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Abstract

This paper explores the steady state welfare implications of permanent transfers in a two-country, two-sector overlapping generations model. At the golden rule and with Walrasian stability, we demonstrate that the change in the (static) terms of trade *always* works in favor of a transfer paradox. The conditions under which the transfer paradox is obtained are independent of factor intensity rankings and also whether the donor or recipient has the higher savings propensity. In contrast, conditions under which a change in the intertemporal terms of trade delivers a Pareto-improving transfer depend upon both of the above and also on the initial position of the world capital-labor ratio relative to the golden rule.

Key Words: Transfer paradox, Pareto-improving transfers, two-sector overlapping generations model

JEL Classification: F11 , F35, F43, O19, O41

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1. Introduction

The transfer problem has been with us for almost one hundred years; its durability is surprising. Its continuing appearance in the economic literature can certainly be accounted for by the variety of issues that, in their most elemental form, involve international redistributions of income. The first and perhaps the most well-known example of the problem was described in an exchange between Keynes and Ohlin regarding the effects of the post-first-world-war reparations that vanquished Germany was asked to pay. Later issues, such as the effects of foreign aid to developing economies, oil price shocks, and the Latin American debt crisis, have also been cast in terms of international transfers.

Early discussions of the transfer problem spawned a large literature which made intensive use of static trade models. That is, transfers were modeled in terms of a donor country transferring real income to a recipient country in a two-sector, static environment. In these models, the transfer has an obvious, direct effect on welfare of both donor and recipient – the recipient gains and the donor loses. The problematic aspect of the transfers, however, arises from consideration of a second, indirect effect that is channeled through the relative price of the produced goods. In situations where the transfer creates an excess demand for the donor's exportable, and raises its price by enough to outweigh the direct effect of the transfer, the donor will be better off (and the recipient worse off). In the literature, this event is referred to as the `transfer paradox'.

Samuelson (1947) argued that the transfer paradox was not possible when excess demands followed a Walrasian adjustment process. That is, even when the secondary effect of a transfer works against the direct effect, stability requirements preclude it from being of sufficient magnitude to override the `normal' effect. Subsequent authors have shown that a transfer paradox might occur provided that other distortions are present (Bhagwati and Brecher (1982), Bhagwati, Brecher and Hatta (1985)) or there is a third economic agent acting as a bystander to the transfer (see Gale (1974), Bhagwati, Brecher and Hatta (1983), and Jones (1984)). Thus, as far as the static literature is concerned, a transfer paradox requires at least a three-agent setting or a distortion. Absent these, there is no transfer paradox.

The static literature expanded in several directions (and, indeed, continues to do so)[1] but in the context of the Latin American debt problem it was felt that transfers affected saving-- as well as investment-- decisions and thus a dynamic formulation would be more appropriate (see Eaton (1989)). The first attempts to construct a dynamic analysis of the transfer problem made use of a one-sector overlapping generations model. In this setting, the indirect effects of the transfer work via the interest rate; that is, via the intertemporal terms of trade. In addition, since a competitive equilibrium need not be Pareto-efficient in the overlapping generations model, the change in the intertemporal terms of trade also reflects whether the transfer moves the world economies towards or away from the golden rule capital-labor ratio. Galor and Polemarchakis (1987) find that permanent, steady state transfers can produce the transfer paradox in the one-sector environment. However Haaparanta (1989), while investigating the welfare effects of temporary transfers, argued that this paradox was incompatible with initial capital-labor ratios below the golden rule. In other words, dynamic inefficiency—which acts like an intertemporal distortion, was required to obtain the transfer paradox. Haaparanta further demonstrated that a transfer paradox could be reconciled with dynamic efficiency, but only if the model further incorporated pre-existing public debt. Tan (1998), who also examined permanent transfers in the one-sector overlapping generations model, later argued that if transfers are made from rich to poor countries, then the transfer paradox may not arise.[2]

Even a cursory look at the existing literature would convince anyone that the two ways of approaching the transfer problem, namely the earlier trade theoretic static literature and the more recent dynamic literature, have very little by way of a common framework. The former emphasizes the effect of transfers on the static terms of trade, ignoring issues of savings and investment altogether. The latter is

[1] See e.g, Yano and Nugent (1999) for the inclusion of nontraded goods, Kemp and Shimomura (2003) for interdependent utility functions and Turunen-Red and Woodland (1988) for multilateral transfers.

[2] There are others that have analyzed the transfer problem in a dynamic context. In an infinite-lived agent setting, see Brock (1996), Yano (1991), and Gombi and Ikeda (2002), and, in a two-period model see Djajic, Lahiri and Raimondos-Moller (1998). Neither of these approaches allow for an intertemporal terms of trade to cause changes in intergenerational welfare. It is also true that in an infinite-lived representative agent setting, with different but fixed rates of time preference for the two countries, the existence of a steady state is in question. If a steady state exists, then both economies cannot be diversified (see Stiglitz (1970)).

invariably done in a one-commodity framework and therefore, by construction, the only terms of trade that it can address is the intertemporal terms of trade.

Our purpose in this paper is to build an analytically tractable model which unifies these two literatures and this we achieve by casting the problem in a two-country, two-sector overlapping generations model—we do this by setting up an open economy version of Galor (1992). [3] Such a framework allows us to incorporate both the static terms of trade and the intertemporal terms of trade in a single model. In order to facilitate comparison with the results of the static models, we assume that trade is balanced in each period.[4]

We also restrict our attention to the steady state effects of a permanent transfer.[5] By confining ourselves to a comparison of steady states rather than the entire paths related to those steady states, we intentionally avoid considering the welfare of many (generations of) agents. A transfer paradox in that setting would, in our view, have been less significant since the static literature already obtains a transfer paradox in a multiple agent setting. Steady states, on the other hand, allow us to focus on the welfare of only two agents while also allowing for capital accumulation.

We show that welfare changes attributed to the transfers can be summarized by three effects. The first term is the direct effect of the transfer, the second is the within period, or static, terms of trade effect and the final term is the intertemporal terms of trade effect. We proceed by first shutting down the last channel (which is tantamount to assuming that we are at the golden rule capital-labor ratio) so as to emphasize the magnitude of the static terms of trade effect in relation to the direct effect of the transfer. Here we can show that the effects of a transfer are zero-sum; ie, if the recipient of the transfer gains then the donor must lose and vice versa.

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[3] We follow Galor (1992) by assuming the two sectors produce consumption and investment goods. We could have alternatively assumed the two sectors produced traded and non-traded goods. However, this would have implied either forgoing a terms of trade effect or introducing two distinct traded goods in addition to the non-traded goods. In such a framework, what constitutes investment is also arbitrary.

[4] There are two ways of closing the model, either by allowing for capital mobility or by assuming balanced trade. To stay as close as possible to the static literature, we chose the latter route.

[5] Galor and Polemarchakis (1987) and Tan (1998) also restrict attention to the steady state effects of a permanent transfer. Haaparanta (1989) and Yano (1991), on the other hand, analyze the effects of a temporary transfer. Our framework is well-suited to look at the (transitory) effects of a temporary transfer. Preliminary derivations can be obtained from the authors.

Also, we show that the transfer-induced change in the static terms of trade always work in favor of the transfer paradox. Finally, we show that a transfer paradox may be possible with Walrasian price-adjustment, and without pre-existing distortions, bystanders and/or public debt.

We then open the third channel to incorporate the possibility of the initial steady state being away from the golden rule. Since in a diversified, trading world economy, the commodity price ratio is common to both the countries, so are the factor prices. Any change in the interest rate would accordingly improve or worsen the welfare of both countries as the world capital labor-ratio moves towards or away from the golden rule capital-labor ratio. This channel therefore suggests the possibility that transfers may be Pareto-improving.

2. Model

There are two equally sized trading countries that are identical except with reference to their discount factors. Both countries are populated by overlapping generations of two-period- lived agents with logarithmic utility functions. Each economy has constant returns to scale technologies for the production of two goods, a consumption good, C, and an investment good, I, using capital and labor inputs. At all dates, it is assumed that both countries are diversified in production and that trade is balanced between them. The relative price of the investment good is denoted by p. Into this environment we introduce (starting from an initial value of zero) a permanent transfer from one country to the other. Let the donor country be denoted by D and the recipient country by R.

2.1 Preferences, budget constraints

During each time period t, an equal number of two-period lived agents are born in countries *D* and R. Let c_t^y and c_{t+1}^o respectively denote the consumption of a member of generation t while young and old. While young, each member of generation t will inelastically supply one unit of labor in exchange for the wage, w_t. From this wage, residents of country *D* will immediately transfer τ units to their counterparts in country R. Also while young, residents of both countries purchase capital goods so as to earn a return which finances old age consumption. Each resident of country *j, j=D, R,* will maximize utility, given by $U^j(c_t^y, c_{t+1}^o) = c_t^y {c_{t+1}^o}^{\beta_j}$, where β^j denotes the discount factor for residents of country j.[6] The consumption choices for this individual are constrained by a lifetime budget constraint, $c_t^y + c_{t+1}^o / \rho_{t+1} = w_t^j$, where $w_t^D = w_t - \tau$, $w_t^R = w_t + \tau$, $\rho_{t+1} = r_{t+1} / p_t$ is the return on capital owned from periods t to $t+1$, r_{t+1} is the rental paid on capital services during period $t+1$. The solution to this optimization problem can be described by an individual savings function, s^j, where $s^j(w^j{}_t, \rho_{t+1}) \equiv w^j{}_t - c_t^y = \sigma^j w^j{}_t$ and $\sigma^j \equiv \beta^j / (1+\beta^j)$ denotes the constant savings rate for residents of country j. Also, $c_{t+1}^o = \rho_{t+1} s^j(w^j{}_t, \rho_{t+1}) = \sigma^j w_t{}^j \rho_{t+1}$.

2.2 Technologies

For each sector, there is a neoclassical production function that, as mentioned above, is linearly homogeneous. In addition, it is assumed that the Inada conditions are satisfied for each production function and there is no factor intensity reversal. Furthermore, to simplify the algebra, capital is assumed to depreciate completely in one period.[7]

3. Preliminaries

In this section we present several preliminary derivations that will be useful in establishing the existence of a transfer paradox. Time subscripts are omitted from the notation below, in preparation for our focus on the steady state. However, in section 3.3, where stability arguments require a time indexation of variables, such notation will be added as appropriate.

3.1 Indirect utility function

For the maximization exercise of the household specified in Section 2.1, we can derive indirect utility functions which are increasing functions of the wage and return on capital. Let the indirect utility function for the residents of the donor country

[6] These preferences imply that savings are independent of future interest rates, or equivalenetly, that the intertemporal consumption substitution elasticity is unity. For a two period model, allowing for variable elasticity, see Djajic, Lahiri and Raimondos-Moller (1998).

[7] This implies that r_t is the gross rate of return on capital. Since we focus on the steady state, this assumption is innocuous.

in the steady state be denoted by $V^D \equiv V(w^D, \rho)$ where $w^D = w - \tau$ and $\rho = \frac{r}{p}$.

Then, the steady state welfare effect of the transfer is given by

$$\frac{dV^D}{d\tau} = V_w \frac{dw^D}{d\tau} + V_\rho \frac{d\rho}{d\tau}$$

Roy's Identity implies that $V_\rho = \frac{s^D(w^D, \rho)}{\rho} V_w$ and, by the envelope theorem, $\frac{dw}{dp} + k^D \frac{dr}{dp} = I(p, k^D)$ or, equivalently, $\frac{dw}{dp} = I(p, k^D) - k^D \frac{dr}{dp}$. These substitutions together with the assumption of balanced trade imply that the effect of the transfer on the steady state indirect utility of the donor is given by

$$\frac{dV^D}{d\tau} = V_w \left[-1 + \frac{dp}{d\tau} \left(\left(I(p, k^D) - k^D \right) - k^D (\rho - 1) \frac{dr}{dp} \frac{p}{r} \right) \right] \tag{3.1}$$

Similar steps, beginning with $w^R = w + \tau$ imply that the effect of the transfer on the recipient is given by:

$$\frac{dV^R}{d\tau} = V_w \left[1 + \frac{dp}{d\tau} \left(\left(I(p, k^R) - k^R \right) - k^R (\rho - 1) \frac{dr}{dp} \frac{p}{r} \right) \right]. \tag{3.2}$$

The first bracketed term in each of the above equations represents the direct effect of a transfer on welfare, and is thus positive for the recipient and negative for the donor. The remaining pair of terms are respectively the static and intertemporal effects of a transfer-induced change in the relative price of the investment good. We shall refer to these effects as the static and intertemporal terms of trade effects.

Our approach towards examining possibilities for a transfer paradox is to first look at the static terms of trade effect in isolation, which amounts to artificially shutting down the last term. To do so, we employ the assumption that the world economy is initially at the golden rule capital stock, so that $\rho = 1$. This allows us to (temporarily) maintain the maximum comparability of our results with those obtained in the static literature. After this initial focus on the static terms of trade effect, we will then relax our assumption to examine the implications of the intertemporal terms of trade effect.

Below, we consider both a transfer paradox, where the donor gains and the recipient loses ($dV^D / d\tau > 0$ and $dV^R / d\tau < 0$) and a Pareto-improving transfer, where both parties gain ($dV^D / d\tau > 0$ and $dV^R / d\tau > 0$).

3.2 Production

It will be shown that the effects of a transfer depend crucially on elasticities of substitution in production so we use cost functions to highlight these. Both countries have identical technologies and are assumed to be diversified in production. Though factors of production, namely labor and previously invested capital, are not traded internationally, national factor markets are assumed to be competitive. Trade in consumption and investment goods imply that both countries share a common world relative price of investment goods, p. Then, in both countries, we have

$$a_{LC}w + a_{KC}r = 1 \tag{3.3}$$

$$a_{LI}w + a_{KI}r = p \tag{3.4}$$

where the a_{ij} are functions of the wage-rental ratio (except when technologies are Leontief). Given a common p, these two equations determine common factor prices, w and r, and thus imply that the a_{ij} are identical across countries.

Logarithmically differentiating these conditions (see appendix) yields the Stolper-Samuelson effects

$$\eta_{wp} \equiv \frac{\hat{w}}{\hat{p}} = \frac{-\theta_{KC}}{\Delta} \tag{3.5}$$

$$\eta_{rp} \equiv \frac{\hat{r}}{\hat{p}} = \frac{\theta_{LC}}{\Delta} \tag{3.6}$$

where an η is an elasticity. Here it is the elasticity of the noted factor payment with respect to the relative price of the investment good, a ` ^' denotes the percentage change, and θ_{ij} is factor i's cost share of good j. That is,

$$\theta_{LC} = a_{LC}w; \quad \theta_{LI} = a_{LI}\, w/p$$

$$\theta_{KC} = a_{KC}r; \quad \theta_{KI} = a_{KI}r/p$$

Furthermore, in the absence of factor intensity reversals,

$$\Delta \equiv \theta_{LC} - \theta_{LI} = \theta_{KI} - \theta_{KC} \underset{<}{\overset{>}{}} 0 \tag{3.7}$$

as the consumption good is respectively labor- or capital-intensive. Thus, $\eta_{wp} < 0$ (> 0) and $\eta_{rp} > 0$ (< 0) as the consumption good is labor- (capital-) intensive.

For each economy, there are two factor markets (for labor and capital respectively). The full-employment conditions are given in per worker terms for $j = D, R,$ by

$$a_{LC}C^j + a_{LI}I^j = 1$$

$$a_{KC}C^j + a_{KI}I^j = k^j$$

where k^j is the capital-labor ratio of country j. These equalities imply that

$$a_{LC}C + a_{LI}I = 1$$

$$a_{KC}C + a_{KI}I = k$$

where $C = \left[C^D + C^R\right]/2$ and $I = \left[I^D + I^R\right]/2$ are world average consumption and investment goods per worker, and $k = \left[k^D + k^R\right]/2$ is the world capital-labor ratio.

Differentiating the world resource constraints yields the elasticity of supply for the investment good with respect to its price,

$$\eta_{Ip} \equiv \left.\frac{\hat{I}}{\hat{p}}\right|_{\hat{k}=0} = \left[\frac{\lambda_{LC}}{\Delta}\delta_K + \frac{\lambda_{KC}}{\Delta}\delta_L\right]\Big/\Omega = \frac{\lambda_{LC}\delta_K + \lambda_{KC}\delta_L}{\Omega\Delta} > 0 \qquad (3.8)$$

where the λ_{ij} are the employment shares of factor in industry j, and $\delta_L \equiv \lambda_{LC}\theta_{KC}\varepsilon_C + \lambda_{LI}\theta_{KI}\varepsilon_I$, $\delta_K \equiv \lambda_{KC}\theta_{LC}\varepsilon_C + \lambda_{KI}\theta_{LI}\varepsilon_I$. Also, $\varepsilon_j = \dfrac{\hat{a}_{Kj} - \hat{a}_{Lj}}{\hat{w} - \hat{r}}$, $j = C, I$, is the elasticity of substitution for sector j. Finally, it is useful to note that

$$\Omega \equiv \lambda_{LC} - \lambda_{KC} \underset{<}{\overset{>}{}} 0 \qquad (3.9)$$

as the consumption good is respectively labor- or capital-intensive and also that Ω and Δ are always of the same sign. Also, we have the Rybczynski effect

$$\eta_{Ik} \equiv \left.\frac{\hat{I}}{\hat{k}}\right|_{\hat{p}=0} = \frac{\lambda_{LC}}{\Omega} \underset{<}{\overset{>}{}} 0 \qquad (3.10)$$

as the consumption good is respectively labor- or capital-intensive.

3.3 Existence and stability

Both consumption and investment goods are tradable. Using Walras' law, we consider the market for the latter. Country j, $j = D, R,$ maximizes gross domestic product at each date subject to its resource constraints and subsequently supplies $I^j = I(p_t, k_t^j)$ units of produced investment goods to the world market at date t. Note that constant returns to scale implies that the world per capita supply of investment goods can be expressed by $I(p_t, k_t) = [I(p_t, k_t^D) + I(p_t, k_t^R)]/2$. Also at date t, let k_{t+1}^j denote the per capita demand for investment goods to be used in production by country j at date $t+1$. Further, let $k_{t+1} \equiv [k_{t+1}^D + k_{t+1}^R]/2$ denote world per capita demand for investment goods at time t. Thus, the world market-clearing condition for investment goods is given by

$$k_{t+1} = I(p_t, k_t) \tag{3.11}$$

Trade, however, is assumed to be balanced. Thus, for $j = D, R$, $s^j(w^j{}_t, \rho_{t+1}) = \sigma^j w_t^j = p_t k_{t+1}^j$ and

$$\left[\sigma^D \frac{w_t^D}{p_t} + \sigma^R \frac{w_t^R}{p_t}\right]/2 = I(p_t, k_t). \tag{3.12}$$

Recalling that $\tau = 0$ in our initial equilibrium, and that diversification implies factor price equalization, this equation describes equally the equilibrium of a closed world economy. Thus, existence of that equilibrium follows directly from Galor (1992).

Traditional static models of transfers show that under Walrasian stability the transfer paradox cannot occur in a two-country framework. In a two-sector overlapping generations model dynamic stability is possible even when the Walrasian stability condition is not met. Since our primary objective is to produce results in a framework that is most akin to the static model, we shall focus exclusively on a set-up that satisfies Walrasian stability.

To identify the conditions under which Walrasian stability is ensured, we begin by noting a fundamental distinction between one- and two-sector models. In a one-sector model, stability is typically defined using a market for financial capital, which cannot be considered separately from the market for physical capital goods. That is, savings in any period is typically identified as the supply of capital and the demand for capital ownership is derived from the marginal productivity of the subsequent period's capital stock. In this setting, the return on capital ownership is the intertemporal price that clears the market for capital. In a two-sector framework, however, there is instead a market for physical capital (investment) goods that is separate and distinct from the market for financial capital. In this market, savings constitute the demand for investment goods and there is also a very clearly defined supply of investment goods. The relative price of the investment good is then the appropriate market-clearing price.

Walrasian stability requires that at the initial equilibrium world excess demand per capita for investment goods is decreasing in p. At the initial equilibrium, (3.12) becomes $\sigma \frac{w_t}{p_t} = I(p_t, k_t)$ so that, given k_t, $\partial\left(\sigma w/p_t\right)/\partial p_t - \partial I(p_t,k_t)/\partial p_t < 0$. Using elasticities, the Walrasian stability condition can also be rewritten as

$$\eta_{wp} - 1 - \eta_{Ip} < 0 \tag{3.13}$$

We now proceed to examine dynamic stability. With full depreciation, the evolution of the capital stock is governed by $k_{t+1} = I_t$ at the initial equilibrium. So,

$$\hat{k}_{t+1} = \hat{I}_t = \eta_{Ip}\hat{p}_t + \eta_{Ik}\hat{k}_t$$

The world capital market-clearing condition gives $\hat{w}_t = \hat{p}_t + \hat{I}_t$. Substituting into the previous equation gives

$$\hat{k}_{t+1} = \eta_{Ik} \frac{\eta_{wp} - 1}{\eta_{wp} - 1 - \eta_{Ip}} \hat{k}_t$$

and therefore dynamic stability requires

$$\left| \eta_{Ik} \frac{\eta_{wp} - 1}{\eta_{wp} - 1 - \eta_{Ip}} \right| < 1.$$

Note that the numerator is negative regardless of the factor intensity assumption and Walrasian stability implies that the denominator is negative. Thus, the bracketed term is always positive and the dynamic stability condition can alternatively be expressed as

$$0 < (1 - \eta_{wp})(1 - \eta_{Ik}) + \eta_{Ip} \tag{3.14}$$

Under either factor intensity assumption, the first term is negative and the second term is positive. Throughout, we will assume both Walrasian and dynamic stability.

Thus, for the dynamic stability condition to be satisfied, we require η_{Ip} to be sufficiently large. As η_{Ip} is increasing in the elasticities of substitution, ε_C and ε_I (see(3.8)), they also cannot be too small.

4. Transfers and the static terms of trade

We begin by restricting our attention to the case where the initial steady state is at the golden rule; that is, where $\rho = 1$ by assumption.[8] By so doing, we are forcing the effect of a transfer through the singular channel emphasized in the static literature; namely, the static terms of trade. Using (3.1) , (3.2) and (3.11), and an immediate implication of this assumption is that

$dV^D / d\tau = -dV^R / d\tau$.

To verify the existence of the transfer paradox, it is therefore sufficient to identify conditions under which the donor's welfare is improved by the transfer. More specifically, we are looking for conditions that yield

$$\frac{dp}{d\tau}\left(I(p,k^D)-k^D\right)>1 \tag{4.1}$$

This inequality implies a static terms of trade effect associated with the transfer that operates contrary to, and also outweighs, the direct income effect.

Next, we analyze the above inequality in two stages. The first focuses on the effect of the transfer on the relative price of the investment good, $dp/d\tau$. The second focuses on the bracketed expression in (4.1), which represents the pattern of trade.

4.1 Static terms of trade

In a steady state, (3.12) can be expressed $\sigma w - D_\sigma \tau = pI(p,k)$ where $\sigma \equiv \left[\sigma^D + \sigma^R\right]/2$ and $D_\sigma \equiv \left[\sigma^D - \sigma^R\right]/2$. Differentiation then yields

$$-\frac{D_\sigma d\tau}{\sigma w} = \left[-\eta_{wp} + 1 + \eta_{Ip}\right]\hat{p} + \eta_{Ik}\hat{k}. \tag{4.2}$$

Previous derivations imply that at the steady state, $\hat{k} = \hat{I} = \eta_{Ik}\hat{k} + \eta_{Ip}\hat{p}$, so that

$$\frac{\hat{k}}{\hat{p}} = \frac{\eta_{Ip}}{1-\eta_{Ik}} \gtrless 0 \tag{4.3}$$

[8] Section 6 will discuss cases where $\rho \neq 1$.

as the consumption good is, respectively, labor or capital intensive. Substituting into (4.2), and evaluating at the initial value of the transfer, $\tau = 0$, gives

$$\frac{dp}{d\tau} = -\frac{D_\sigma}{k}\left[\frac{1-\eta_{Ik}}{\left(1-\eta_{wp}\right)\left(1-\eta_{Ik}\right)+\eta_{Ip}}\right] \tag{4.4}$$

Note that dynamic stability implies that the denominator of the bracketed term is positive. The sign of the numerator depends upon factor intensity rankings. When the consumption good is labor-intensive, the numerator is negative and $\operatorname{sgn} dp/d\tau = \operatorname{sgn} D_\sigma$. When the consumption good is capital intensive, the numerator is positive and $\operatorname{sgn} dp/d\tau = -\operatorname{sgn} D_\sigma$.

4.2 Pattern of trade

Recalling (4.1), the effect of the transfer on steady state welfare depends also on the pattern of trade. The following, rather intuitive result can be easily verified and is left to the interested reader,

$$I(p,k^D) - k^D = -\frac{D_\sigma}{\sigma^2}\eta_{wp}k\,. \tag{4.5}$$

This equality shows that the donor exports investment goods in two cases: i) when it has a higher savings rate than the recipient and the consumption good is labor intensive and ii) when it has a lower savings rate than the recipient and the consumption good is capital intensive. There are two other cases to be considered, under which the donor instead exports the consumption good. This equality, together with the corresponding implication for the recipient, asserts that the high-saving country, regardless of whether it is the donor or the recipient, will always export the capital-intensive good, whether that good happens to be the consumption or the investment good. And, vice versa for the low saving country.

4.3 Static terms of trade and welfare

Substituting (4.4) and (4.5) into the left hand side of (4.1), the overall static terms of trade effect for the donor is given by

$$\frac{dp}{d\tau}\left(I(p,k^D)-k^D\right) = \left(\frac{D_\sigma}{\sigma}\right)^2\left[\frac{\eta_{wp}\left(1-\eta_{Ik}\right)}{\left(1-\eta_{wp}\right)\left(1-\eta_{Ik}\right)+\eta_{Ip}}\right] \tag{4.6}$$

and its negative gives that of the recipient. Clearly, the numerator of the second term is positive regardless of the factor intensity assumption. Dynamic stability implies that the denominator is also positive. Thus, under the assumption of both Walrasian and dynamic stability, the donor's terms of trade effect is positive regardless of the ranking of savings rates or the factor intensity assumption. More to the point, for both the donor and the recipient, the terms of trade effect *always* works against the direct income effect of the transfer, and thus in favor of a transfer paradox.[9]

This result is at odds with the familiar. In the static formulation of the transfer paradox, the effect of a transfer on the terms of trade is determined entirely by the respective marginal propensities to consume of the donor and recipient. Thus, only when the recipient has a higher propensity to spend on the donor's export will there be a possibility for the donor's terms of trade to improve. But, in the static framework, preference attributes need not have any bearing on the pattern of comparative advantage. In our dynamic context, a large relative propensity to save is associated with a relative abundance of capital in the steady state and thus a comparative advantage in whichever good is capital-intensive. In other words, in the dynamic setting, the ranking of the two countries' marginal propensities to save is inextricably linked with the steady state pattern of comparative advantage.

Recalling the origin of (4.6), it is helpful to note that (4.4) and (4.5) both have the sign of $\operatorname{sgn} D_\sigma$ when the consumption good is labor intensive and both have the sign of $-\operatorname{sgn} D_\sigma$ when the consumption good is capital intensive. Thus, for a given factor intensity assumption, the ranking of savings rates determines whether or not the donor exports the investment good and also whether the transfer has increased or decreased the relative price of the investment good, p. If, for example, the consumption good is labor intensive and the donor has the higher savings rate, then the donor exports the investment good and the donor's terms of trade improve. If, under the same factor intensity assumption, the donor instead has the lower savings rate, then the donor imports the investment good and its terms of trade, now $1/p$, also improve. Analogous arguments can be made under the assumption of a capital-intensive consumption good. To sum, the circumstances under which the relative

[9] The balanced trade assumption implies that $I(p,k^D)-k^D=-\left(I(p,k^R)-k^R\right)$.

price of capital falls as a result of the transfer are the same circumstances under which the donor becomes a steady state importer, rather than an exporter, of the investment good. This explains why the donor's terms of trade always improve as a result of a transfer.

Another way to view these results is via (4.2), which provides insight with regard to the difference between static and dynamic frameworks in examining the welfare effects of transfers. To make the comparison precise, note that the static model has two consumption goods; hence, the investment good in our model is to be interpreted as the second of the consumption goods. Also, σ^j is the marginal propensity to spend on the second good by the residents of country j. If the model were not dynamic, the second term-- reflecting steady state effects on capital accumulation--would be zero. In this case, if the donor had the higher savings rate, a transfer would imply an unambiguous deterioration in the donor's terms of trade under Walrasian stability. As the earlier literature suggested, this would reinforce rather than negate the direct effects of the transfer and obviate the possibility of a transfer paradox. In a dynamic model, however, the effects on steady state capital accumulation must also be taken into account and are reflected by a non-zero second term. Thus, for the same scenario just described, a transfer of income to the low-saving country now reduces world steady state capital accumulation. When the investment good is capital intensive, the scarcity thus introduced has a positive influence on the donor's terms of trade (see (4.3)) and consequently reintroduces the possibility of the transfer paradox.

5. Transfer paradox

Of course, a positive terms of trade effect is not enough to deliver the transfer paradox. In addition, the terms of trade effect must be `sufficiently large.' That is, it remains to be shown that the static terms of trade effect can dominate the direct income effect of a transfer. This section approaches this in two ways. First, a general argument is constructed. Second, a special case is considered where both industries have a common elasticity of substitution. Under this simplification, it is possible to describe a precise range within which this elasticity is consistent with a transfer paradox.

5.1 General argument

The transfer paradox requires a demonstration of conditions under which (4.1), now expressed by

$$\left(\frac{D_\sigma}{\sigma}\right)^2\left[\frac{\eta_{wp}\left(1-\eta_{Ik}\right)}{\left(1-\eta_{wp}\right)\left(1-\eta_{Ik}\right)+\eta_{Ip}}\right]>1, \qquad (5.1)$$

holds. We begin by noting that $\left(D_\sigma/\sigma\right)^2<1$. To proceed, however, we focus instead on the magnitude of the second bracketed term in the above equation. The critical insight is that should $\left(1-\eta_{wp}\right)\left(1-\eta_{Ik}\right)+\eta_{Ip}\to 0$, then (5.1) will be generously satisfied—it will in fact tend to $+\infty$. Thus, it is sufficient to establish conditions under which $\left(1-\eta_{wp}\right)\left(1-\eta_{Ik}\right)+\eta_{Ip}\to 0$.

Recalling that the denominator of (5.1) is the expression for dynamic stability, we first substitute (3.5), (3.8) and (3.10) into that expression to describe the requirements that dynamic stability places on the elasticities of substitution. The denominator of the bracketed expression in (5.1) can now be expressed

$$\left(\frac{\theta_{KI}}{\Delta}\right)\left(\frac{-\lambda_{KC}}{\Omega}\right)+\left(\frac{\lambda_{LC}\delta_K+\lambda_{KC}\delta_L}{\Omega\Delta}\right).$$

Thus, since $\Omega\Delta>0$, dynamic stability requires the elasticities of substitution to satisfy

$$-\theta_{KI}\lambda_{KC}+\lambda_{LC}\left(\lambda_{KC}\theta_{LC}\varepsilon_C+\lambda_{KI}\theta_{LI}\varepsilon_I\right)+\lambda_{KC}\left(\lambda_{LC}\theta_{KC}\varepsilon_C+\lambda_{LI}\theta_{KI}\varepsilon_I\right)>0 \qquad (5.2)$$

It is thus clear that dynamic stability requires that elasticities of substitution for either one or both industries must be \`sufficiently large', regardless of the factor intensity assumption. That is, we first seek any pair of ε_j that satisfy

$$\theta_{KI}\lambda_{KC} = \lambda_{LC}\left[\lambda_{KC}\theta_{LC}\varepsilon_C+\lambda_{KI}\theta_{LI}\varepsilon_I\right]+\lambda_{KC}\left[\lambda_{LC}\theta_{KC}\varepsilon_C+\lambda_{LI}\theta_{KI}\varepsilon_I\right].$$

Then, increasing either or both of the ε_j slightly will deliver both dynamic stability and the transfer paradox. If the ε_j are increased too much, however, the denominator of (5.1) will deviate significantly from zero and, if large enough, the quotient becomes too small to deliver the transfer paradox.

To sum, the transfer paradox obtains at the golden rule when elasticities of substitution are sufficiently large, as required for the terms of trade to have the needed sign, but not too large, so as also to deliver a change in the terms of trade of sufficient magnitude so as to dominate the direct income effect of the transfer. Note further that the conditions on the ε_j that deliver the transfer paradox (with stability) are independent of the factor intensity assumption.

5.2 Common elasticities of substitution

If we assume that both industries have identical elasticities of substitution, now denoted by ε, (5.2) implies that the dynamic stability condition can now be expressed as

$$\varepsilon > \frac{\lambda_{KC}\theta_{KI}}{\lambda_{KC}\theta_{KI} + \lambda_{LC}\theta_{LI}}.$$

Requirements for a transfer paradox can be expressed by

$$\left(\frac{D_\sigma}{\sigma}\right)^2 \frac{\lambda_{KC}\theta_{KC}}{-\theta_{KI}\lambda_{KC} + \varepsilon\left(\lambda_{LC}\theta_{LI} + \lambda_{KC}\theta_{KI}\right)} > 1.$$

Recalling that the denominator is positive under dynamic stability, the restrictions on ε that imply both stability and the observance of a transfer paradox are together given by

$$\frac{\lambda_{KC}\theta_{KI}}{\lambda_{KC}\theta_{KI} + \lambda_{LC}\theta_{LI}} < \varepsilon < \frac{\lambda_{KC}\theta_{KI} + \lambda_{KC}\theta_{KC}\left(\frac{D_\sigma}{\sigma}\right)^2}{\left(\lambda_{LC}\theta_{LI} + \lambda_{KC}\theta_{KI}\right)} \qquad (5.3)$$

Note several things about the range for a common elasticity of substitution. First, the range is nonvacuous provided only that the donor and recipient have differing savings rates. Second, the transfer paradox does not depend upon whether it is the donor or the recipient that has the higher savings rate. Third, the range for the elasticity of substitution does not depend upon a particular factor intensity ranking.[10]

[10] Those familiar with two-sector trade models may recognize a friction that develops in this case between the observance of a transfer paradox and the observance of steady state factor price equalization. More particularly, as made evident by (5.3), the larger the difference in savings rates of the donor and recipient, the larger is the range of elasticities that are consistent with a transfer paradox. However, steady state factor price equalization requires that the difference between the savings rates of the donor and recipient countries not be too large, otherwise the steady state, country specific capital-labor ratios may fall outside the cone of diversification (see Cremers (2001)). This friction is not, however, sufficient to negate the possibility of a transfer paradox that is internally consistent with the assumptions of the model. Numerical examples can be provided on request that satisfy both requirements for the transfer paradox and also the requirements for factor price equalization.

6. Intertemporal terms of trade effect

We now return to (3.1) and (3.2), and consider cases where $\rho \neq 1$, that is, cases where the world steady state capital labor ratio is either above or below that associated with the golden rule. As previously noted, we can then tap the model for its intertemporal, or dynamic terms of trade, effect. From the two equations, it can be immediately noticed that, unlike the static terms of trade effect, the dynamic terms of trade effect is symmetric for the two countries and is expressed by

$$-k^R(\rho-1)\eta_{rp}\frac{dp}{d\tau} \tag{5.4}$$

Whether this effect brings a simultaneous gain or loss to the donor and recipient clearly depends upon both the positioning of the world steady state relative to the golden rule and upon the factor intensity assumption. Moreover, recalling that the sign of $dp/d\tau$ depends upon the ranking of savings rates for the donor and recipient countries-- in addition to the factor intensity assumption—it is evident that signing the intertemporal terms of trade effect will be quite taxonomic. In this respect, the intertemporal terms of trade effect is unlike the static terms of trade effect, which was positive for the donor and negative for the recipient in all characterizations of savings rates and factor intensities.

To avoid a tedious detailing of cases and associated signs for this effect, we instead focus only on what we consider to be the most interesting case that arises upon inclusion of the intertemporal terms of trade effect. Accordingly, let us assume that $\rho > 1$, so that there is no overaccumulation of capital. Then if, in addition, the donor has the higher savings rate and the consumption good is capital-intensive, then the term given by (5.4) is positive. That is, the intertemporal terms of trade effect will in this case work to the benefit of both the donor and recipient. Overall, if the first pair of terms delivered the transfer paradox, then this additional positive effect on welfare implies that the donor is unambiguously better off and the recipient now *may* be better off as a result of an international transfer. On the other hand, if the first pair of terms did not deliver the transfer paradox, then the recipient has an unambiguous welfare improvement and the donor *may* have a welfare improvement. Regardless of whether or not a transfer paradox arises from the static terms of trade effect, the overall welfare implication allows for the possibility that the transfer makes both donor and recipient better off; that is, an international transfer of income may be Pareto-improving.

7. Conclusion

This paper explores the welfare implications of international transfers in the context of a two-sector overlapping generations model. Within this framework, it has been possible to provide an analysis that incorporates both the static effects described by the early trade theoretic literature and also the dynamic effects explored by dynamic one-sector models. It is demonstrated that the effects of an international transfer on the static terms of trade always work in favor of a transfer paradox, though elasticities of substitution can neither be too large nor too small for the transfer paradox to arise. Moreover, neither Walrasian nor dynamic stability are sufficient to rule out the possibility of a transfer paradox. In contrast, the intertemporal terms of trade effect is shown to depend upon factor intensity assumptions, a ranking of savings rates for the donor and recipient, and also on the position of the world economy relative to the golden rule. It is possible, however, for the transfer to produce a Pareto-improvement when steady states away from the golden rule are considered.

APPENDIX

To show that $\eta_{wp} \equiv \frac{\hat{w}}{\hat{p}} = \frac{-\theta_{KC}}{\Delta}$ and $\eta_{rp} \equiv \frac{\hat{r}}{\hat{p}} = \frac{\theta_{LC}}{\Delta}$, differentiate (3.3) and (3.4) to get

$$\theta_{LC}\hat{w} + \theta_{KC}\hat{r} = 0$$

$$\theta_{LI}\hat{w} + \theta_{KI}\hat{r} = \hat{p}$$

or,

$$\begin{bmatrix} \theta_{LC} & \theta_{KC} \\ \theta_{LI} & \theta_{KI} \end{bmatrix} \begin{bmatrix} \hat{w} \\ \hat{r} \end{bmatrix} = \begin{bmatrix} 0 \\ \hat{p} \end{bmatrix}$$

where θ_{ij} is factor 's cost share of good j

$$\theta_{LC} = a_{LC}w; \theta_{LI} = a_{LI}\, w/p;$$

$$\theta_{KC} = a_{KC}r; \theta_{KI} = a_{KI}r/p$$

and

$$\theta_{LC} + \theta_{KC} = 1$$

$$\theta_{LI} + \theta_{KI} = 1$$

or, equivalently, $\sum_i \theta_{ij} = 1$.

Then, using Cramers' rule,

$$\eta_{wp} \equiv \frac{\hat{w}}{\hat{p}} = \frac{-\theta_{KC}}{\Delta}$$

$$\eta_{rp} \equiv \frac{\hat{r}}{\hat{p}} = \frac{\theta_{LC}}{\Delta}$$

where, making use of $\sum_i \theta_{ij} = 1$,

$$\Delta \equiv \theta_{LC} - \theta_{LI} = \theta_{KI} - \theta_{KC}$$

and where $\Delta \gtrless 0$ as the consumption good is respectively labor- or capital-intensive.

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