results of comparison of forecast accuracy of different models chosen by the econometric tests. Section 7 discusses the implications of results for univariate forecasting of macroeconomic variables and multivariate modelling. Section 8 concludes the paper with suggestions for further research.

2 Models for seasonal macroeconomic time series

The models for seasonal time series are extensions of the commonly used integration-cointegration models, to accommodate seasonality. One way to incorporate seasonality in the univariate models is to allow seasonal roots in AR and/or MA polynomials. In such models, the non-stationarity of seasonality is captured by seasonal unit roots in the AR polynomial; and long-run relations among such series, if any, are reflected in seasonal cointegration. On the other hand, if the seasonality in univariate time series is modelled by allowing the AR and/or MA parameters to vary with the seasons, one gets the periodic model and the long-run relations among such series are reflected in periodic cointegration. This section discusses each of these approaches briefly.

2.1 Seasonal Integration-Cointegration Model

The simplest way to capture seasonality in univariate models is to modify the the AR(I)MA models to have seasonal frequency roots in the AR and/or MA polynomials. This gives rise to what are known as SAR(I)MA models. Deterministic seasonality is incorporated in such models by allowing the mean to vary with the seasons. The non-stationary seasonality, which is found to be present in many macroeconomic time series, is taken care of by allowing some or all of the seasonal roots of AR polynomial to lie on the unit circle. In the presence of non-stationary seasonality the conventional methodology of testing for integration and cointegration (possibly with inclusion of seasonal dummies in the test equations) yields spurious results, and therefore this type of analysis is valid only after removing the seasonal unit roots, if any. This is to some extent taken care of in the conventional analysis by seasonally adjusting the data, either by the X-11 (lately, X-12) filter or by the seasonal differencing operator. However, these filters assume the presence of all the seasonal roots on the unit circle, leading to overdifferencing in case only some (or none) of them are on the unit circle. Further, even for those frequencies, roots corresponding to which are on the unit circle, removal of those roots by the relevant filter is not necessarily the best strategy. Two or more series may have common non-stationary seasonality at those frequencies so that some linear combination of theirs may not have unit roots at those frequencies. The latter implies that there is some long-run relation among the seasonal patterns of the two series. Filtering the unit root non-stationarity in this case would not only lead to model misspecification[2], but would also lead to loss of information about the long-run comovements of the series, reflected in their common seasonals.

It is therefore better to test each series for unit roots at each of the seasonal frequencies separately. The filter should then be chosen so as to remove these seasonal unit roots. For multivariate modelling, one should explore the possibility of linear combinations of the

[2] From the theory of cointegration it is well known that in such a situation the vector of the filtered series does not have an invertible VMA representation. One implication of this is that the vector does not have a finite order VAR representation, and that is why Ermini and Chang (1996) argue that seasonally adjusted data are, in most of the cases, not fit for VAR analysis, since the latter is based on the assumption of existence of finite order VAR representation, at least approximately.

series which do not have unit roots at the frequencies for which all the individual series are found to have unit roots. If there are some such linear combinations these should be used for modelling relations among them.

Specifically, consider a zero-mean ARMA(p, q) process

$$\phi(L)y_t = \theta(L)\epsilon_t \tag{1}$$

The usual approach to incorporate seasonality is to allow for seasonal frequency roots in $\phi(z)$ and/or $\theta(z)$. Seasonal integration arises when either or all of these seasonal roots of the polynomial $\phi(z)$ are on the unit circle, but this is not the case with $\theta(z)$. Thus, the above process has simple integration when $\phi(z)$ has root 1, while it is said to be seasonally integrated when $e^{-\frac{2\pi j}{S}i}$ is a root, where S is the number of observations in a year, $j = 1, \ldots, (S-1)$ and $i = \sqrt{-1}$. In the seasonal integration-cointegration approach one tests for these unit roots and after identifying them, proceeds with univariate analysis using the filter suggested by these unit roots. Multivariate analysis, on the other hand, proceeds by testing for linear combinations of these series for which the roots are not on the unit circle. In case there are such linear combinations it indicates some long run relation among seasonals in these series and this information is incorporated in the models.

A number of tests have been suggested over the past one-and-a-half decades for testing for unit roots at seasonal frequencies, starting with the seminal test by Hylleberg *et al.* (1990)(referred to as HEGY test, henceforth), which was extended to monthly data by Franses (1991) and Beaulieu and Miron (1993). Similarly, Engle *et al.* (1993) introduced the tests for seasonal cointegration. Lee (1992) and Johansen and Schaumburg (1999) have developed a more general modelling strategy for non-stationary seasonal time series by extending Johansen's strategy for series with zero-frequency unit roots.

Though seasonal integration-cointegration models are able to capture seasonals (even non-stationary), and also the common seasonals among different time series, there are a number of problems with this class of models:

- First, these models cannot capture the periodically varying dynamics, which is the feature of many macroeconomic time series (see, for example, Osborn (1988) and Birchenhall *et al.* (1989), among others). In fact, it becomes a natural characteristic for most of the economic time series once one allows for periodic variation in consumers' tastes/technology/production adjustment costs, resulting in periodic variation in consumers' demand functions and/or firms' cost functions.

- Second, the seasonal integration model does not allow for interactions among seasonal and other components. Though it allows for changing seasonality and trend, these changes are independent of each other. On the other hand, earlier studies have shown that changes in seasonals are not independent of business cycles and the two types of cycles are related to each other. These relations are documented in, among others, Ghysels (1997), Franses (1995a), Cecchetti, Kashyap and Wilcox (1997), Canova and Ghysels (1994). Ghysels (1988) proves this point analytically for dynamic models.

- Finally, there is the question of economic plausibility. Though one can interpret the first difference as the growth rate, it is difficult to interpret on economic grounds the filters suggested by the presence of unit roots at different (pairs of) seasonal frequencies[3]. On the other hand, the seasonal differencing filter (which can be interpreted as the annual growth rate) implies that all the seasonal roots are on the unit circle (along with the zero frequency root), giving rise to S stochastic trends, which implies that the S seasons are governed by S different stochastic trends. In such a situation, there is no cointegrating relation among the seasons of the year. This allows the different seasons to wander arbitrarily away from each other, thus allowing 'spring to become summer'. However, this is not the case with many macroeconomic time series, implying that for most macroeconomic time series with non-stationary seasonality, the filters implied by seasonal unit root tests are difficult to justify on economic grounds.

In view of these, it is clear that an alternative framework is required for macroeconomic time series with non-stationary seasonality. This class of models should be able to capture three features: slowly changing seasonality, interdependence of seasonal and non-seasonal components, and periodically varying dynamics. That is where the periodic models appear useful.

2.2 Periodic Integration-Cointegration Model

Periodic models are obtained by allowing the ARMA parameters in the AR(I)MA models to vary with the seasons. This takes care of all the three problems with the seasonal integration-cointegration framework discussed above.

[3]For example, the presence of unit roots at the frequency $\pm\frac{5\pi}{6}$, which corresponds to 5 and 7 cycles per year for monthly data, suggests that the filter required for stationarity is $(1-\sqrt{3}L+L^2)$, which is difficult to interpret on economic grounds.

Looking at model in eq. (1), a periodic model is obtained by simply allowing the parameters in the AR and/or MA polynomials to vary with the seasons. Thus, a general periodic model may be written as[4]

$$\phi_s(L)y_t = \theta_s(L)\epsilon_t \tag{2}$$

However, since small (less that S) order MA models cannot capture annual dependence of observations, which is a prominent feature of seasonal time series, only periodic AR (PAR) models are common in econometrics.

In periodic models, since each season has a different ARMA structure, the moments vary with the seasons. Due to this periodic variation in the moments, the series is not stationary even if the roots are outside the unit circle for all the seasons. Therefore, periodic models are analysed in a multivariate VAR framework, in terms of 'vector of quarters' (VQ). The latter is the vector obtained by stacking the four annual series corresponding to the four quarters of the year in a vector. The stationarity properties of the series are then defined in terms of this vector. Thus, if the process

$$\phi_s(L)y_t = \epsilon_t \tag{3}$$

may be written as

$$\Phi(L)Y_T = E_T \tag{4}$$

where $Y_T = [y_{1T} \quad y_{2T} \quad y_{3T} \quad y_{4T}]'$, T being the year in which the observation t falls and y_{sT} being the observation for s^{th} season for the year T, the series is said to be periodically stationary if this vector is stationary. As can be easily seen from the theory of cointegrating VARs, the condition for this is that $\Phi(1)$ is non-singular[5]. Equivalently, if the above is written in VECM form as

$$\Delta Y_T = \Pi Y_{T-1} + \Gamma_1 \Delta Y_{T-1} + \cdots + \Gamma_{T-P+1}\Delta Y_{T-P+1} + E_T \tag{5}$$

the condition for stationarity is that the matrix Π has full rank. It may be seen that there are three ways in which this matrix can be singular, leading to three types of non-stationarity in periodic models:

[4] Here the variance of ϵ_t may also be periodically varying. It can be shown that periodically varying variance of ϵ_t gives rise to periodic unconditional heteroskedasticity for y_t even in a non-periodic model, while in a periodic model, y_t exhibits periodic unconditional heteroskedasticity even if ϵ_t is homoskedastic.

[5] Unlike the standard VARs, in this case $\Phi(0)$ (let's say Φ_0) $\neq I$, due to relations among the quarters of the series for same year. However, it may be seen easily that Φ_0 is a triangular matrix (since expression for any quarter contains only the quarters preceding it) and hence nonsingular and therefore eq. (4) can be multiplied by Φ_0^{-1} to obtain VAR in standard form. Further, though the condition for stationarity of this VAR would be that $\Phi_0^{-1}\Phi(1)$ being non-singular, this condition is equivalent to $\Phi(1)$ being non-singular in view of the non-singularity of Φ_0.

1. The first type of non-stationarity arises when $(1-z)$ is a common factor of $\phi_s(z)$ for all s. In this case, application of the filter $(1-L)$ to all the four quarters would lead to stationarity, implying that simply taking first difference of the series would render the series periodically stationary. This is the $I(1)$ case[6].

2. When $(1-z^4)$ is a common factor of $\phi_s(z)$ for all s, the seasonal differencing filter is required to render the series stationary, and nothing short of that would do, since the series has all the four roots (zero as well as seasonal frequencies) on the unit circle. This is the case of seasonal integration.

3. Finally, it is also possible that neither of the above is a common factor in all the individual polynomials, but still the matrix $\Phi(1)$ is singular. This type of non-stationarity can arise only in periodic models and to deal with this one needs periodically varying filters. This type of non-stationarity is known as *periodic integration.*

For example, if $\phi_s(z) = (1-\alpha_s z)$, then $|\Phi(z)| = (1-\alpha_1\alpha_2\alpha_3\alpha_4 z)$. For this specification, $|\Phi(1)|$ will be zero for every value of α_s $\forall$ s such that $\alpha_s^4 = 1$. This covers all the cases of seasonal and non-seasonal unit roots mentioned in case (1) above. However, it will be zero whenever the four α's have product 1, even when they are not equal. In the latter case, first difference would not give a stationary series, while fourth difference would lead to overdifferencing, even though $\Phi(L) = (1-L)$ and hence $\Phi(L)$ contains the seasonal difference operator[7]. The latter is similar to modelling each series in first difference in a VAR, which amounts to overdifferencing in case the cointegrating rank is greater than zero. The appropriate filter in such cases would be periodically varying, and would be given by the cointegrating vectors among the different quarters. It may be seen easily that in this case $[1 \quad -\alpha_s]$ is the cointegrating vector for every s and therefore the filter required to render the series stationary would be $(1-\alpha_s L)$.

This can be seen more clearly in the following way. In terms of equation (4), here we have (given that $\Phi(L) \equiv \Phi_0 - \Phi_1 L - \Phi_2 L^2 - \cdots - \Phi_P L^P$)

$$\Phi_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\alpha_2 & 1 & 0 & 0 \\ 0 & -\alpha_3 & 1 & 0 \\ 0 & 0 & -\alpha_4 & 1 \end{bmatrix}, \Phi_1 = \begin{bmatrix} 0 & 0 & 0 & -\alpha_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{6}$$

[6]Another case possible is one in which $(1-e^{\frac{2\pi j}{4}i}z)$ with $j=1$ and/or 2 is common across the AR polynomials for all quarters. This type of non-stationarity can be similarly dealt with by applying the required filter to the entire series.

[7]Since first lag for the vector corresponds to seasonal lag for individual series and hence first difference for vector implies seasonal difference for individual series.

It may be seen clearly that the ECM reprsentation for this would be

$$\Delta Y_T = (\Phi_0^{-1}\Phi_1 - I)Y_{T-1} + \Phi_0^{-1}E_T \tag{7}$$

where

$$\Phi_0^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha_2 & 1 & 0 & 0 \\ \alpha_2\alpha_3 & \alpha_3 & 1 & 0 \\ \alpha_2\alpha_3\alpha_4 & \alpha_3\alpha_4 & \alpha_4 & 1 \end{bmatrix} \tag{8}$$

so that the matrix Π in eq. (4) will be

$$\Pi = \begin{bmatrix} -1 & 0 & 0 & \alpha_1 \\ 0 & -1 & 0 & \alpha_1\alpha_2 \\ 0 & 0 & -1 & \alpha_1\alpha_2\alpha_3 \\ 0 & 0 & 0 & \alpha_1\alpha_2\alpha_3\alpha_4 - 1 \end{bmatrix}$$

In case of periodic integration, as stated above, $\alpha_1\alpha_2\alpha_3\alpha_4 - 1 = 0$ and it is clear that in that case the above matrix will have rank 3, and there will be three cointegrating vectors among the quarters. Following Granger representation theorem, it can be written as $\alpha\beta'$ where α and β are 4×3 matrices. In this particular case these are given by

$$\alpha = \begin{bmatrix} \frac{1}{\alpha_2} & \frac{1}{\alpha_2\alpha_3} & \alpha_1 \\ 0 & \frac{1}{\alpha_3} & \alpha_1\alpha_2 \\ 0 & 0 & \alpha_1\alpha_2\alpha_3 \\ 0 & 0 & 0 \end{bmatrix} \text{and } \beta' = \begin{bmatrix} -\alpha_2 & 1 & 0 & 0 \\ 0 & -\alpha_3 & 1 & 0 \\ 0 & 0 & -\alpha_4 & 1 \end{bmatrix} \tag{9}$$

This shows clearly that $[1 \quad -\alpha_s]$ are cointegrating vectors.

To see the implications of periodic integration clearly, we apply Granger representation theorem to eq. (4), to get

$$Y_T = C\Phi_0^{-1}\sum_{i=0}^{T-1} E_{T-i} + C^*\Phi_0^{-1}E_T + Y_0 \tag{10}$$

where $C = \beta_\perp(\alpha_\perp'\beta_\perp)^{-1}\alpha_\perp'$ with $\alpha_\perp = [0\ \ 0\ \ 0\ \ 1]'$ and $\beta_\perp = [1\ \ \phi_2\ \ \phi_2\phi_3\ \ \phi_2\phi_3\phi_4]'$. $C\Phi_0^{-1}$ here displays the impact of accumulated shocks and is given by

$$\begin{bmatrix} 1 & \phi_1\phi_3\phi_4 & \phi_1\phi_4 & \phi_1 \\ \phi_2 & 1 & \phi_1\phi_2\phi_4 & \phi_1\phi_2 \\ \phi_2\phi_3 & \phi_3 & 1 & \phi_1\phi_2\phi_3 \\ \phi_2\phi_3\phi_4 & \phi_3\phi_4 & \phi_4 & 1 \end{bmatrix}$$

This matrix has rank 1 and therefore can be written as ab' where

$$a = \begin{bmatrix} 1 \\ \phi_2 \\ \phi_2\phi_3 \\ \phi_2\phi_3\phi_4 \end{bmatrix} \text{and } b' = \begin{bmatrix} 1 & \phi_1\phi_3\phi_4 & \phi_1\phi_4 & \phi_1 \end{bmatrix}$$

This shows that the effect of accumulated shocks is given by $ab' \sum_{i=0}^{T-1} E_{T-i}$. Thus, effectively the effect of accumulated shocks on the vector is $b' \sum_{i=0}^{T-1} E_{T-i}$ and effect on s^{th} season is given by the corresponding element of the vector a multiplied by this quantity. Hence, in case of periodic integration, the four seasons follow four different stochastic trends, leading to stochastic seasonality.

The above specification also makes clear the difference between seasonal integration and periodic integration. The only case of seasonal integration that is generated by one unit root is that of the Nyquist frequency unit root. That is obtained by setting $\alpha_s = -1 \ \forall \ s$ in the above discussion. It is clear that in that case the alternate seasons have the same stochastic trend, while in the other two seasons it is given by the mirror image of this trend. As a result, the stochastic seasonality can be removed by taking the sum of consecutive quarters, or in other words, by applying the $(1 + L)$ filter, which does not vary periodically. Hence, the four seasons do not have entirely different dynamics, as is the case with periodic integration.

In multivariate analysis, periodic integration opens up a large number of possibilities with regard to cointegration[8] (see Ghysels and Osborn (2001) for a lucid discussion). One important case is the one in which one series is non-periodically integrated (at zero/some seasonal frequency) and the other is periodically integrated. It can be seen easily that in this situation, if the two series cointegrate, the cointegrating vectors can only be periodically varying. This possibility is important in view of the fact that many time series exhibit deterministic seasonality but stochastic trends, while many others exhibit periodic integration.

3 Characterising Seasonality in Indian macroeconomic time series

Analysis of a number of macroeconomic time series for various countries over the past one and a half decades has shown that the two classes of models discussed above capture seasonals in most of the aggregate time series quite well. Seasonality is non-stationary in many cases, thus invalidating the practice of dealing with seasonality by including seasonal intercept dummies in the models which otherwise ignore seasonality altogether (see, for example, Osborn (1990), Lee and Siklos (1991), and Hylleberg, Jorgensen and Sorensen (1993), among others). However, in most of the cases where seasonality was

[8] In error correction models, we may have periodically varying adjustment coefficients also.

initially found to be generated by seasonal unit roots, it was verified later that the series were in fact periodically integrated. For example, Franses and Romijn (1993) found that many series which were reported by Osborn (1990) as seasonally integrated were in fact periodically integrated. Similar results were obtained by Franses (1994) and Wells (1997b) for the series found to be seasonally integrated by Engle *et al.* (1993) and Wells (1997a) respectively[9]. Further, using one type of model when the other is in fact appropriate leads to deterioration in forecast accuracy as well as spurious inference for parameters of interest. For example, Franses (1991) shows that application of the seasonal differencing filter when seasonality is in fact deterministic, leads to deterioration in forecast accuracy. Clements and Smith (1997)[10] report similar results in case the presence of deterministic shifts in seasonal patterns leads to spurious detection of periodic structure in the time series. Even though the tests detect periodicity, the use of periodic model leads to a fall in forecast accuracy since the series is not in fact generated by a periodic DGP.

In view of this, it becomes very important to study the univariate properties of the time series prior to using them, to see what type of models are required to capture the seasonal patterns. Though there have been two studies for India in this direction (Nachane and Lakshmi, 2002 and Sinha and Kumawat, 2004), the analysis in both of these is incomplete. The former relies for lag selection (for augmenting the test equation) on the approach suggested by Beaulieu and Miron (1993), which is suspected to choose too parsimonious lag structure, as pointed out by Hylleberg *et al.* (1993) and Rodrigues and Osborn (1999), among others. The latter is limited in its coverage of models, although it finds evidence of nonstationarity. Specifically, even though it finds that the index of industrial production is seasonally integrated, it leaves open the possibility that the unit roots found are due to periodic integration. The same is true for a few other variables also, which are integrated at some (pairs of) seasonal frequencies. Another aspect which this study ignores is the detection and treatment of outliers, which may affect the properties of the tests substantially. Haldrup, Montanes and Sanso (2000) show that outliers bias the HEGY test results towards rejection. Finally, neither of these two studies explores

[9] This is not unexpected, however, since the misspecified homogeneous model (MHM) corresponding to a periodically integrated model would contain the seasonal difference operator in the AR component with a large MA component The MHM is the constant coefficient model which generates the same second order moments as exhibited by a series generated by the given periodic model. See the discussion in Osborn (1991) and Ghysels and Osborn (2001). However, due to the presence of large MA component, which is known to increase the size of unit root tests substantially, unit roots may be found at only a few frequencies, even though the AR component has all seasonal roots on unit circle.

[10] 'Forecasting Seasonal UK Consumption Components', Unpublished paper, Department of Economics, University of Warwick.

the implications of its results for out-of-sample forecasts, which is an important objective of the present study.

The following section contains the details of methodology adopted in this paper.

4 Econometric Methodology and Data

In this section we describe the econometric methodology used in this paper, followed by description of the data used.

4.1 Econometric Methodology

From the discussion in the previous section it is clear that periodic models are the most general models which embed both, the seasonal integration-cointegration models and the simple integration-cointegration models, as special cases. The neglect of periodicity, on the other hand, not only leads to misspecification of the models but it hides some important information relating to periodic variation in the parameters of interest. Similarly, the presence of seasonal unit roots, even in non-periodic models, invalidates the analysis which is based on the assumption that the seasonality can be taken care of by simply including seasonal dummies in the relevant equations (as shown by Abeysinghe (1991, 1994) and Franses *et al.* (1995), among others). It would therefore be better to start with periodic models and then move to simpler models if the characteristics of the data permit. This is the strategy followed in the present study. It is based on the results of Franses and Paap (1994), Boswijk and Franses (1996), Boswijk, Franses and Haldrup (1998, referred to as BFH henceforth) and Paap and Franses (1999). It consists of the following steps:

1. In the first step the order of PAR is determined. This is done by estimating the equation

$$y_t = \sum_{s=1}^{4} \alpha_s D_{st} + \sum_{s=1}^{4} \beta_s D_{st} T_t + \sum_{s=1}^{4} \phi_{1s} D_{st} y_{t-1} + \cdots + \sum_{s=1}^{4} \phi_{ps} D_{st} y_{t-p} + \epsilon_t \qquad (11)$$

   where $D_{st}, s = 1, \ldots, 4$ are seasonal dummies and T_t is the year in which the observation falls. It should be noted that we have to allow for periodically varying intercepts and trends since even if the series does not have periodically varying intercepts and trends, the presence of periodically varying autoregressive coefficients causes these components to be periodically varying in the reduced from presented in eq. (11). The number of lags is chosen using the following strategy. Initially the model is estimated with 1 to 12 lags and values of AIC and SBC are compared for all these specifications. Following Franses and Paap (1994), we choose the model suggested by the latter in case the two criteria suggest different specifications. Joint significance of the lag next to the one chosen on the basis of the information criteria is then tested and included if it is significant. This is done till we find that the lag next to that included is insignificant. This is done in view of the fact that at times the model chosen by AIC or SBC suffers from misspecification. This model is then subjected to a number of misspecification tests[11].

2. If residuals from some equation are not normally distributed, outlier dummies are added. If even then evidence of significant autocorrelation or ARCH is found, higher lags are considered.

3. The specification thus chosen is subjected to test for periodic parameter variation. This is simply the F-test for the hypothesis of the autoregressive parameters in (11) being equal across different quarters.

4. If the hypothesis of no periodicity is rejected, then we proceed along the following lines:

   - Test for periodic integration, starting with largest number of unit roots possible (4 in case the order of PAR is 4 or greater; equal to order of PAR otherwise) using the methodology suggested by BFH, which relies on a series of likelihood ratio tests. The test equations are estimated using non-linear least squares (NLS); or weighted NLS, if there is evidence of periodic heteroskedasticity.
   - Having determined the number of unit roots, the hypothesis of non-periodic integration is tested.
   - Finally the hypotheses of restrictions on deterministic components, i.e., no quadratic trends (only in case of periodic integration), common linear trends,

[11]The following checks were applied: the Jarque-Bera test for normality (sufficient number of outlier dummies were added, whenever required, to ensure that the residuals do not reject the hypothesis of normality); the LM tests for first and first to fourth order autocorrelation; the LM tests for first and first to fourth order ARCH; the F-test for first order periodic autocorrelation, which is given by the joint significance of the lagged residual terms in the regression

$$r_t = \sum_{s=1}^{4} \alpha_s D_{st} + \sum_{s=1}^{4} \beta_s D_{st} T_t + \sum_{s=1}^{4} \phi_{1s} D_{st} y_{t-1} + \cdots + \sum_{s=1}^{4} \phi_{ps} D_{st} y_{t-p} + \sum_{s=1}^{4} \rho_s D_{st} r_{t-1} + \eta_t \tag{12}$$

r_t being the residuals from (11); and finally, the test for periodic heteroskedasticity, which is simply the F-test for significance of seasonal dummies in the regression of squared residuals on intercept and three seasonal dummies.

etc. are tested.

(All the equations involved in testing for periodic integration are given in Appendix II.)

5. If the hypothesis of no periodicity is not rejected, then we proceed by applying the HEGY test for seasonal unit roots. This test is based on the equation

$$y_{4t} = \pi_1 y_{1t-1} + \pi_2 y_{2t-1} + \pi_3 y_{3t-2} + \pi_4 y_{3t-1} + \epsilon_t \qquad (13)$$

where $y_{1t} = (1 + L + L^2 + L^3)y_t$, $y_{2t} = -(1 - L + L^2 - L^3)y_t$, $y_{3t} = -(1 + L^2)y_t$ and $y_{4t} = (1 - L^4)y_t$. The equation is augmented by deterministic components and lags of the dependent variable depending on the characteristics of the series[12]. The lag augmentation here is taken to be that suggested by AIC and SBC. This is increased further if the next lag is significant or if the equation shows misspecification on the basis of misspecification tests mentioned above (out of those tests, the test for periodic autocorrelation is not carried out here). Outlier dummies are also included whenever the residuals are not normally distributed and it appears that this is due to some outlier(s). Finally, in case the aforementioned criteria select less than four lags, the test is also carried out with four-lag specification, This is in view of the results of Taylor (1997) and Rodrigues and Osborn (1999), which indicate that the results of the test are highly sensitive to lags and we should include at least four lags in case of quarterly data[13].

4.2 Data

This study covers four variables. Index of industrial production (IIP) is the most comprehensive measure of real economic activity for India, that is available at high frequency for a sufficiently long period. Though the GDP series is also available now at quarterly frequency, this is available only since 1996 and hence is not sufficient for our purpose. Two

[12]For a discussion of the deterministic components and lags of the dependent variable, see Sinha and Kumawat (2004).

[13]Detection of significant periodic heteroskedasticity raises an additional statistical issue, since the original HEGY tests are based on the assumption of homoscedasticity. Albertson and Aylen (1996) suggest one modification to the HEGY test equation to accommodate periodic heteroskedasticity, but properties of that modified version have not yet been tested. However, Burridge and Taylor (1999) have shown that under many patterns of periodic heteroskedasticity, the properties of the HEGY tests remain unaffected. Many other patterns increase the size of the tests as compared to specified level of significance, mainly for those pertaining to the annual frequency; though the effect is reduced to a great extent if one relies on the F test for the pair of complex unit roots. Therefore, we stick to the HEGY equation without any change, with the modification that for inference we use the standard errors suggested by White's heteroskedasticity-corrected covariance matrix.

measures of money supply are also considered, broad money (M3) and narrow money (M1). This takes into account the possibility of differing behaviour of aggregates and their components with respect to seasonality, as reported by Lee and Siklos (1997) for the USA. Finally we study the consumer price index for industrial workers (CPI). The data series cover the period from 1981Q1 to 2004Q2. However, only the observations for the period ending in 2001Q4 are used for estimation. The remaining are reserved for out-of-sample forecast comparison. The data on consumer price index has been taken from the website of the Central Statistical Organisation (CSO), Government of India, while the other three series are from the Reserve Bank of India Database on the Indian Economy. All the series have been considered in natural logarithm.

5 Discussion of results

5.1 IIP

A preliminary idea about the nature of movements in a variable can be obtained from the plots and autocorrelation functions of the series, and this is true for the nature of seasonality as well. However, in case of seasonal time series, we not only look at the plots of the level and first difference of a time series, but also at the so-called VQ plots. The latter are obtained by plotting together the four annual series corresponding to the four quarters of the year. Similarly, apart from the simple autocorrelation function, we also look at the periodic autocorrelation function.

A look at the plots of IIP suggests clear seasonal patterns which change in a systematic but non-monotonic manner over the sample period, thus ruling out pure deterministic seasonality. The autocorrelation function exhibits clear-cut seasonality, possibly non-stationary, in first differences. The most interesting observations, however, come from the periodic autocorrelation function. First, the autocorrelation functions are periodically varying (based on the recursive periodic autocorrelations). Second, at times their values exceed unity, thus indicating periodic unconditional heteroskedasticity for the series in question. As discussed in subsection 2.2 above, this can be due to either periodic conditional heteroskedasticity or periodic variation in the autoregressive parameters, or both. Nevertheless, these two observations confirm that there is some form of periodicity in the DGP for IIP. Thus the plots give strong indication of non-stationarity of seasonality and also periodicity.

With respect to the econometric tests, PAR of order 2 with an outlier dummy for the period 1992Q1 clears all the misspecification tests. However, there is evidence of significant periodic heteroskedasticity[14]. The square roots of maximum likelihood estimates of error variance for the four quarters are 0.025, 0.026, 0.014 and 0.016 respectively, indicating that the error variance in the first two quarters is almost three times that in the last two quarters. The test for periodic parameter variation (which is now based on weighted least squares, in view of significant periodic heteroskedasticity) clearly rejects the null hypothesis of no periodic parameter variation. In view of this result we test for periodic integration, starting with two unit roots (again, in view of significant periodic heteroskedasticity, the equations presented in Boswijk and Franses (1996), BFH and Paap and Franses (1999) were modified to accommodate heteroskedasticity. The modified equations are given in Appendix II). The hypothesis of two unit roots is strongly rejected, while that of one unit root is not rejected even at 10%. The hypotheses of this root being non-seasonal (1 or -1) and that of no quadratic trends[15] are also strongly rejected (see Table 1, Appendix I). Thus we conclude that the appropriate model for IIP is a PAR(2) model with one unit root, which is periodic.

Though our testing strategy shows clear presence of periodic parameter variation, it is possible that this is due to some untapped feature of the series. For instance, Clements and Smith (1997) found that the presence of a structural break in the seasonal pattern caused them to detect periodic parameter variation spuriously. Similarly, Proietti (1998) has also shown that the tests for periodicity do not have sufficiently large power. Finally, even when we are sure that the true model for IIP is periodic, there remains the question of whether allowing for periodicity, and consequently increasing the size of the model substantially, leads to any significant gains in forecast accuracy. Therefore, it is imperative that we try to model the series in constant parameter framework to see if that model performs better in terms of forecast accuracy. For this the HEGY test was applied to this series[16], the results for which are presented in Table 2 in Appendix I.

[14]The results of misspecification tests are not given here but may be obtained from the authors on request.

[15]This test follows the discussion in Franses and Paap (1999). It can be carried out in either of two ways. First, we can test for the joint hypothesis of periodic integration and no quadratic trends, against the unrestricted model, followed by the test of simple hypothesis of periodic integration. The rejection of the former but not the latter indicates periodic integration and quadratic trends. Second, we can test simple hypothesis of periodic integration and then test the hypothesis of no quadratic trends conditional on periodic integration. We have followed both the routes and got the same result.

[16]In view of the periodic heteroskedasticity, the test for this variable is based on heteroskedasticity-corrected standard errors. Further, the test was carried out using both the seasonal trends and non-seasonal trend specifications. Finally, in view of the poor power of the t-tests for lag augmentation, the tests were carried out on the basis of two specifications: first, the specification suggested by the strategy outlined in the previous section; and second, four-lag specification.

The results show that the zero frequency unit root is present while the annual frequency unit roots are not. The evidence for biannual frequency is not that clear. The fact that the series shows periodic heteroskedasticity provides some clue here. Even though the type of heteroskedasticity found in our case does not affect the distributions of test statistics, we do not have any statistical evidence for the fact that the heteroskedasticity is of this form. Therefore, it may be better to look at the results in light of the effects of periodic heterokedasticity in general. The results of Burridge and Taylor (1999) show that the general effect of periodic heteroskedasticity is to increase size as compared to the specified level of significance. Hence, even though the results seem to suggest rejection of the unit root at this frequency, we cannot reject it with much confidence and conclude that there is a possibility of two unit roots for IIP, one corresponding to the zero frequency and the other for the frequency π.

Thus we conclude that the seasonality in IIP is essentially non-stationary. While the data suggest that there is significant periodic parameter variation in the AR parameters, even when we ignore the periodic parameter variation we find evidence of non-stationary seasonality, reflected in the biannual frequency unit root.

5.2 M3

Unlike IIP, the plots of M3 do not indicate any non-stationarity of seasonality, nor is there any indication of periodicity. The absence of periodicity is supported by a formal test also, implying that we can test for unit roots in a non-periodic framework. However, it has to be done using the HEGY test, to see if there are any seasonal unit roots as well. Unlike IIP, we carry out this test only in non-seasonal linear trend specification, since the plots do not suggest seasonally varying drifts. Further, since the lag selection strategy chooses only one lag, we carry out the test with four lags also. The results here are a bit surprising in that the specification with just one lag rejects the unit root hypothesis even at zero frequency while the one with four lags is not able to reject it at the Nyquist frequency either. It is very difficult to settle this question on the basis of this information and therefore it is better left to be settled by forecast comparison. However, there is no doubt regarding the absence of unit root at the annual frequency, implying that the use of the seasonal difference operator is not justified.

Thus, the results of the seasonal unit root test do not provide any clear answer to the question of whether the seasonality in M3 is deterministic or stochastic, though they clearly reject the need for a seasonal difference filter to render the series stationary.

5.3 M1

The plots and autocorrelation functions for M1 are largely similar to those for M3. The hypothesis of periodicity is clearly rejected. However, it was very difficult to find a specification free of all types of misspecification, for the HEGY test. The specification that was chosen had six lags of the dependent variable and one outlier dummy for the period 1992Q1 (the same period for which we had to include an outlier dummy for IIP). The test results indicate that while the zero and Nyquist frequency unit roots are not rejected at all, those corresponding to the annual frequency are rejected at 2.5% only. The hypothesis of no seasonal unit roots (tested using the F_{234} statistic) is rejected at 5% only, while that of no unit roots (tested by F_{1234}) is not rejected at all, indicating non-stationarity of seasonality. However, the specification having six lags may lead to loss of power of the test, and the rejection of unit roots at this small level even under this specification clearly shows rejection of unit roots at this frequency.

Thus, for M1, we conclude that while the zero and π frequency roots are on the unit circle (implying that seasonality in this variable is non-stationary), those corresponding to the frequencies $\pm\frac{\pi}{2}$ are not. Though this indicates non-stationarity of seasonality, this result has to be seen in light of the fact that the lag polynomial here includes a large number of lags (six), which may cause the test to have low power.

5.4 CPI

The last variable studied here is the CPI. The preliminary tools do not indicate any periodicity, and this result is further endorsed by the formal econometric test for periodic parameter variation[17]. The HEGY test clearly rejects the unit roots at the annual frequency, while that at the zero frequency is not rejected at all. As for the Nyquist frequency, the specification with one lag rejects it outright, while the one with four lags rejects it only at 10%, a level too large for this test in view of the multiple testing problem[18].

Thus, for CPI the unit root tests do not provide a clear answer to the question of

[17] It may be noted here that to arrive at a specification free from all the types of misspecification we have to include two outlier dummies, for the periods 1998Q1 and 1999Q1.

[18] For a detailed discussion of the levels of significance in case of multiple hypothesis testing, see Sinha and Kumawat (2004), among others.

whether the seasonality is deterministic or stochastic. However, it makes clear that the seasonal difference filter is certainly redundant.

From the above discussion it is clear that though there is evidence of non-stationarity of seasonality in the variables considered, in some of the cases the tests do not provide any clear answer. One way to resolve this issue would be to compare the forecasts from the alternative models between which the unit root tests are not able to differentiate. This is attempted in the following section.

6 Forecast Comparison

The discussion in the previous section indicates that the four series discussed here exhibit different types of seasonality. In this section we compare the forecasting performance of different models to see which model has the best performance in terms of out-of-sample forecasts, for each variable. This is important not only for generating out-of-sample forecasts, but also it serves as a confirmatory check for the models chosen by the econometric tests. The latter is necessitated by the the fact that the econometric tests are sometimes not able to differentiate between different models having similar properties and hence it becomes very important to apply other checks. Therefore, after having selected the model using the data through 2001Q4, one to four quarters ahead dynamic forecasts were generated recursively for the period 2002Q1 through 2004Q2, thus getting 10 one quarter ahead, 9 two quarters ahead, 8 three quarters ahead and 7 four quarters ahead forecasts for each variable. These forecasts were then evaluated using two criteria:

1. As a first check, Theil's inequality statistic was computed for each set of forecasts. This is given by
$$U = \frac{\sqrt{\frac{1}{n}\sum_{t=1}^{n}(y_t - \hat{y}_t)^2}}{\sqrt{\frac{1}{n}\sum_{t=1}^{n}(y_t - y_{t0})^2}} \tag{14}$$
where n is the number of forecasts, $\hat{y}_t$ is the forecast for period t, y_t is the actual value for period t, and y_{t0} is the *naive* forecast for period t, which is simply the value of the variable in the last observation in the sample used for estimating the model. The values of this statistic were computed for all the models, for all the horizons.

2. The significance of difference between the Theil's U statistic from alternate models was then tested using the Modified Diebold-Mariano Statistic suggested by Harvey *et al.* (1997). This is necessitated by the fact that even though the U statistic shows

difference between forecasting performance of two models, it ignores the random character of the forecasts and therefore, it may show very large differences between forecast accuracy which might be due to purely random factors.

The statistic is given by

$$S_1^* = \left[\frac{n+1-2h+n^{-1}h(h-1)}{n}\right]^{1/2} \left[\hat{V}(\bar{d})\right]^{-1/2} \bar{d} \tag{15}$$

where

$$\bar{d} = \frac{1}{n}\sum_{t=1}^{n}(e_{1t}^2 - e_{2t}^2) \tag{16}$$

and

$$\hat{V}(\bar{d}) = \frac{1}{n}\left[\hat{\gamma}_0^* + \frac{2}{n}\sum_{k=1}^{h-1}(n-k)\hat{\gamma}_k^*\right] \tag{17}$$

with

$$\hat{\gamma}_k^* = \frac{1}{n-k}\sum_{t=k+1}^{n}(d_t - \bar{d})(d_{t-k} - \bar{d}) \tag{18}$$

Here e_{1t} and e_{2t} are the forecast errors from the two models being compared, h is the forecast horizon and n is the number of forecasts generated for that forecast horizon. The value of this statistic is compared with relevant one-tailed critical value from the t distribution with $(n-1)$ degrees of freedom.

The details of the models used for forecast comparison are given in Table 3 in the Appendix I. The results for the four variables are presented in Tables 4A, 4B, 4C and 4D respectively, and are discussed in the following subsections.

6.1 IIP

The values of the U statistic presented in Table 4A show that among the non-periodic models, the model with four unit roots performs better than the model with just one unit root at 1, 2 and 3-quarters ahead forecast horizons, though the differences in the values of the statistic are not very large. These differences between the values of the statistic, favouring the model with four unit roots, suggest that the seasonality in IIP is not stationary. Comparing with the periodic model, the forecasts produced by the seasonal difference model are far worse than those produced by the periodic model for one-quarter-ahead horizon, while at the other three horizons the former produces better forecasts as compared to the latter. Testing for the significance of the difference between the forecast accuracy of seasonal and periodic models using the MDM test, we reject at 5% the hypothesis of equal forecast accuracy against the alternative of the periodic model producing better forecasts, for one-quarter-ahead horizon. For other horizons, it is not possible to reject this hypothesis.

It may thus be concluded that while the periodic model produces significantly more accurate forecasts at the one-quarter-ahead horizon, its performance is not significantly worse than the non-periodic models at other horizons and therefore the periodic model is the best for forecasting IIP.

6.2 M3

The results presented in Table 4B show that the model for first difference performs better than the other two models in terms of the Theil's inequality statistic. This indicates that the non-rejection of the Nyquist frequency unit root (noted in the previous section) was due to poor power of this test caused by the large number of lags in the augmentation polynomial. Using the MDM test for the null hypothesis of equal forecast accuracy of the models with one and four unit roots, we do not reject the null hypothesis at one and two quarters horizons, but reject it in favour of the simple integration model at 10% for three-quarters-ahead horizon and at 1% for four-quarters-ahead horizon; thus discarding the seasonal difference operator in favour of first difference. The sufficiency of the first difference is further supported by the fact that when applied to the first and second difference models (the latter implies unit root at the biannual frequency also, in addition to the zero frequency), the MDM test favours the former at 1% for two-quarters-ahead horizon and at 10% for one and three-quarters-ahead horizons[19]. Hence, we conclude that the forecasts produced by the model with two unit roots are inferior to those produced by the model with only one unit root, and thus seasonality in this variable may be captured more accurately by the latter.

The results thus indicate that the seasonality in this variable is not non-stationary and the model with only zero frequency unit root produces more accurate forecasts than the ones with seasonal unit roots.

[19] At four quarters ahead horizon, we find a problem with the MDM test: the variance estimate turns out to be negative and hence it is not possible to take square root of this. Diebold and Mariano (1994) treat negative variance estimates as zero and automatically reject the null hypothesis of equal forecast accuracy. Though not reported here, the mean of difference between squared forecast errors (the $\bar{d}$ statistic, whose significance is being tested) is negative for this case also, like the other three forecast horizons, indicating better performance by the model with one unit root.

6.3 M1

The results for M1 are different from those for M3. In terms of the U statistic, the performance of the model with four unit roots is comparable to that of the model with one unit root at one, two and four quarter horizons, and is better at the three-quarters horizon. This indicates that overall, the former is capturing the seasonal patterns better than the latter, implying in turn some degree of non-stationarity of seasonality in this variable. The MDM test, however, does not reject the hypothesis of equal forecast accuracy for any horizon, neither for the model with four unit roots against one (non-seasonal) root, nor for two unit roots against one unit root, and hence is not able to distinguish between alternative models.

Thus, we may conclude that overall, the model with seasonal unit roots produces at least as accurate forecasts as the model with just a non-seasonal unit root.

6.4 CPI

The results here are different from those for the other three variables. In terms of the U statistic, the model with one unit root performs better than the one with four unit roots[20] at one-quarter-ahead horizon. The performance of the two is comparable at the two-quarter horizon, while at the other two horizons the model with four unit roots performs better. This indicates that the seasonality in this variable may be of non-stationary nature. The MDM test, however, again does not favour any model, at any horizon.

Hence, we may conclude that the seasonal difference model generates at least as accurate forecasts as does the model with just one unit root, as in the case of M1.

7 Implications of results

The results of this paper have implications for forecasting as well as macroeconomic modelling. The main implications may be summarised as follows:

- One important implication of periodic integration in IIP is that though both trend and seasonality are stochastic, the seasonality is not independent of the other components and therefore it is not possible to decompose this series into mutually or-

[20] The roots of the MA component in the latter model are found to be near the unit circle, thus indicating overdifferencing caused by the fourth difference filter.

thogonal seasonal and non-seasonal components (see Franses (1995b) and Ooms and Franses (1997)). Hence, seasonal adjustment would be a senseless exercise.

- Though IIP is found to be periodically integrated, it has less than four unit roots. This implies that while the first difference would not be sufficient to render the series stationary, the seasonal difference would lead to non-invertibility. Therefore, the proper way to render the series stationary is by means of periodically varying filters.

- Among the two measures of money supply, we find some evidence of non-stationary seasonality in narrow money, while the seasonality in broad money is stationary. As the former is a component of the latter, the results show that the individual components of money supply exhibit non-stationary seasonality but these random changes in the seasonals in different components are related to each other, giving rise to stationary seasonality in the aggregate.

- The finding that all the four unit roots are not on the unit circle for any of the variables considered here implies that the seasonal difference filter would lead to overdifferencing in all of these.

- For the purpose of forecasting, the results show that while the periodic model (with periodic integration) would give the best forecasts for IIP, for broad money, the model with zero frequency unit root and stationary seasonality gives most accurate forecasts. In narrow money and consumer price index, the model with four unit roots produces at least as accurate forecasts as the one with only zero frequency unit root.

- Another important result is that the IIP shows periodic conditional heteroskedasticity with error variance in the first and second quarters being almost three times that in the last two quarters. This indicates that the forecasts are more likely to be off the mark in these two quarters. The other implication of this is that any change appearing in the IIP in these two quarters needs to be taken with greater caution as it is more likely to be random[21]. Finally, given the fact that the first and second quarters are, respectively, the highest and lowest economic activity quarters for the Indian economy, this implies that the seasonal range has very large variance.

[21]This is further supported by the fact that almost all the outlier dummies required for different variables correspond to these two quarters, as is clear from Table 3 in Appendix I.

High volatility in the lowest economic activity quarter is not a new phonomenon. Cecchetti, Kashyap and Wilcox (1997) suggest that this might be due to the fact that firms face upward sloping and convex marginal cost curves, which make shifting of production to low-activity quarters in the times of high demand the optimal policy. Due to this, the amount of seasonal variation varies with the level of economic activity, in that at the time of high economic activity, some amount of production will shift to the low economic activity quarter, thus reducing total seasonal variation in output. This is reflected in increased volatility in the lowest economic acitivity quarter. Obviously, any model which does not capture this interaction between seasonal variation and average level of economic activity would show large error variance in the low-activity quarter.

The reason for high volatility in the first quarter is less clear. The primary reason for the industrial production in India achieving its intra-year peak in the first quarter may be the demand generated by the harvesting of the Kharif crop (the crop which is sown in the months of monsoon and harvested over October-November), which causes a lagged surge in demand. The Kharif crop also provides major inputs to many industries. However, the quantum as well as distribution of monsoon rains is highly erratic, causing wild fluctuations in Kharif output and hence also demand for industrial products generated by this income. This erratic nature of monsoon is therefore likely to result in larger variance in industrial output in the first quarter. Again this feature may be captured by some model which models seasonal fluctuations as a function of aggregate economic activity.

- For macroeconomic modelling, the existence of periodic integration in IIP has one important implication. It implies that the cointegrating vector of IIP with any series that has non-periodic integration will be periodic in nature. One potential example of this is the relation between money supply and output. This result becomes all the more important for the Indian economy in view of the fact that the sub-annual frequency data for GNP is not available for the period before 1996 and therefore IIP is taken as the measure of real economic activity in many studies involving the period before 1996.

8 Conclusions

The three types of variables considered here are found to have different nature of seasonality. However, with the exception of broad money, the other three variables exhibit stochastic seasonality, invalidating the practice of deseasonalisation using regression on seasonal dummies. On the other hand, the finding of less than four unit roots in all the series invalidates blind use of the seasonal differencing filter also. Taken together, these two facts imply that the appropriate way to proceed would be to first test for nature of seasonality.

For the purposes of forecasting, we conclude that the model with periodic integration is recommended for industrial production. For broad money, the model with first difference and seasonal dummies would give the best forecasts. For the other two variables considered here, namely, narrow money and the consumer price index, there may be some gains by using the seasonal differencing filter. For macroeconomic modelling, we find that the relation of industrial production with any series with non-periodic integration will essentially be periodic, since the former is periodically integrated.

An examination of the plot of the first difference of IIP suggests that the seasonal range changes in a systematic manner, being at its peak in the early 1990's. This suggests, in turn that though the seasonal patterns in this variable are subject to changes, these changes may not be entirely random; i.e., there is still some information in these changes. There is some systematic variation, which might be captured by a non-stochastic process, though possibly non-linear, and such non-linear models might provide a better description of the series. This is further reinforced by the finding of periodic conditional heteroskedasticity. Similarly, for both the measures of money supply, we find that fairly large AR/MA polynomials are required to capture the underlying dynamics. Again this may be due to some movements not captured by linear models. Thus it may be appropriate to check these series for non-linear dynamics. Preliminary results support this proposition.

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Appendix I

Table 1: Tests for Periodic Integration in IIP

	Model	LLF	RSS	LR
(A)	Unrestricted	-116.3544	82.00293	—
(B)	2 Unit roots	-148.1170	177.9410	63.5251[1]
(C)	1 Unit root	-116.4676	82.22967	0.226417[2]
(D)	1 Unit root, no quadratic trend	-161.334	245.6297	89.9591[3]
				89.733[4]
(E)	Root = 1	-124.6287	100.3403	16.32218[5]
(F)	Root = -1	-160.4798	240.5635	88.02437[6]

[1] vs. Model (A). Significant at 1%. (Critical value: 23.46, see Osterwald-Lenum (1992), Table 2).

[2] vs. Model (A). Insignificant. Critical value at 10%: 2.57 (see the reference in note 1).

[3] vs. Model (A). Significant at 1% (Critical value: 16.42, see table V, Johansen (1994)).

[4] vs. Model (C). Significant at 1% (Critical value: 6.6349, χ^2 with 1 d.f.).

[5] vs. Model (C). Significant at 1%. (Critical value: 11.345, χ^2 with 3 d.f.).

[6] vs. Model (C). Significant at 1%. (Critical value: 11.345, χ^2 with 3 d.f.).

Table 2: Tests for Seasonal Unit Roots

Variable	Specification[1]	Lags[2]	t_1	t_2	F_{34}	F_{234}	F_{1234}
IIP	$SD+T$	2	-2.09195	-3.00212[b]	17.859217[d]	19.720265[d]	24.601713[d]
		4	-2.37715	-3.31388[c]	14.306791[d]	18.916602[d]	24.358414[d]
	$SD+ST$	1	-1.81773	-2.86164	29.934948[d]	32.537338[d]	32.944497[d]
		4	-2.50035	-3.63819[b]	25.803085[d]	37.146001[d]	40.241701[d]
M3	$SD+T$	1	-4.433418[d]	-2.682248[a]	9.64968[d]	8.69205[d]	18.50940[d]
		4	-2.72569	-1.81035	7.63754[c]	6.77794[b]	6.92446[b]
M1	$SD+T$	6	-0.60918	-1.63473	8.39513[c]	6.14345[b]	4.72545
CPI	$SD+T$	1	-0.90951	-5.05396[d]	38.88970[d]	35.65421[d]	31.66993[d]
		4	-0.95369	-2.67915[a]	26.65437[d]	24.42195[d]	18.31843[d]

[1] Deterministic components in the test equation. SD stands for seasonal intercept dummies, T for deterministic linear trend, and ST for seasonally varying deterministic linear trend.

[2] Number of lags of dependent variable used to augment the test equation.

[a] Significant at 10%. Critical values: For SD+T specification – from Franses and Hobijn (1997); for SD+ST specification – from Smith and Taylor (1998).

[b] Significant at 5%. Critical values: from sources indicated in the note (a).

[c] Significant at 2.5%. Critical values: from sources indicated in the note (a).

[d] Significant at 1%. Critical values: from sources indicated in the note (a).

Table 3: Models used for forecast comparison

Variable	1 UR[1]	2 UR	SI	PI
IIP	SD[2], ST, AR(1,3,4) and MA(3), outlier dummies for the periods 1991q2 and 1992q2	SD, ST, AR(1,2) and MA(1,2)	I, with AR(1) and MA(4), outlier dummies for 1988q2, 1990q1 , 1991q2 and 1993q1,	SD, ST, outlier dummy for 1992q1
M3	SD, MA(5,7)	SD, AR(1,2) and MA(1,2)	Intercept(non-seasonal), AR(1), MA(5,7) with multiplicative seasonal MA term	NA
M1	SD, AR(2,4) two outlier dummies for 1991q4 and 1994q2	SD, AR (1,2,3), MA(1,2), outlier dummy for 1992q4	I, AR(1 to 5), MA (1 to 3) with multiplicative seasonal MA term, one outlier dummy for 1992q4	NA
CPI	SD, AR(1) outlier dummies for 1998q1 and 1991q1	SD, AR (1 to 4), MA (1) outlier dummies for 1998q1 and 1991q1	AR(1,2), MA(4)	NA

[1] In this and all the following tables, the following notation has been used for different models: 1 UR – Model for first differenced series; 2 UR – Model for the series filtered with $(1 - L^2)$ filter; SI – Model for seasonally differenced series; PI: Model for series filtered with quasi-differences (periodically varying).

[2] The following notation has been used for deterministic variables: I for intercept, SD for seasonal intercept dummies, T for linear trend, ST for seasonally varying linear trend.

Table 4A: Forecast Comparison for IIP

Horizon	**Theil's U Statistic**				MDM Statistic
	1 UR	2 UR	SI	PI(1)	SI vs. PI(1)
One-quarter	0.248903	0.276052	0.238006	0.144581	1.70635[a]
Two-quarter	0.26084	0.300496	0.245956	0.302244	0.08446
Three-quarter	0.236478	0.318377	0.22558	0.244374	0.23439
Four-quarter	0.209376	0.355668	0.244771	0.314438	0.03445

[a] Significant at 5% (One tailed t-test with 9 d.f.)

Table 4B: Forecast Comparison for M3

Horizon	**Theil's U Statistic**			**MDM Statistic**	
	1 UR	2 UR	SI(1)	1 UR vs. SI	1 UR vs. 2 UR
One-quarter	0.316145	0.336942	0.32999	-0.6498	-1.5868[a]
Two-quarter	0.291317	0.295588	0.314293	-0.6631	-3.6689[b]
Three-quarter	0.270396	0.290879	0.293269	-1.6406[c]	-1.4797[d]
Four-quarter	0.265514	0.285079	0.279737	-5.8878[e]	—[f]

[a] Significant at 10% (One tailed t-test with 9 d.f.)

[b] Significant at 1% (One tailed t-test with 8 d.f.)

[c] Significant at 10% (One tailed t-test with 7 d.f.)

[d] Significant at 10% (One tailed t-test with 7 d.f.)

[e] Significant at 1% (One tailed t-test with 6 d.f.)

[f] Test statistic could not be computed due to negative estimate of variance of the measure of distance.

Table 4C: Forecast Comparison for M1

Horizon	**Theil's U Statistic**			**MDM Statistic**	
	1 UR	2 UR	SI(1)	1 UR vs. SI	2 UR vs. SI
One-quarter	0.253062	0.261475	0.258106	-0.79734	-0.04168
Two-quarter	0.198354	0.204703	0.194444	-0.15003	0.03020
Three-quarter	0.182964	0.178247	0.159625	-0.07350	-0.10722
Four-quarter	0.160844	0.159787	0.158827	-0.19266	-0.22265

Table 4D: Forecast Comparison for CPI

Horizon	**Theil's U Statistic**			MDM Statistic
	1 UR	2 UR	SI	1 UR vs. SI
One-quarter	0.550981	0.616182	0.578576	0.56650
Two-quarter	0.521704	0.549677	0.529958	0.06571
Three-quarter	0.510269	0.554031	0.497343	0.33992
Four-quarter	0.533768	0.571626	0.488944	0.49554

Appendix II
Econometric Methodology

This paper focuses on two classes of models, seasonal integration and periodic integration models. To test for the former we employ the HEGY test (suggested by Hylleberg *et al.* (1990)). For the latter, we rely on the test strategy suggested by Boswijk and Franses (1996) and Boswijk, Franses and Haldrup (1998, referred to as BFH henceforth), and extended it to include deterministic components as suggested by Paap and Franses (1999). The following two sections discuss these tests.

Testing for seasonal unit roots

The HEGY test has been documented very widely in the literature, including a number of textbooks and review articles on time series; and therefore does not need to be mentioned in detail. There are three issues, however, which are worth mentioning:

- In case of IIP we find evidence of periodic heteroskedasticity. Burridge and Taylor (1999) show that periodic heteroskedasticity does not generally affect the properties of the HEGY test. In some cases, however, it leads to a rise in the test size. Albertson and Aylen (1996) suggest one modification to the HEGY test to accommodate periodic heteroskecasticity, but the properties of that have not been studied as yet. We therefore stick to the HEGY test without any modification, except the fact that for the purpose of testing we use heteroskedasticity-corrected standard errors.
- Smith and Taylor (1998) argue that the HEGY test with just periodically non-varying linear trend is not similar under seasonally varying drifts. In case plots suggest seasonally varying trends, the test equation should include the seasonally varying linear deterministic trends. Therefore, for IIP we use two specificatoins: one with seasonally varying intercepts but non-seasonal linear deterministic trends; and the other with seasonally varying intercepts as well as linear deterministic trends.
- Finally, in view of the multiple testing problem we also apply the tests for unit roots at all frequencies and also at all seasonal frequencies. These tests were suggested for quarterly data by Ghysels, Lee and Noh (1994).

Testing for periodic integration

Boswijk and Franses (1996) suggested a test for periodic integration, which was extended to allow for multiple unit roots by BFH. BFH suggest that one should start with the largest number of unit roots and then move towards smaller number of unit roots[22]. This is done using a sequence of likelihood ratio tests. We therefore use the unrestricted PAR model and the restricted models corresponding to different numbers of unit roots. These models are given below:

- **Unrestricted Model:** Denoting a p^{th} order PAR polynomial as $\phi_{p,s}(L)$, the unrestricted model is given by

$$\phi_{p,s}(L)y_t = \epsilon_t \tag{19}$$

  This is estimated by applying OLS (since if ϵ_t is normally distributed, OLS is equivalent to maximum likelihood) to the equation

$$y_t = \sum_{s=1}^{4} \varphi_{1s} D_{st} y_{t-1} + \ldots + \sum_{s=1}^{4} \varphi_{ps} D_{st} y_{t-p} + \epsilon_t \tag{20}$$

  where $\phi_{p,s}(L) \equiv (1 - L - \cdots - L^p)$ and $D_{st}, s = 1, 2, 3, 4$ are the seasonal dummies.

- **Model with four unit roots:** This model is given by

$$\phi_{p-4,s}(L)(1 - L^4)y_t = \epsilon_t \tag{21}$$

  which can be estimated in the same way as done in case of unrestricted model above. The test for periodic integration with four unit roots is then simply the likelihood ratio test for the model (19) vs. (21).

- **Three unit roots:** For this case, the restricted model is given by

$$\phi_{p-3,s}(L)(1 - \gamma_{1s}L - \gamma_{2s}L^2 - \gamma_{3s}L^3)y_t = \epsilon_t \tag{22}$$

  with the following set of restrictions:

$$\begin{array}{rclrclrcl} \gamma_{11}\gamma_{34} & = & 1, & \gamma_{21}\gamma_{34} & = & -\gamma_{14}, & \gamma_{31}\gamma_{34} & = & -\gamma_{24} \\ \gamma_{12}\gamma_{24} & = & -\gamma_{34}, & \gamma_{22}\gamma_{24} & = & 1, & \gamma_{32}\gamma_{24} & = & -\gamma_{14} \\ \gamma_{13}\gamma_{14} & = & -\gamma_{24}, & \gamma_{23}\gamma_{14} & = & -\gamma_{34}, & \gamma_{33}\gamma_{14} & = & 1. \end{array} \tag{23}$$

  Under the assumption of normality of the error process, maximum likelihood estimation of this is equivalent to NLS estimation. Therefore, this model is estimated using NLS. The test for periodic integration with three unit roots is simply the likelihood ratio test for the above model against the unrestricted model (19).

[22] Since in an I(1) process with quarterly frequency the largest number of unit roots possible is four, one starts with a test for four unit roots.

- **Two unit roots:** The restricted model is given by

$$\phi_{p-2,s}(L)(1-\gamma_{1s}L-\gamma_{2s}L^2)y_t)=\epsilon_t \tag{24}$$

with the restrictions

$$\begin{aligned}\gamma_{11} &= -\frac{\gamma_{13}}{\gamma_{23}\gamma_{24}}\\ \gamma_{21} &= \frac{1}{\gamma_{23}}-\frac{\gamma_{13}\gamma_{14}}{\gamma_{23}\gamma_{24}}\\ \gamma_{12} &= -\frac{\gamma_{14}\gamma_{23}}{\gamma_{13}\gamma_{14}+\gamma_{24}}\\ \gamma_{22} &= \frac{1}{\gamma_{13}\gamma_{14}+\gamma_{24}}\end{aligned} \tag{25}$$

Again the model is estimated using NLS and the hypothesis of two unit roots tested by likelihood ratio test against the unrestricted model (19).

- **One unit root:** The restricted model is given by

$$\phi_{p-1,s}(L)(1-\phi_sL)y_t=\epsilon_t \tag{26}$$

with the restriction that

$$\prod_{s=1}^{4}\phi_s=1. \tag{27}$$

Equations estimated for IIP: The above methodology as suggested by BFH is based on two assumptions: (i) zero mean process, and (ii) homoskedasticity of error process. Neither of these is true in our case. Our IIP series (which was found to have periodically varying dynamics) has non-zero mean with upward trend. It also has periodic heteroskedasticity. Due to the former, we have to modify the test equations along the lines of Paap and Franses (1999). The periodic heteroskedasticity, on the other hand, implies that maximum likelihood estimation is not equivalent to NLS but is equivalent to weighted NLS. The latter is, in turn, equivalent to NLS applied to the equation obtained by multiplying the dependent variable and the regression function by appropriate weights, which are functions of maximum likelihood estimates of the error variances.

These equations are presented below[23]. Since in our case, the order of PAR is 2, there can be at most 2 unit roots. Therefore, we do not present the equations corresponding to 3 and 4 unit roots.

- **Unrestricted Model:** Taking $p=2$ and adding deterministic terms, the eq. (19) becomes

$$y_t=\alpha_s^*+\beta_s^*T_t+\phi_{1s}y_{t-1}+\phi_{2s}y_{t-2}+\epsilon_t \tag{28}$$

[23]In addition to the above two changes, we also have to include an outlier dummy for the period 1992Q1.

A slightly different parameterisation of this is

$$(1-\psi_s L)(y_t - \alpha_s - \beta_s T_t - \alpha_s y_{t-1}) = \epsilon_t. \tag{29}$$

This is unrestricted model, and we estimate it by applying NLS to the equation

$$\frac{y_t}{\sum_{s=1}^{4} D_{st}\hat{\sigma_s}} = \sum_{s=1}^{4}\alpha_s\frac{D_{st}}{\hat{\sigma_s}} + \sum_{s=1}^{4}\beta_s\frac{D_{st}T_t}{\hat{\sigma_s}} + \sum_{s=1}^{4}\phi_s\frac{D_{st}y_{t-1}}{\hat{\sigma_s}} + \sum_{s=1}^{4}\psi_s\frac{D_{st}}{\hat{\sigma_s}}(y_{t-1} - \alpha_{s-1} - \beta_{s-1}T_{t-1} - \phi_{s-1}y_{t-2}) + \delta OD_t. \tag{30}$$

where $\hat{\sigma_s}$ are the square roots of maximum likelihood estimates of the periodic error variance, and OD_t is the outlier dummy for the period 1992Q1. This model appears as model (A) in Table 1 in Appendix I.

- **Periodic Integration with two unit roots** Taking $p = 2$ and adding the deterministic variables to (24), we get

  $$(1 - \gamma_{1s}L - \gamma_{2s}L^2)y_t = \alpha_s + \beta_s T_t + \epsilon_t \tag{31}$$

  This was estimated by applying NLS to the equation

  $$\frac{y_t}{\sum_{s=1}^{4} D_{st}\hat{\sigma_s}} = \sum_{s=1}^{4}\alpha_s\frac{D_{st}}{\hat{\sigma_s}} + \sum_{s=1}^{4}\beta_s\frac{D_{st}T_t}{\hat{\sigma_s}} + \sum_{s=1}^{4}\gamma_{1s}\frac{D_{st}y_{t-1}}{\hat{\sigma_s}} + \sum_{s=1}^{4}\gamma_{2s}\frac{D_{st}y_{t-2}}{\hat{\sigma_s}} + \delta OD_t \tag{32}$$

  under the restrictions specified in eq. (25). This model is referred to as Model (B) in Table 1.

- **Periodic Integration with one unit root:** It may be seen that the restriction for 1 unit root, given in eq. (27) can be imposed in eq. (28) to give the model with one unit root. We estimated this by applying NLS to eq. (30), with

  $$\phi_4 = \frac{1}{\phi_1\phi_2\phi_3}. \tag{33}$$

  This is the model (C) in Table 1.

- **Periodic Integration with one unit root and no quadratic trends :** This model is obtained by substituting into eq. (30) the restriction for one unit root (eq. (33)) and the following restriction for no quadratic trends

  $$\beta_1 + \phi_1\phi_3\phi_4\beta_2 + \phi_1\phi_4\beta_3 + \phi_1\beta_4 = 0 \tag{34}$$

  This is referred to as Model D in Table 2.

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