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ABSTRACT

While the focus of mainstream economic analysis of liability rules remains on negligence liability, recently some legal scholars have argued for the sharing of liability. In this paper, our first objective is contribute to the debate regarding the desirability of the sharing of liability for the accident loss. To this end, we study the implications of various approaches toward liability assignment for the existence and efficiency of equilibria. In particular, we analyze the proposal of Calabresi and Cooper (1996). Contrary to what is suggested in the literature, we show that the sharing of liability when parties are either both negligent or both non-negligent does not threaten the existence of equilibria. Moreover, it does not dilute the incentives for the parties to take the due care. Our second objective is to extend the efficiency analysis beyond Shavell (1980, 1987) and Miceli (1997), to search for the second-best liability rules. We show that each of the standard liability rules fails to be efficient even from a second-best perspective. Furthermore, we show that second-best efficiency requires loss sharing between non-negligent parties. As corollaries to our main results, we reexamine some of the existing claims regarding the existence and efficiency of equilibria under liability rules.

KEY WORDS: Liability rules, negligence liability, comparative liability, accident loss, loss sharing, social welfare, first best, second best, Nash equilibrium

JEL CLASSIFICATION: C62; D62; K13

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1. Introduction

This paper has two main objectives. The first objective is to contribute to an important and current debate. This debate is regarding the desirability as well as the implications of the sharing of liability between parties involved in an accident. In this context, we analyze the proposal by Calabresi and Cooper (1996). Our second objective is to extend the efficiency analysis of liability rules beyond Shavell (1980, 1987) and Miceli (1997), in order to search for the second-best liability rules. This exercise is important since in view of Shavell (1980, 1987) no liability rule can achieve the first-best outcome.

In regard to the debate on the desirable attributes of liability, it is interesting to note that the focus of the mainstream economic analysis of liability rules has been on negligence liability. Under negligence-based liability rules, when both parties are non-negligent, only one party bears the entire accident loss.[1] Also, if both parties happen to be negligent, generally, one party bears the entire accident loss.[2] As a result, under negligence-based liability, depending on care levels, a party faces either full liability or no liability at all. Some recent works have criticized this attribute of negligence-based liability. These works argue that the negligence-based approach toward liability totally neglects the causal contributions of the parties involved. Therefore, it does not form a convincing basis for liability assignment, particularly when both the injurer and the victim are non-negligent. [See, Calabresi and Cooper (1996); Parisi and Fon (2004).][3]

Calabresi and Cooper (1996) and Honoré (1997) recommend proportionate or comparative apportionment of liability as an alternative basis for liability assignment. Calabresi and Cooper (1996) argue that courts and juries have shown an inclination toward com-

[1]This party is the victim under the rule of negligence, the rule of comparative negligence, and the rule of negligence with the defense of contributory negligence. Under the rule of strict liability with the defense of contributory negligence, on the other hand, the injurer is liable for the entire accident loss when both parties are non-negligent.

[2]This is true for all negligence-based rules, except the rule of comparative negligence.

[3]For criticisms of economic modeling of liability rules on various grounds see Grady (1989), Kahan (1989), Mark (1994), Burrow (1999), and Wright (2002)

parative division of liability. In contrast to negligence-based liability, under comparative liability, when parties are either both negligent or both vigilant, they share liability for the accident loss.[4] Because of this attribute, comparative liability is said to be consistent with the principle of equity, which requires loss spreading between parties (Honoré, 1997).[5] Moreover, some studies show that this approach is being used by some courts in many countries, including France, Germany, Japan and the United States. [See, e.g., Calabresi and Cooper (1996), Yoshihsa (1999), Grimley (2000), Yu (2000), and Parisi and Fon (2004).]

However, the proposal for comparative liability has also met with its share of criticisms. In an interesting work, Parisi and Fon (2004) study the implications of the sharing of liability when parties are either both negligent or both vigilant. Assuming that care levels as well as activity levels of the parties affect the accident loss, Parisi and Fon (2004) argue that, the sharing of liability dilutes the incentives for the parties to take the due (efficient) care. Moreover, they argue that under comparative sharing of liability an equilibrium may or may not exist.[6] In contrast, as several studies show, even when care levels as well as activity levels of the parties affect the expected accident loss, under negligence based liability rules the equilibrium outcome is unique and well defined [see Shavell (1980, 1987), Miceli (1997), Parisi and Fon (2004)].[7]

[4]Proportional or comparative loss sharing also takes place under the rule of comparative negligence. However, under this rule, loss sharing takes place only when both parties are negligent; not when both parties are non-negligent. For an analysis of this rule when *only* care levels affect the accident loss see Schwartz, G. (1978), Landes and Posner (1980), Cooter and Ulen (1986), Haddock and Curran (1985), Rubinfeld (1987), and Rea (1987). For a critical review of some of these works see Liao and White (2002), and Bar-Gill and Ben-Shahar (2003).

[5]Honoré (1997) has argued that the morality of tort law requires that liability of a party should be proportional to the parties causal contribution. Also see, Calabresi (1965, and 1970).

[6]When only the care levels of the parties affect the accident loss, comparative liability does not dilute incentives for the parties to take efficient care. Moreover, equilibrium under comparative liability is unique and efficient. See Singh (2006 b).

[7]Restricting attention only to care levels, many works show that the equilibria under negligence based liability rules are well defined and efficient. For example, see Polinsky (1989), Landes and Posner (1987), Shavell (1987), Arlen, J (1990), Miceli (1997), Jain and Singh (2002), Cooter and Ulen (2004), Singh (2006a), etc. For a comprehensive account of the positive theory of torts doctrines see Hylton (2001), and Geistfeld (2001). Dharmapala and Hoffmann (2005) extend the model of bilateral care to consider the

It is in the context of these claims that we analyze the implications of the two approaches toward liability assignment, especially for the existence and efficiency properties of equilibria under liability rules. Our framework of analysis is more general than the standard framework. We assume that care has several aspects, some of which are verifiable before a court while others are not. For example, the probability of motor vehicle accidents depends not only on how carefully drivers drive but also on how much they drive. While care taken by a driver may be verifiable, the amount of driving undertaken by a driver on a particular day is not. We assume that the choices of verifiable aspects as well as non-verifiable aspects of care by the parties affect the expected accident loss.

We show that the sharing of liability when parties are either both negligent or both negligent does not dilute the incentives in the sense indicated in the literature. In fact, it can have a welfare enhancing effect.[8] In addition, we show that the sharing of liability does not threaten the existence of equilibrium under liability rules. Following the mainstream assumptions, we demonstrate the existence of equilibria for a general class of liability rules. We show that in these equilibria, care level of each party is equal to or greater than the due level of care. That is, being vigilant is a dominant strategy for the parties.

The second main objective of the paper is to analyze liability rules from a second-best efficiency perspective. This issue is important, since in view of Shavell (1980, 1987) no liability rule can achieve the first-best outcome.[9] That is, no liability rule can induce an equilibrium in which parties opt for socially optimal care with respect to verifiable as well as non-verifiable aspects of care.[10] The inefficiency arises due to the fact that courts can base liability only on verifiable aspects of care.

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interdependence of the costs of care.

[8]Assuming that only the care levels of the parties affect the accident loss, Singh (2006 a) shows that regardless of whether parties are negligent or not, and a party is solely negligent or not, liability can be shared between the parties, without diluting their incentives to take efficient care.

[9]Cf. Hindley and Bishop (1983) and DeMeza (1986).

[10]For an analysis of this issue in the context of firms and consumers see Polinsky (1980).

However, in the existing works, the response to the impossibility of first-best liability rule has been either to restrict the analysis to the comparison of standard liability rules, or to undertake the analysis out of the purview of liability rules. For example, in Landes and Posner (1987), Shavell (1980, 1987) and Miceli (1997), the exercise of finding the second-best liability rules is restricted only to a comparison of the standard rule of negligence with that of the rule of strict liability with the defense of contributory negligence. Emons (1990), Emons and Sobel (1991), and Goerke (2002) have explored the issue of the second-best efficiency of liability rules. The problem with these works, however, is that the solutions and mechanisms proposed in these works are not only difficult to implement, they are in fact violative of the essential features of the law of torts.[11]

In contrast, we explore the nature of the second-best liability rules in a framework that captures the essential features of liability assignment under the law of torts; namely, (i) the issue of liability arises only if the victim suffers harm, (ii) the liability payments made by the injurer are received by the victim, and (iii) liability of a solely negligent injurer is equal to the harm sustained by the victim. Within the confines of liability rules, we show that each of the standard liability rules fails to be efficient even from a second-best perspective. We show that the economic efficiency requires loss sharing when both the parties are non-negligent.[12] In fact, depending on the context, second-best efficiency may require either loss sharing between non-negligent parties or an increase in the due care standards or both.

Our results are also relevant for some of the existing propositions in the literature. Some works have argued that, when care levels as well as activity levels affect the expected

[11]In Emons and Sobel (1991) the liability payments made by the injurer are, generally, different from from the actual harm suffered by the victim. In Goerke (2002) injurers are required to make (liability) payments even when there is no harm and legal standard or care is met with. Moreover, in Emons (1990), and Emons and Sobel (1991) the analysis considers only the activity levels and not the care levels of the parties.

[12]Rubinfeld (1987), and Emons (1990) show that loss sharing leads to welfare gain if the injurers are non-identical. However, Rubinfeld (1987) considers only care level of the injurers, and Emons (1990), as we noted earlier, focuses only on activity levels not on care levels.

accident loss, the negligence-based rules, e.g., the rule of negligence and the rule of strict liability with the defense of contributory negligence, induce equilibria in which the injurer and the victim opt for care levels that are appropriate from the view point of the first-best efficiency. As a result, it is implicitly suggested that the rule of negligence and the rule of strict liability with the defense of contributory negligence are at least as good as it can get, given that the first-best outcome cannot be achieved. [see text books on Law and Economics, e.g., Miceli (1997 p. 29), Cooter and Ulen (2004, pp. 332-33), and research papers Dari Mattiacci (2002), Parisi and Fon (2004), also see Delhaye (2002).] In this paper, we review both of these propositions. We show that neither of these two claims is correct. In particular, we show that none of the standard liability rules, including the rule of negligence and the rule of strict liability with the defense of contributory negligence, induces an equilibrium in which both the parties opt for care levels that are appropriate from the view point of the first-best efficiency.

Section 2 introduces the framework of analysis that outlines the notations and assumptions made in the paper. In Section 3, we investigate the implications of the above-mentioned two approaches to liability assignment for the existence and properties of equilibria under liability rules. In Section 4 we extend the analysis to search for the second-best liability rules. We conclude in Section 5 with remarks on the nature of results in the paper.

2. Framework of Analysis

We consider accidents resulting from the interaction of two parties who are strangers to each other. Both parties are assumed to be rational and risk-neutral. Each party's behaviour potentially contributes to the accident costs. However, when an accident takes place, the entire loss falls on one party to be called the victim; the other party being the injurer. For example, we can think of motor vehicle drivers as injurers and pedestrians or bicyclists as victims. Each party decides on several aspects of care. Drivers choose, e.g., how carefully to drive as well as how much to drive. Likewise, pedestrians decide on how carefully to walk on roads and how much to walk. Some aspects of care are verifiable before a court, while others are not. The choice of verifiable as well as non-verifiable aspects of care by the parties affect the expected accident loss. To facilitate exposition and compare our results with those in the literature, we assume that care has only two aspects. We call verifiable aspect of care as the 'care level' and non-verifiable aspect as the 'activity level'.[13] The elements contributing to the overall social cost of accident are the loss that is borne by the victim in the event of an accident and the cost of care taken by the parties.

Following the standard notations, we denote by:

x care level as well as the cost of care for the injurer,

y care level as well as the cost of care for the victim,

s activity level for the injurer,[14]

t activity level for the victim,

$X = \{x \mid x$ is some feasible level of care for the injurer $\}$,

$Y = \{y \mid y$ is some feasible level of care for the victim$\}$,

$S = \{s \mid s$ is some feasible level of activity for the injurer $\}$,

$T = \{t \mid t$ is some feasible level of activity for the victim$\}$,

u the benefit function for the injurer,

v the benefit function for the victim,

π the probability of accident,

D the loss suffered by the victim in the event of an accident, $D \geq 0$.

L the expected accident loss.

We assume that while x and y are verifiable, s and t are not. In addition, we assume:

(A1): u is a function of s and x; $u = u(s, x)$. Benefits to the injurer increase with his activity level but at a decreasing rate, i.e., u is an increasing and strictly concave function of s. Care is costly to the injurer, hence u is a decreasing function of x, for all $s \in S$.[15] Likewise,

(A2): v is a function of t and y; v=$v(t, y)$. v is an increasing and strictly concave function of the victim's activity level, t. v is a decreasing function of y, for all $t \in T$.

[13]For example, while the care taken by a driver may be verifiable, the amount of driving is not.

[14]Our analysis can be generalized to consider s, x, etc. as vectors rather than scalers.

[15]This and other assumptions are for expository convenience. In a more general model, the benefits to the injurer do not necessarily have to decrease with every verifiable aspect of care, and increase with every non-verifiable aspect of care.

(A3): The expected accident loss L is a function of s, x t and y; $L = L(s, x, t, y)$. L is a non-increasing function of care level of each party, and is an increasing function of both s and t. That is, a larger care by either party, given the care level of the other party, results in lesser or equal expected accident loss. Increase in activity level of the either party causes an increase in the expected accident loss. $L(.) = 0$ when $s = 0$ or $t = 0$.

In the literature, it is generally taken that $L = st\pi D$. Both π and D are assumed to be decreasing functions of x and y. (See, e.g., Landes and Posner 1987, Shavell 1987; Miceli 1997; Parisi and Fon 2004). Alternatively, it can be assumed that $L = \pi(s, x, t, y)D(x, y)$, where π is a decreasing function of x and y, and an increasing function of both s and t. To make the analysis tractable, later on we will assume that $L = \phi(s, t)\pi(x, y)D(x, y)$, i.e., $L = \phi(s, t)l(x, y)$, where $l(x, y) = \pi(x, y)D(x, y)$, and $\phi(.) = 0$ when $s = 0$ or $t = 0$. For the ease of comparison, wherever relevant, we shall use the standard specification, i.e., $L = stl(x, y)$, as well. Note that all these specifications satisfy assumption (A3), and our specification of L is more general than the standard specification; the latter is a special case of our specification.

(A4): Social benefits from the activity of a party are fully internalized by that party.

(A5): Social goal is to maximize the net social benefits from the activities of the parties; the net social benefits are the sum of the benefits to the two parties minus the total social costs of accident.

Therefore, the social optimization problem is given by:

$$\max_{(s,x,t,y)\in S\times X\times T\times Y} \quad u(s, x) + v(t, y) - L(s, x, t, y). \tag{1}$$

(A6): The benefit, cost, and expected loss functions are such that there is a unique tuple $((s, x), (t, y))$ that is socially optimal. We will denote this tuple by $((s^*, x^*), (t^*, y^*))$. In other words, net social benefits are maximized, if the injurer chooses s^* as his activity level and x^* as his care level, and the victim simultaneously opts for t^* as his activity level and y^* as his care level.

(A7): $((s^*, x^*), (t^*, y^*)) >> ((0, 0), (0, 0))$. That is, social efficiency requires positive care level and activity level from each party.

(A8): The legal due care standard (i.e., the negligence standard) for the injurer, wherever applicable (say under the rule of negligence), is set at x^*. Similarly, the legal negligence standard of care for the victim, wherever applicable (say under the rule of strict liability with defense) is set at y^*.

It should be noted that assumptions (A1)-(A8) are standard assumptions.[16] In Section 4, (A8) is relaxed.

Definition 1: *Liability Rules*: Depending on the context and the care levels of the victim and the injurer, a liability rule uniquely determines the proportions in which they are to bear the accident loss. Therefore, a liability rule can be considered as a rule or a mechanism that determines the proportions, q, in which the injurer bears the accident loss, as a function of the parties' care levels.[17] Formally, for given X and Y, a liability rule is a function f:

$$f : X \times Y \mapsto [0, 1] \text{ such that; } f(x, y) = q(x, y).$$

Clearly, $1 - q$ is the proportion of D that is borne by the victim.

For a party, payoff from engaging in the activity depends on its activity level, its care level as well as the proportion of accident loss the party is required to bear under the liability rule in force. Therefore, the choice of care and activity levels by a party depends on the rule in force, as well as on the choice of the care and the activity levels by the other party. Suppose, it is given that the victim has opted for some $t \in T$ as his activity level and some $y \in Y$ as his care level. Now, if the injurer opts for s as activity level and x as care level, in the event on an accident, the liability rule will require him to bear $q(x, y)D(x, y)$ out of the total accident loss $D(x, y)$; remaining loss, i.e., $D(x, y) - q(x, y)D(x, y)$, will be borne by the victim. In other words, given $(t, y) \in T \times Y$ opted by the victim, if the injurer chooses a pair $(s, x) \in S \times X$, his expected liability is $q(x, y)L(s, x, t, y)$. The

[16] E.g., see Shavell (1987), Miceli (1997), Cooter and Ulen (2004), and Parisi and Fon (2004).

[17] Note that q depends on only the care levels and not on the activity levels, i.e., on verifiable and not on non-verifiable aspects of care.

injurer being rational and risk-neutral will choose a pair (s, x) that maximizes his expected payoff. Formally, given $(t, y) \in T \times Y$ opted by the victim and the liability rule in force, the problem facing the injurer is

$$\max_{(s,x)\in S\times X} \quad u(s, x) - q(x, y)L(s, x, t, y). \tag{2}$$

Likewise, given $(s, x) \in S \times X$ opted by the injurer, the problem facing the victim is

$$\max_{(t,y)\in T\times Y} \quad v(t, y) - (1 - q(x, y))L(s, x, t, y), \tag{3}$$

where $q(x, y) \in [0, 1]$ and is determined by the relevant liability rule.

3. Results regarding the first-best efficiency

In view of Shavell (1980, 1987) no liability rule can achieve $((s^*, x^*), (t^*, y^*))$ as a Nash equilibrium (N.E.). That is, there cannot be a liability rule that can achieve the first-best outcome.[18] In this section, we explore the properties of equilibria that may exist under various possible liability rules. Particulary, we examine the choice of care levels and activity levels by the parties, as compared to the levels that are optimal from the social point of view. While undertaking this exercise, we assume that under every liability rule there exists an equilibrium. Later on, we demonstrate the existence of pure strategy Nash equilibria for a large class of liability rules.

3.1 **Liability of a solely negligent party**

An essential feature of the negligence-criterion based liability rules is captured by the following property.

Property (P1):

$$(\forall x \in X)(\forall y \in Y)[(x \geq x^* \& \;\; y < y^* \Rightarrow \;\; q = 0) \text{ and } (x < x^* \& \;\; y \geq y^* \Rightarrow \;\; q = 1)].$$

[18] The inefficiency arises due to the fact that liability rules do not take activity levels of the parties into account.

Property (P1) says that liability assignment under a liability rule is such that: A non-negligent party has no liability, if the other party is negligent. That is, whenever the injurer is negligent and the victim is not, the victim receives full compensation for the loss. If the victim is negligent and the injurer is not, the victim bears the entire loss.

To start with, we investigate the behavior of parties under rules that satisfy Property (P1). We show that under a liability rule that satisfies Property (P1), the parties cannot both be negligent in a N.E., no matter how the liability is assigned when both parties are negligent. In other words, in any N.E., $x < x^*$ and $y < y^*$ can never hold.

To see why, take any $((s,x),(t,y))$ such that $x < x^*$ and $y < y^*$. Suppose, the injurer opts for (s,x) and the victim for (t,y). At $((s,x),(t,y))$, let $q(x,y)$ be the injurer's share of loss, where $0 \leq q(x,y) \leq 1$. So, $1 - q(x,y)$ is share of the victim in accident loss. As a result, at $((s,x),(t,y))$, the expected payoff of the victim is

$$v(t,y) - (1 - q(x,y))L(s,x,t,y).$$

On the other hand, given that (s,x) is opted by the injurer, if the victim instead opts for (t^*,y^*), then the injurer will be solely negligent. In that case, in view of (P1), the injurer's liability is full and that of the victim is none. Therefore, given that (s,x) is opted by the injurer, if the victim opts for (t^*,y^*), his payoff will be $v(t^*,y^*)$. Similarly, at $((s,x),(t,y))$ the expected payoff of the injurer is $u(s,x) - q(x,y)\ L(s,x,t,y)$. But, given that (t,y) is opted by the victim, should the injurer instead opt for (s^*,x^*), his payoff will be $u(s^*,x^*)$. At $((s,x),(t,y))$ if

$$u(s^*,x^*) > u(s,x) - q(x,y)\ L(s,x,t,y),$$

a unilateral deviation by the injurer to (s^*,x^*) is strictly profitable. In that case, $((s,x),(t,y))$ cannot be a N.E. Thus, if $((s,x),(t,y))$ is a N.E., then a unilateral deviation by the injurer to (s^*,x^*) cannot be strictly profitable. Therefore, assume that

$$u(s,x) - q(x,y)\ L(s,x,t,y) \geq u(s^*,x^*). \tag{4}$$

Since $((s,x),(t,y)) \neq ((s^*,x^*),(t^*,y^*))$, by assumption, we know that

$$u(s^*,x^*) + v(t^*,y^*) - L(s^*,x^*,t^*,y^*) > u(s,x) + v(t,y) - L(s,x,t,y). \tag{5}$$

Subtracting $u(s^*, x^*)$ from the LHS and $u(s, x) - q(x, y)L(s, x, t, y)$ from the RHS of (5), in view of (4), we get

$$v(t^*, y^*) - L(s^*, x^*, t^*, y^*) > v(t, y) - (1 - q(x, y))L(s, x, t, y). \tag{6}$$

Now, since $L(s^*, x^*, t^*, y^*) \geq 0$, from (6) we have

$$v(t^*, y^*) > v(t, y) - (1 - q(x, y))L(s, x, t, y).$$

That is, given $(s, x < x^*)$ opted by the injurer, the payoff of the victim is strictly greater if he chooses (t^*, y^*) rather than (t, y), i.e., the victim is better off opting (t^*, y^*) rather than (t, y). Again, $((s, x), (t, y))$ cannot be a N.E.

In other words, under a liability rule satisfying (P1), from any $((s, x), (t, y))$ such that $x < x^*$ & $y < y^*$, either the injurer finds unilaterally deviation to (s^*, x^*) profitable, or the victim finds unilaterally deviation to (t^*, y^*) profitable. Hence, if a liability rule satisfies Property (P1), then any $((s, x), (t, y)))$, such that $x < x^*$ & $y < y^*$, cannot be an equilibrium. Formally, we can make the following claim.

Lemma 1 *Under a liability rule satisfying (P1),*
$(\forall((s, x), (t, y)))$ $[x < x^*$ & $y < y^* \Rightarrow ((s, x), (t, y))$ *cannot be a N.E.* $]$.

In fact, Property (P1) enables us to make further deductions about the behaviour of the parties with respect to their choice of care levels. Suppose a liability rule satisfies Property (P1). When $x \geq x^*$ and $y < y^*$, the victim is solely negligent. In such an event, due to Property (P1), the injurer has no liability. So, for given s his payoff is $u(s, x)$. Note that $u(s, x)$ deceases with x. Therefore, regardless of the s opted by him whenever $x > x^*$, the injurer can increase his payoff simply by reducing x until he reaches at x^*. This means that if the victim opts for some y such that $y < y^*$, the injurer is better off opting x^* rather than any $x > x^*$. As a result, any tuple $((s, x), (t, y))$, such that $x > x^*$ & $y < y^*$, cannot be a N.E. Similarly, under a rule that satisfies Property (P1), a tuple $((s, x), (t, y))$, such that $x < x^*$ & $y > y^*$, cannot be a N.E. Therefore, we have the following result.

Lemma 2 *Under a liability rule satisfying (P1),*
$\forall((s, x), (t, y))$ $[[(x > x^*$ &$y < y^*) or (x < x^*$ & $y > y^*)]$ $\Rightarrow ((s, x), (t, y))$ *cannot be a N.E.*$]$

Let,

$s_p^* =$ the activity level that maximizes $u(s, x)$ when $x = x^*$.

$t_p^* =$ the activity level that maximizes $v(t, y)$ when $y = y^*$.

That is, s_p^* is the optimum activity level for the injurer when he simply opts for x^* as care level but does not bear the accident costs at all. Likewise, for t_p^*. It is easy to show that $s_p^* > s^*$ and $t_p^* > t^*$.

Remark 1: When a liability rule satisfies Property (P1), in the region of $x \geq x^*$ and $y < y^*$, $u(s, x)$ is uniquely maximized at (s_p^*, x^*). Therefore, under a liability rule that satisfies Property (P1), when $x \geq x^*$ and $y < y^*$, a tuple $((s, x), (t, y))$ can be a N.E. only if $(s, x) = (s_p^*, x^*)$. Similarly, under a rule satisfying Property (P1), when $x < x^*$ and $y \geq y^*$, a tuple $((s, x), (t, y))$ can be a N.E. only if $(t, y) = (t_p^*, y^*)$.

Remark 2: It should be noted that while establishing the claims in Lemmas 1 and 2, we have considered a very general form of L function. Therefore, these claims hold for any $L(s, x, t, y)$ that satisfies Assumption (A3). In particular, Lemmas 1 and 2 are equally valid for continuous as well as discrete care and activity levels.

At these stage, for the ease of illustration, we make the functional forms more specific. We assume the following.

Assumption (A9): S, X, T, Y are convex subsets of $\Re_+$, $u(s, x) = u(s) - sx$, $v(t, y) = v(t) - ty$, where $u(s)$ and $v(t)$ are increasing and strictly concave functions. Further, $L(s, x, t, y) = \phi(s, t)l(x, y)$, where $l(x, y)$ is a decreasing and strictly convex function for all x and y; and, $\phi(s, t)$ is an increasing and strictly convex function such that $\phi_{st}(s, t) > 0$, $\phi_{ts}(s, t) > 0$, for all s and t.

For the specification of functions as in (A9), the social optimization problem is:

$$\max_{(s,x,t,y) \in S \times X \times T \times Y} \quad u(s) + v(t) - sx - ty - \phi(s, t)l(x, y).$$

For this specification, we can show the following: When the injurer opts for x^*, a choice of some $y < y^*$ cannot be an optimum choice for the victim. Likewise, if the victim opts for y^*, a choice of some $x < x^*$ cannot be an optimum choice for the injurer. In fact, the following claim holds.

Theorem 1 *If (A9) holds and $\phi_{ij}(i,j) > 0$ is large, then under a liability that satisfies Property (P1),* $\quad (\forall((s,x),(t,y)))\ [((s,x),(t,y))$ *is a N.E.* $\Rightarrow (x \geq x^* \& \ y \geq y^*)]$.

Theorem 1 says that under a liability rule that satisfies Property (P1), in equilibrium no party can be negligent; both the parties will be non-negligent. For a formal proof see Appendix B. For an informal argument, first of all note that the term $\phi_{ij}(.)$ captures the extent to which an increase in the activity level of a party increases the contribution of the other party's activity toward the expected accident loss. Now, take any liability rule f that satisfies Property (P1). In view of the arguments presented for Lemmas 1 and 2, a tuple $((s,x),(t,y))$ cannot be a N.E. if $x < x^*$ and $y < y^*$, or if $x > x^*$ and $y < y^*$, or if $x < x^*$ and $y > y^*$. Therefore, to prove the claim in Theorem 1, it will be sufficient if we can show that under the rule, a tuple $((s,x),(t,y))$ such that $x = x^*$ and $y < y^*$, or $x < x^*$ and $y = y^*$ cannot be a N.E. When $x = x^*$ and $y < y^*$, the victim is solely negligent. Therefore, due to Property (P1), the injurer's liability is zero. This means that the injurer has strong incentives to engage in an excessive level of activity. The excessive activity on the part of the injurer further increases the costs of accident, and it is the victim who bears the entire cost. Therefore, in order to decrease the accident costs, a solely negligent victim has incentive to increase his care level. In addition, excessive activity level of the injurer enhances the productivity of the victim's care, providing the victim with additional incentives to take even greater care. As the formal proof shows, when $\phi_{ij}(.)$ is large, the victim is better of opting a care level that is at least y^*. Likewise, when $x < x^*$ & $y = y^*$, i.e., when the injurer is solely negligent, the victim opts for an excessive activity level and no x that is less than x^* can be an optimal choice for the injurer.

When is $\phi_{ij}(.)$ sufficiently large, so that Theorem 1 holds? It can be checked that if we take $\phi(s,t) = st$, then $\phi_{ij}(.)$ is large enough; indeed, more than what is necessary. Here, it is worth mentioning that the specification of functional forms in the standard literature is a special case of the specification in ($A9$); the standard literature, in particular, assumes that $\phi(s,t) = st$. (e.g., see Landes and Posner (1987), Shavell (1987), Miceli (1997), and Dari Mattiacci (2002).

Let us see how the claim in Theorem 1 stands as compared to the relevant claim in literature. Note that, among other things, all of the negligence criterion based rules discussed in the literature, e.g., the rule of negligence, the rule of negligence with the defense of contributory negligence, the rule of the comparative causation under negligence,[19] and the rule of strict liability with the defense of contributory negligence, satisfy Property (P1). Therefore, as an implication of Theorem 1, under these rules the parties cannot be negligent in an equilibrium. Theorem 1 shows that the relevant claim in Parisi and Fon (2004) is not valid, where it is suggested that under the rule of the comparative causation under negligence, in equilibrium one or both the parties can be negligent.

Remark 3: It should be noted that while proving Lemmas 1, 2, and Theorem 1, we have used only Property (P1). Therefore, how a liability rule assigns liability when parties are either both negligent or both non-negligent has no implications for the validity of Lemmas 1, 2 and Theorem 1.

In view of Theorem 1, search for the existence of an equilibrium under a liability rule can be restricted to the region where $x \geq x^*$ and $y \geq y^*$.

Let,

$$q^* = f(x^*, y^*) = q(x^*, y^*).$$

Property (P2): $(\forall x \in X)(\forall y \in Y)[x \geq x^* \& y \geq y^* \Rightarrow f(x,y) = q^* \ ]$.

(P2) says that when both the parties are vigilant (non-negligent), the shares in which they are required to bear the accident loss remain the same, regardless of the degrees of vigilance of the parties. However, it should be noted that Property (P2) allows q^* and

[19]Under the rule of the comparative causation under negligence, when a party is found solely negligent, the entire loss is borne by this party. Accident loss is shared between the parties in cases where parties are either both negligent or when both are non-negligent (see Parisi and Fon (2004).)

therefore the shares to be any number between (and including) 0 and 1.

All liability rules whether based on the negligence criterion or not, satisfy Property (P2). The standard negligence-based liability rules are such that $q^* = 0$ or 1. That is, when both the parties are non-negligent, only one party is fully liable for the accident loss; this party is the injurer if $q^* = 1$, and the victim if $q^* = 0$. For these rules, we have the following claim.

Lemma 3 *Suppose (A9) holds with $\phi(s,t) = st$. If a liability rule satisfies Properties (P1) and (P2) with $q^* \in \{0,1\}$, then under the rule:*

(i) $((s,x),(t,y))$ is a N.E. $\Rightarrow (x \neq x^*$ or $y \neq y^*)$;

(ii) when $q^* = 0$, for some $y > y^*$ & $t < t^*$, $((s_p^*, x^*),(t,y))$ is a N.E.; and

(iii) when $q^* = 1$, for some $s < s^*$ & $x > x^*$, $((s,x),(t_p^*, y^*))$ is a N.E.

Lemma 3 shows that under standard negligence-based liability rules, including the rule of negligence and the rule of the strict liability with the defense of contributory negligence, a tuple $((s,x^*),(t,y^*))$ cannot be N.E. That is, regardless of their choice of activity levels, in equilibrium, the injurer and the victim will not simultaneously opt for x^* and y^*. For a formal proof see Appendix B. For an intuitive argument, consider a liability rule that satisfies Properties (P1) and (P2). First, assume that when the injurer opts for x^* and the victim opts for y^*, the victim bears the entire accident loss, i.e., $q^* = 0$. This, in view of Properties (P1) and (P2), implies that regardless of the care level and activity level chosen by the victim, if the injurer opts for x^*, his liability is zero. Under such a rule, in order to avoid liability, the injurer will opt for x^* as care level. But, as his liability is zero, his activity level, s_p^*, is excessive. Since, the victim bears the entire cost of accident, he has incentives to increase his care and reduce his activity level. Excessive activity level of the injurer further strengthen these incentives. Indeed, as the formal proof shows, the victim opts for a care level that is greater than y^* and an activity level that is less than t^*. The argument when $q^* = 1$ is analogous.

Lemma 3 stands in contrast to the relevant claims in the existing literature. In Miceli (1997 p. 29), Cooter and Ulen (2004, pp. 332-33), Dari Mattiacci (2002), Parisi and Fon

(2004), among others, it is argued that under the rule of negligence as well as under the rule of strict liability with the defense of contributory negligence, the injurer and the victim opt for x^* and y^*, respectively. Lemma 3 shows that this claim is not valid in general. It is interesting to note that the claim has been made for the same specifications as in the Lemma.

3.2 Sharing of liability between non-negligent parties

In the following, we consider the implication of sharing of accident loss for the existence of equilibria under liability rules. When both the parties are negligent, we allow sharing of liability for accident loss in *any* arbitrary manner. When both the parties are non-negligent, liability can be shared in any proportions subject to the restriction imposed in Property (P2). To start with, let us consider the behaviour of the parties in the region $x \geq x^*$& $y \geq y^*$.

Lemma 4 *Take any liability rule. In the region $x \geq x^*$& $y \geq y^*$, the injurer's activity level is at most s_p^*, the victim's activity level is at most t_p^*. Moreover, there exist care levels $\hat{x}$ and $\hat{y}$, such that the care level opted by the injurer is always less than $\hat{x}$, and the care level opted by the victim is always less than $\hat{y}$.*

For a formal proof see Appendix B. As was demonstrated earlier, in the region $x \geq x^*$, the injurer opts for (s_p^*, x^*) if his liability is zero. If he is liable for some or whole of the accident loss, he internalizes at least part of the externality. As a result, his activity level is moderated, i.e., less than s_p^*. Likewise, as long as $y \geq y^*$, the victim never opts for an activity level that is greater than t_p^*. Now, if we assume that marginal productivity of care approaches zero as care level becomes very high,[20] then there exist care levels, say $\hat{x}$ and $\hat{y}$, such that the injurer's choice of care is always less than $\hat{x}$, and the victim's choice of care is always less than $\hat{y}$.

In view of Lemma 4 and the discussion in the previous paragraph, in the region of $x \geq x^*$ and $y \geq y^*$, we can meaningfully restrict attention to the following choice sets:

$$\bar{S} = \{s \in S \mid s \leq s_p^*\},\ \bar{X} = \{x \in X \mid x^* \leq x \leq \hat{x}\},$$

[20]That is, we assume that $\lim_{i\to\infty} \|l_i(.)\| = 0$. This assumption is needed on technical ground.

$$\bar{T} = \{t \in T \mid t \leq t_p^*\}, \bar{Y} = \{y \in Y \mid y^* \leq y \leq \hat{y}\},$$

where $\hat{x}$ and $\hat{y}$ are as in Lemma 4.

Definition 2: *Restricted Liability Rule* : Take any liability rule f. We define $\bar{f}$ to be the restricted liability rule associated with f, if

$$\bar{f} : \bar{X} \times \bar{Y} \mapsto [0, 1] \text{ and } (\forall (x, y) \in \bar{X} \times \bar{Y})[\bar{f}(x, y) = f(x, y)],$$

and choice set of the injurer is $\bar{S} \times \bar{X}$, and for the victim choice set is $\bar{T} \times \bar{Y}$.

That is, $\bar{f}$ is the restricted liability rule associated with f, if $\bar{f}$ assigns liability in the same way as does f. However, the injurer's choice of care and activity levels are restricted to sets $\bar{X}$ and $\bar{S}$, and that of the victim are restricted to sets $\bar{Y}$ and $\bar{T}$.

Property (P3): For all $x \geq x^*$ and $y \geq y^*$, q is a function of x and y, such that

$$q_x \leq 0, \, q_{xx} \geq 0, \, q_y \geq 0, \text{ and } q_{yy} \leq 0.$$

Informally put, (P3) allows the liability of non-negligent parties to change with the changes in their degrees of vigilance. However, it requires liability of a party to, *ceteris-paribus*, decrease at a decreasing rate with an increase in its care level. Note that Property (P2) is a special case of Property (P3).

In the following, we show that under every liability rule that satisfies Properties (P1) and [(P2) or (P3)] there exists a N.E., in which the injurer care level is at least x^* and the victim's care level is at least y^*.

Lemma 5 *For every liability rule f that satisfies Property (P2) or (P3),* $\exists((s, x), (t, y)) \in (\bar{S} \times \bar{X}) \times (\bar{T} \times \bar{Y})$ *such that* $((s, x), (t, y))$ *is a N.E. of* $\bar{f}$.

Take any liability rule f that satisfies Property (P2) or (P3). Lemma 5 says that under $\bar{f}$, i.e., under the restricted liability rule associated with f, there exists at least one N.E., say, $((s, x), (t, y))$ such that $x^* \leq x < \hat{x}$ and $y^* \leq y < \hat{y}$. The claim is an application of a result in Game Theory. For proof see Appendix B.

Consider a liability rule f that satisfies Properties (P1) and [(P2) or (P3)]. By Lemma 5, there exists at least one tuple, say, $((\bar{s}, \bar{x}), (\bar{t}, \bar{y}))$ such that $x^* \leq \bar{x} < \hat{x}$, $y^* \leq \bar{y} < \hat{y}$ and $((\bar{s}, \bar{x}), (\bar{t}, \bar{y}))$ a N.E. under $\bar{f}$. The following theorem shows that every N.E. of the restricted rule $\bar{f}$ is also a N.E. of the original liability rule f.

Theorem 2 *Suppose (A9) holds. For every liability rule f satisfying Properties $(P1)$ and $[(P2) or (P3)]$, $\exists((\bar{s}, \bar{x}), (\bar{t}, \bar{y}))$ such that $\bar{x} \geq x^*$ & $\bar{y} \geq y^*$, and $((\bar{s}, \bar{x}), (\bar{t}, \bar{y}))$ is a N.E.*

A formal proof is provided in Appendix B. First of all, let $((\bar{s}, \bar{x}), (\bar{t}, \bar{y}))$ be a N.E. under $\bar{f}$. Clearly, $x^* \leq \bar{x} < \hat{x}$ and $y^* \leq \bar{y} < \hat{y}$. Now, given that $(\bar{t}, \bar{y})$ is opted by the victim, consider a choice of (s, x), where $x \geq x^*$, by the injurer. Since, irrespective of the liability rule, the injurer never opts for any $x \geq \hat{x}$ (Lemma 4), therefore, $x^* \leq x < \hat{x}$. Also, note that for any (x, y) where $x^* \leq x < \hat{x}$ and $y^* \leq y < \hat{y}$, the liability assignments under f and $\bar{f}$ are exactly the same. Therefore, $((\bar{s}, \bar{x}), (\bar{t}, \bar{y}))$ is a N.E. under $\bar{f}$ implies the following: Under f, given that $(\bar{t}, \bar{y})$ is opted by the victim, the injurer's payoff cannot be greater at any (s, x), where $x \geq x^*$, than his payoff at $(\bar{s}, \bar{x})$. Likewise, under f, given that $(\bar{s}, \bar{x})$ is opted by the injurer, for any (t, y), where $y \geq y^*$, the victim's payoff cannot be greater at (t, y) than his payoff at $(\bar{t}, \bar{y})$. Therefore, to prove that $((\bar{s}, \bar{x}), (\bar{t}, \bar{y}))$ is a N.E. under f when choices are not restricted, we just have to show that given that $(\bar{s}, \bar{x})$ is opted by the injurer, a unilateral deviation to some (t, y), where $y < y^*$, cannot be profitable for the victim; and given that $(\bar{t}, \bar{y})$ is opted by the victim, a unilateral deviation to some (s, x), where $x < x^*$, cannot be profitable to the injurer. Note that such deviations imply full liability for the deviating party, and, as the formal proof demonstrates, are not profitable. Theorem 2 is proved by assuming that $\phi(st) = st$. However, arguments in the proof of Theorem 1 suggest that the claim is likely to hold in a more general context as long as $\phi_{ij}(.)$ is large.

Theorem 2 says that as long as a liability rule satisfies Properties (P1) and [(P2) or (P3)], the sharing of liability between non-negligent parties does not threaten the existence of equilibrium. More specifically, there exists at least one equilibrium, regardless of the proportions in which liability is shared between non-negligent parties. Moreover, in this equilibrium the injurer's care level is at least x^*, and the victim's care level is at least y^*.

Theorems 1 and 2 also imply that how liability is assigned when both parties are negligent has no implications for the equilibrium outcome; i.e., it does not affect the behaviour of the parties.

4. Second Best Liability Rules

Suppose, the legal standards of care for the injurer and the victim are x^* and y^*, respectively. In view of Lemma 5 and Theorem 2, we can meaningfully restrict our attention to the properties of equilibria in the region of $x \geq x^* \& y \geq y^*$; more specifically to the properties of equilibria under restricted liability rules. For the ease of exposition, in this section we assume that every restricted liability rule has only one N.E.

First, consider a liability rule that satisfies Properties (P1) and (P2). Under such a rule, when $x \geq x^* \&\ y \geq y^*$, the objectives of the injurer and the victim are given by

$$\max_{(s,x)\in S\times X} \quad u(s) - sx - q\phi(s,t)l(x,y),$$

and

$$\max_{(t,y)\in T\times Y} \quad v(t) - ty - (1-q)\phi(s,t)l(x,y),$$

respectively. Here, $q = q(x^*, y^*)$ is constant; a real number between (and including) 0 and 1, uniquely determined for a liability rule.

Since the rule satisfies Properties (P1) and (P2), in view of Theorem 2, there is a profile, say $((s,x),(t,y))$, such that $x \geq x^* \&\ y \geq y^*$ and $((s,x),(t,y))$ is a N.E. under the rule. At the equilibrium $((s,x),(t,y))$, activity levels of the injurer and the victim satisfy the following equations:

$$u'(s) = x + q\phi_s(s,t)l(x,y), \tag{7}$$

$$v'(t) = y + (1-q)\phi_t(s,t)l(x,y), \tag{8}$$

respectively. From (7), we see that s is a function of q, along with t, x, y, i.e., we can write $s = s(q,t,x,y)$. From (8), we get $t = t(q,t,x,y)$. Also, for given $s, t \ \& \ y$, the injurer chooses x that minimizes $sx + q\phi(s,t)l(x,y)$. Therefore, x is a function of q, along with s, t, y, i.e., $x = x(q,s,y,t)$. Moreover, assuming $s > 0 \ \& \ t > 0$, if $x > x^*$, then in equilibrium x is such that

$$1 + \frac{q\phi(s,t)}{s} l_x(x,y) = 0. \tag{9}$$

Likewise, $y = y(q, s, x, t)$. If $y > y^*$, then in equilibrium y is such that

$$1 + \frac{(1-q)\phi(s,t)}{t} l_y(x,y) = 0. \tag{10}$$

From (7)-(10) is clear that:

$$\partial s/\partial q < 0; \ \ \partial t/\partial q > 0; \tag{11}$$

$$\partial x/\partial q > 0, \text{ when } x > x^*; \ \partial x/\partial q \geq 0, \text{ when } x = x^*;^{21} \tag{12}$$

$$\partial y/\partial q < 0, \quad \text{when } y > y^*; \ \partial y/\partial q \leq 0, \quad \text{when } y = y^*; \tag{13}$$

Therefore, at the equilibrium $((s,x),(t,y))$, $\partial s/\partial q < 0$; $\partial t/\partial q > 0$; $\partial x/\partial q \geq 0$; $\partial y/\partial q \leq 0$.

The following theorem shows that the second best efficiency requires loss sharing between the parties, when both the parties are non-negligent.

Theorem 3 *If a liability rule satisfies Properties (P1) and (P2) and the legal standards are as in (A8), then the second best q is such that* $(\forall x \geq x^*)(\forall y \geq y^*)[q(x,y) \in (0,1)]$.

Theorem 3 says that if the legal standards for the injurer and the victim are at x^* and y^*, respectively, and if the rule satisfies Properties (P1) and (P2), then second best efficiency requires loss sharing whenever parties are non-negligent. For a formal proof see Appendix B. Informal argument is as follows. Consider a liability rule that satisfies Properties (P1) and (P2). Due to (P2), for all $x \geq x^*$ and $y \geq y^*$, $q(x,y) = q(x^*,y^*)$. If $q(x^*,y^*) = 0$, as under the rule of negligence, then by Lemma 3, $((s_p^*, x^*), (t,y))$, where $t < t^*$ and $y > y^*$, is a unique N.E. That is, the injurer's care level is x^*, but his activity level is excessive at s_p^*. On the other hand, the victim takes too much care and opts for too little activity level, as compared to the first-best levels. Now, if the non-negligent injurer is required to bear

[21] That is, in the non-negligence region an increase in the injurer's liability cannot lower his care level. Note that, in equilibrium $x = x^*$ implies that $q < 1$ (Lemma 3). Due to Property (P1), a dercrease in care level from x^* makes him fully liable. Therefore, a small increase in his liability cannot induce him to reduce his care level below x^*.

a very small fraction of the accident loss, it will leave his care level at x^*, but will reduce his activity level making it closer to the first-best level, s^*. Sharing of liability means that the victim's care will come down and his activity level will go up little bit. As the proof shows, the net result of these changes is an increase in the social welfare. Likewise, it can be shown that $q(x^*, y^*) = 1$ is not second best.

Two observations are important for the result shown in Theorem 3. First of all, it should be noted that the equilibrium outcome and, therefore, the social welfare depends on q; the proportions of loss sharing in the non-negligence region. The proof of Theorem 3 shows that the optimum q depends on the context at hand. Note that q has greater degree of freedom under (P3) than under (P2). Therefore, in principle, a higher social welfare can be achieved under a liability rule that satisfies (P3), i.e., when liability assignment is sensitive to the levels of vigilance adopted by the parties.

Second, Theorem 3 says that the sharing of liability improves incentives related to non-verifiable aspects of care without diluting those related to verifiable aspects. This argument is true for a context broader that we have formally captured. For instance, we have assumed that benefits to injurers increase with the non-verifiable aspect of care. This is and other simplifications are not necessary for the validity of Theorem 3. To illustrate the point, suppose the care taken by drivers has three aspects; driving speed, regular checking of break oil, and the amount of driving. Assume that while the speed is verifiable, care w.r.t. quality of break oil and the level of driving are not. Note that, now, the benefits to a driver decrease with one non-verifiable aspect (quality of oil) of care. Under negligence liability, drivers will opt for a low level of this aspect of care. Theorem 3 says that the sharing of liability when both driver and pedestrian are non-negligent, induces the drivers to better maintain the quality of break oil.

Remark 4: Note that under every standard liability rule, whether based on negligence criterion or not, when both the parties are non-negligent, only one party bears the accident loss, i.e., either $q = 0$ or $q = 1$. However, Theorem 3 shows that social welfare can be increased by loss sharing between non-negligent parties. Therefore, it follows that none of the standard liability rules is efficient even from a second-best perspective. Also, note that liability can be shared between two negligent parties without any loss of social welfare.

Alternatively, if liability of a non-negligent party is zero, then the social welfare can be improved by increasing the due care level for this party. To see why, let the due care level for the injurer be x^*, and $q = 0$ when $x \geq x^*$. This is the case under the standard rule of negligence. In this case, from Lemma 3, some $((s, x), (t, y))$ such that $s = s_p^* > s^*, x = x^*, y > y^*, \&t < t^*$ is a unique N.E. Also, equilibrium values of $s_p^*, x^*, y, \&\ t$ are such that $u'(s_p^*) = x^*$, and $v'(t) = y + \phi_t(s_p^*, t)l(x^*, y)$. Now if x^* and y are comparable, or if $\phi_t(s_p^*, t)l(x^*, y) \geq x^*$, then $v'(t) > u'(s_p^*)$. This means that social welfare can be increased by making the injurer bear a very small fraction of the accident loss in the form of increased care level and decreased activity level. To this end, consider a very small increase in the due care level for the injurer, i.e., let $\bar{x} > x^*$ (where $\bar{x}$ is very close to x^*) be the new due care level for the injurer. Since $\bar{x}$ is very close to x^*, under the new equilibrium the injurer will opt for $\bar{x}$. His activity level, however, will come down, as is desired. The case when $q = 1$ is analogous.

Therefore, depending on the context, second-best efficiency may require either loss sharing between non-negligent parties or raising of the due care standards or both. However, in some contexts, the former can be better than the latter. In the context of our example, the sharing of liability will induce drivers to watch out the quality of oil as well as reduce the amount of driving. The raising the due care level by lowering of speed limit, on the other hand, may bring down the amount of driving, it may not result in better up keep on break oil.

In principle, the second-best optimization problem can be posed as:[22]

$$\max_{(x,y)\in X\times Y,\ q(x,y)\in[0,1]} \quad u(s) + v(t) - sx - ty - \phi(s,t)l(x,y), \tag{14}$$

such that s and t, respectively, satisfy

$$u'(s) = x + q(x,y)\phi_s(s,t)l(x,y), \tag{15}$$

[22]It should be noted that in (14) not only the domain is unrestricted, liability assignment is also unrestricted.

and

$$v'(t) = y + (1 - q(x, y))\phi_t(s, t)l(x, y). \tag{16}$$

Suppose, there is a unique solution to (14). Let the solution be denoted by x^{**}, y^{**} and $q^{**}(.)$. Let s^{**} and t^{**}, respectively, solve (15) and (16) when $x = x^{**}$, $y = y^{**}$ and $q(x, y) = q^{**}(x^{**}, y^{**})$. Now, the next question is whether a liability rule can achieve $(s^{**}, x^{**}), (t^{**}, y^{**}))$ as an equilibrium outcome.

5. Concluding Remarks

We have shown that if the due care levels are equal to the levels that are appropriate from a first-best perspective, the sharing of liability between parties who are either both negligent or both non-negligent, does not endanger the existence of an equilibrium. More specifically, we have shown that regardless of the share of the parties in accident loss there exists at least one equilibrium (Theorem 2). Moreover, in an equilibrium, care level of each party is at least equal to the due level of care (Theorems 1 and 2). The requirement that a solely negligent party is liable for the entire accident loss, ensures that no party prefers to be negligent. This means that the sharing of liability between parties who are either both negligent or both vigilant does not dilute their incentives to take the due care.

In regard to the question of efficiency of liability rules, within the confines of the law of torts, we have shown that each of the standard liability rules fails to be efficient even from a second-best perspective. Under these rules, one party takes too little care with respect to the non-verifiable aspects of care, while other takes too much of care. Theorem 3 shows that the second best efficiency requires loss sharing between the parties when both parties are non-negligent. Sharing of liability improves parties' incentives with respect to the non-verifiable aspects of care, without spoiling their incentives for the verifiable aspects. The result is an improvement in social welfare. We have shown that depending on the context, second-best efficiency may require either loss sharing between non-negligent parties or raising of the due care standards or both. To sum up, we have shown that the sharing of liability as proposed in Calabresi and Cooper (1996) is desirable from an efficiency perspective.

Our analysis has some limitations too. In Section 4, while exploring the nature of second-best liability rules, we have assumed that under a liability rule the equilibrium outcome is unique. It should be noted that the sharing of liability between non-negligent parties opens up the possibility of the existence multi equilibria. Though the possibility of multiple equilibria does not undermine the argument that *some* sharing of liability between non-negligent parties is desirable, it does introduce complications that we have not fully explored here. This issue needs to be analyzed further.

As a matter of practice, liability is determined only on the basis of the verifiable aspects of care levels of the parties. Therefore, a liability rule can directly control only the verifiable aspects of care; the non-verifiable aspects cannot be controlled directly. At the same time, there two policy tools available under a liability rule; namely, the due care levels in terms of the verifiable aspects, and liability as function of care levels. However, we have considered the implications of the second tool only. This suggests that, at least in theory, the analysis of the nature of second-best liability rules can be extended beyond what we have done here. In particular, it needs to checked whether a solution to (14) can be achieved within the confines of liability rules.

Appendix A

Definition A1: *Liability rules as Strategic Games*: The outcome under a liability rule f can be studied by solving the following strategic game:

$$\Gamma_f = \langle \{injurer, victim\}, ((S \times X), (T \times Y)), (U, V) \rangle.$$

Where, $U(.) = u(s) - sx - q(x,y)\phi(s,t)l(x,y)$, $V(.) = v(t) - ty - (1 - q(x,y))\phi(s,t)l(x,y)$, and $q(x,y) \in [0,1]$ is determined by the liability rule f.

Note that different liability rules generate different games. Let,

$$F = \{f \mid f \text{satisfies (P1) and [(P2) or (P3)]}\}, \quad \text{and}$$

$$\Gamma^0 = \bigcup_{f \in F} \Gamma_f.$$

That is, Γ^0 is the set of all possible games that can be generated by liability rules that satisfy Properties (P1) and [(P2) or (P3)]. Clearly, the sets F and Γ^0 are infinite.

Definition A2: *Restricted Liability rules as Restricted Games*: Take any liability rule f that satisfies Properties (P1) and [(P2) or (P3)]. Suppose, Γ_f is the game generated by f. Let $\bar{f}$ be the restricted liability rule associated with f. The outcome under $\bar{f}$ can be studied by solving the following restricted game:

$$\bar{\Gamma}_f = \langle\{injurer, victim\}, ((\bar{S} \times \bar{X}), (\bar{T} \times \bar{Y})), (\bar{U}, \bar{V})\rangle;$$

where $\bar{S} = \{s \in S \mid s \leq s_p^*\}$, $\bar{X} = \{x \in X \mid x^* \leq x \leq \hat{x}\}$, $\bar{T} = \{t \in T \mid t \leq t_p^*\}$, $\bar{Y} = \{y \in Y \mid y^* \leq y \leq \hat{y}\}$,

$$\bar{U} = u(s) - sx - q(x,y)\phi(s,t)l(x,y),$$

$$\bar{V} = v(t) - ty - (1 - q(x,y))\phi(s,t)l(x,y).$$

$q(x,y)$ is a given constant if f satisfies Property (P2); and if f satisfies Property (P3), then $q_x \leq 0$, $q_{xx} \geq 0$, $q_y \geq 0$, and $q_{yy} \leq 0$. Clearly, $\bar{\Gamma}_f$ is a restriction of Γ_f. Let,

$$\bar{\Gamma}^0 = \bigcup_{f \in F} \bar{\Gamma}_f.$$

We denote,

$\mathrm{B}_i^{\Gamma_f}(.) =$ the best response function for the injurer under the game Γ_f, and

$\mathrm{B}_v^{\Gamma_f}(.) =$ the best response function for the victim under the game Γ_f.

Clearly, $\mathrm{B}_i^{\Gamma_f} : T \times Y \mapsto S \times X$ and $\mathrm{B}_v^{\Gamma_f} : S \times X \mapsto T \times Y$. Therefore, $\mathrm{B}_i^{\Gamma_f}(t,y) \in S \times X$ and $\mathrm{B}_v^{\Gamma_f}(s,x) \in T \times Y$.

That is, if the victim opts for some (t,y), then under the game Γ_f the optimal choice for the injurer is $\mathrm{B}_i^{\Gamma_f}(t,y)$. Likewise, if the injurer has opted for (s,x), then the optimal choice for the victim is $\mathrm{B}_v^{\Gamma_f}(s,x)$.

Appendix B

Proof of Theorem 1: Take any liability rule f that satisfies Property (P1). In view of Lemmas 1 and 2, to prove the claim it will be sufficient if we show that under the rule, a tuple $((s,x),(t,y))$ such that $x = x^*$ and $y < y^*$, or $x < x^*$ and $y = y^*$ cannot be a N.E.

Consider a tuple $((s,x),(t,y))$ such that $x=x^*$ and $y<y^*$. In view of Remark 2, when $x=x^*$ and $y<y^*$, $((s,x),(t,y))$ can be a N.E. only if $(s,x)=(s_p^*,x^*)$, where $s_p^*=s_p(x^*)$. Therefore, to show that a tuple $((s,x),(t,y))$ such that $x=x^*$ and $y<y^*$ cannot be a N.E., we just have to show that tuple $((s_p^*,x^*),(t,y))$, where $y<y^*$, cannot be a N.E. Similarly, to show that a tuple $((s,x),(t,y))$, such that $x<x^*$ and $y=y^*$, cannot be a N.E., we just have to show that tuple $((s,x),(t_p^*,y^*))$, where $x<x^*$ and $t_p^*=t_p(y^*)$, cannot be a N.E.

In view of our assumption that $((s^*,x^*),(t^*,y^*)) >> ((0,0),(0,0))$, the necessary and sufficient first order conditions imply that s^*, x^*, t^* and y^* will, respectively, satisfy the following conditions:

$$u'(s)=x^*+\phi_s(s,t^*)l(x^*,y^*), \tag{A1}$$

$$1+\tfrac{\phi(s^*,t^*)}{s^*}l_x(x,y^*)=0, \tag{A2}$$

$$v'(t)=y^*+\phi_t(s^*,t)l(x^*,y^*), \tag{A3}$$

$$1+\tfrac{\phi(s^*,t^*)}{t^*}l_y(x^*,y)=0. \tag{A4}$$

Suppose the injurer has opted for (s_p^*,x^*). Then, since f satisfies Property (P1), for all $y<y^*$, the problem facing the victim is

$$\max_{(t,y)\in T\times Y} \quad v(t)-ty-\phi(s_p^*,t)l(x^*,y).$$

Therefore, given that (s_p^*,x^*) is opted by the injurer, a pair $(\bar{t},\bar{y})$, such that $\bar{y}<y^*$, can be a best response for the victim, only if $\bar{t}$ and $\bar{y}$ satisfy the following, respectively:

$$v'(\bar{t})=\bar{y}+\phi_t(s_p^*,\bar{t})l(x^*,\bar{y}), \tag{A5}$$

$$1+\tfrac{\phi(s_p^*,\bar{t})}{\bar{t}}l_y(x^*,\bar{y})=0. \tag{A6}$$

Note that $s_p^*>s^* \Rightarrow (\forall t\in T)(\frac{\phi(s_p^*,t)}{t}>\frac{\phi(s^*,t)}{t})$.[23] Now, if $\phi_{st}(s,t)>0$ is sufficiently large, then $\frac{\phi(s_p^*,\bar{t})}{\bar{t}}>\frac{\phi(s^*,t^*)}{t^*}$ will hold.[24] In that case, (A4), in view of $l_y(.)<0$ & $l_{yy}(.)>0$, implies that no $\bar{y}<y^*$ can satisfy (A6), i.e., no $\bar{y}<y^*$ can be a best response for the victim. Therefore, $((s_p^*,x^*),(t,y))$, such that $y<y^*$, cannot be a N.E.

Similarly, we can show that $((s,x),(t_p^*,y^*))$, where $x<x^*$ and $t_p^*=t_p(y^*)$, cannot be a N.E. $\sharp$

Proof of Lemma 3: For the specification of functions as in (A9) with $\phi(s,t)=st$, the

[23]We are assuming that $t>0$. If $t=0$, the victim will not take any care at all.

[24]It is straightforward to see that if $\phi(s,t)=st$ then this relation holds.

social optimization problem is given by:

$$\max_{(s,x,t,y)\in S\times X\times T\times Y} \quad u(s)+v(t)-sx-ty-stl(x,y).$$

Therefore, s^*, x^*, t^*, and y^* simultaneously and respectively solve the following necessary and sufficient first order conditions:

$$u'(s) = x^* + t^* l(x^*, y^*), \tag{A7}$$

$$1 + t^* l_x(x, y^*) = 0, \tag{A8}$$

$$v'(t) = y^* + s^* l(x^*, y^*), \tag{A9}$$

$$1 + s^* l_y(x^*, y) = 0. \tag{A10}$$

Let f be a rule that satisfies Properties (P1)-(P3). Without any loss of generality let $f(x^*, y^*) = q(x^*, y^*) = \bar{q} = 0$. Suppose, $((s, x), (t, y))$ is a N.E. under the rule. Let $x = x^*$; if $x \neq x^*$, there is nothing to prove. $f(x^*, y^*) = 0$, in view of Properties (P1)-(P3), implies that if the injurer opts for a pair (s, x^*), his payoff is $u(s, x^*)$, regardless of the care level and activity level chosen by the victim. As before, $u(s, x^*)$ attains a unique maximum at (s_p^*, x^*), where $s_p^* > s^*$. Therefore, when $f(x^*, y^*) = 0$,

$$((s, x), (t, y)) \text{ is a N.E. and } x = x^* \Rightarrow ((s_p^*, x^*), (t, y)) \text{ is a N.E.}$$

Now, given (s_p^*, x^*) opted by the injurer, the problem facing the victim is

$$\max_{(t,y)\in T\times Y} \quad v(t) - ty - s_p^* tl(x^*, y).$$

Therefore, the victim will choose $t \in T$ and $y \in Y$ that simultaneously satisfy

$$v'(t) = y + s_p^* l(x^*, y), \tag{A11}$$

$$1 + s_p^* l_y(x^*, y) = 0. \tag{A12}$$

A comparison of (A10) and (A12), in view of the fact that $s_p^* > s^*$ and that $l(.)$ is strictly convex, implies that $y > y^*$. This means that regardless of the $t \in T$ opted by the victim, $((s_p^*, x^*), (t, y^*))$ cannot be a N.E. When $f(x^*, y^*) = q(x^*, y^*) = \bar{q} = 1$, an analogous argument shows that $((s, x^*), (t, y^*))$ cannot be a N.E.

Therefore, $[((s, x), (t, y)) \text{ is a N.E.} \Rightarrow (x \neq x^* \text{ or } y \neq y^*)]$.

When $q = 0$ and the injurer has opted for (s_p^*, x^*), the above argument shows that the victim will opt for t and y that satisfy (A11) and (A12), respectively. Note that from (A12) it follows that $y > y^*$. $t < t^*$ follows from (A11), in view of the fact that

$$(\forall y \in Y)[y + s_p^* l(x^*, y) \geq y + s^* l(x^*, y), \& \text{ for } y \neq y^*, y + s^* l(x^*, y) > y^* + s^* l(x^*, y^*)].$$

Now, given (t, y) opted by the victim, the injurer is strictly worse off at any $x > x^*$. If he opts for a $x < x^*$, he will be strictly worse off as he will be liable for the entire accident costs. Therefore, given (t, y) opted by the victim, (s_p^*, x^*) is a unique best choice for the injurer.[25] Hence, when $q = 0$, $((s_p^*, x^*), (t, y))$, where $t < t^*$ and $\bar{y} > y^*$ is a unique N.E.

Similarly, it can be shown that when $q = 1$, for some $s < s^* \& x > x^*$, $((s, x), (t_p^*, y^*))$ is a unique N.E. $\sharp$

Proof of Lemma 4: Take any liability rule. Given the choice of care and activity levels by the victim, the injurer opts for s that solves

$$u'(s) = x + q(x, y)\phi_s(s, t)l(x, y).$$

We know that s_p^* solves $u'(s) = x^*$, and t_p^* solves $v'(t) = y^*$. Since, $q(x, y)\phi_s(s, t)l(x, y) \geq 0$, as long as $x \geq x^*$, the injurer will never opt for any $s > s_p^*$. Similarly, as long as $y \geq y^*$, the victim will never opt for any $t > t_p^*$. Let $\epsilon = \frac{1}{s_p^* t_p^*}$. Clearly, $\epsilon > 0$. Now, the assumption that $\lim_{i \to \infty} \|l_i(.)\| = 0$ implies that there exists x such that $\|l_x(x, 0)\| < \epsilon$. Take any such x and call it $\hat{x}$. So, $\|l_x(\hat{x}, 0)\| < \epsilon$. Moreover, $l_x(.) < 0$ and $l_{xx}(.) > 0$ imply that for all $x \geq \hat{x}$, $\|l_x(x, 0)\| < \epsilon$. For the same reason, there exists y, say $\hat{y}$, such that for all $y \geq \hat{y}$, $\|l_y(0, y)\| < \epsilon$. This in view of $l_{ij}(.) > 0$ implies that $(\forall x \geq \hat{x})(\forall y)[\|s_p^* t_p^* l_x(x, y)\| < 1]$; and $(\forall x)(\forall y \geq \hat{y})[\|s_p^* t_p^* l_y(x, y)\| < 1]$. That is, at and beyond $\hat{x}$ $[\hat{y}]$, marginal decrease in the expected accident loss is less that the cost of care for the injurer [the victim]. Therefore, the care opted by the injurer [the victim] is always less than $\hat{x}$ $[\hat{y}]$. $\sharp$

Proof of Lemma 5: (For notations and terminology, see Appendix A) To prove the claim it is sufficient to show that
$(\forall \bar{\Gamma}_f \in \bar{\Gamma}^0)[\exists((s, x), (t, y)) \in (\bar{S} \times \bar{X}) \times (\bar{T} \times \bar{Y})$ such that $((s, x), (t, y))$ is a N.E. of $\bar{\Gamma}_f]$.

Take any $\bar{\Gamma}_f \in \bar{\Gamma}^0$. The payoffs of the injurer and the victim are $u(s) - sx - q(x, y)\phi(s, t)l(x, y)$, and $v(t) - ty - (1 - q(x, y))\phi(s, t)l(x, y)$, respectively. Notice that when f satisfies Property

[25] Following the literature, we assume that, relative to the cost of care, the accident costs are large so that the injurer will prefer to spend x^* on care rather than bearing the entire accident costs by being negligent.

(P3), q is a continuous function of x and y, for all $x \geq x^* \& \ y \geq y^*$; when f satisfies Property (P2), q is constant and trivially continuous. Also, notice the choice sets of the agents $\bar{S} \times \bar{X}$ and $\bar{T} \times \bar{Y}$ are nonempty compact subsets of $\Re^2_+$. For the choice sets $\bar{S} \times \bar{X}$ and $\bar{T} \times \bar{Y}$: the payoff functions $\bar{U}$ and $\bar{V}$ are continuous in s, x, t, and y; $\bar{U}$ is concave in s and x; and $\bar{V}$ is concave in t and y. Now, these properties of choice sets and payoff functions ensure existence of a N.E. [26] $\sharp$

Proof of Theorem 2: (For notations and terminology, see Appendix A)

We prove the claim by assuming that $\phi(s,t) = st$. Take any $\Gamma_f \in \Gamma^0$, i.e., consider any liability rule that satisfies Properties (P1) and [(P2) or (P3)]. Let $\bar{\Gamma}_f \in \bar{\Gamma}^0$ be the restriction of Γ_f. By Lemma 5, $\exists((s,x),(t,y)) \in (\bar{S} \times \bar{X}) \times (\bar{T} \times \bar{Y})$ such that $((s,x),(t,y))$ is a N.E. of $\bar{\Gamma}_f$. Let $((\bar{s},\bar{x}),(\bar{t},\bar{y}))$ be a N.E. of $\bar{\Gamma}_f$. Clearly $\bar{x} \geq x^* \& \bar{y} \geq y^*$. Now, consider the game Γ_f of which $\bar{\Gamma}_f$ is a restricted game. First, we show that $\mathrm{B}_i^{\Gamma_f}(\bar{t},\bar{y}) \in \bar{S} \times \bar{X}$, and $\mathrm{B}_v^{\Gamma_f}(\bar{s},\bar{x}) \in \bar{T} \times \bar{Y}$. From Lemma 4, when $x \geq x^*$ the choice of s by the injurer is such that $s \leq s_p^*$, and when $y \geq y^*$ the choice of t by the victim is such that $t \leq t_p^*$. Therefore, to prove $\mathrm{B}_i^{\Gamma_f}(\bar{t},\bar{y}) \in \bar{S} \times \bar{X}$, it is sufficient to show that, given that $(\bar{t},\bar{y})$ is opted by the victim, the inurer is worse off opting any $x < x^*$ rather than $\bar{x}$.

Given that $(\bar{t},\bar{y})$ is opted by the victim, if the injurer opts for some $(s,x) \in S \times X$, his payoff is

$$u(s) - s[x + q(x,\bar{y})\bar{t}l(x,\bar{y})].$$

Since $\bar{y} \geq y^*$, $q(x,\bar{y}) = 1$ if $x < x^*$, and $q(x,\bar{y}) = \bar{q} \in [0,1]$ if $x \geq x^*$. Note that regardless of the choice of s by the injurer, as long as $s > 0$, an optimal x minimizes $x + q(x,\bar{y})\bar{t}l(x,\bar{y})$. Likewise, given that $(\bar{s},\bar{x})$ is opted by the injurer, for the victim optimal y minimizes $y + (1 - q(\bar{x},y))\bar{s}l(\bar{x},y)$.

Also note that in the region $x \geq x^* \& y \geq y^*$, $q(x,y) = \bar{q} = q(x^*,y^*)$ is a constant.

Regarding the N.E. $((\bar{s},\bar{x}),(\bar{t},\bar{y}))$, the following four cases are possible.

Case 1: $\bar{x} > x^* \& \bar{y} > y^*$: In view of the above, at the equilibrium $((\bar{s},\bar{x}),(\bar{t},\bar{y}))$, $\bar{x} > x^*$ & $\bar{y} > y^*$ implies that $\bar{x}$ and $\bar{y}$, respectively, solve the following equations:

$$1 + \bar{q}\bar{t}l_x(x,\bar{y}) = 0, \tag{A13}$$

[26]See, Glicksberg (1952), Fan (1952), also see Debreu (1952).

and

$$1 + (1 - \bar{q})\bar{s}l_y(\bar{x}, y) = 0. \tag{A14}$$

The facts that $\bar{x} > x^*$ solves (A13), $l_x(.) < 0$ and that $l_{xx}(.) > 0$ imply that at x^*, $\|\bar{q}\bar{t}l_x(x, \bar{y})\| > 1$, i.e., $\|\bar{t}l_x(x^*, \bar{y})\| > 1$, since $\bar{q} \in (0, 1)$.[27]

Note that $\|\bar{t}l_x(x^*, \bar{y})\| > 1$, $l_x(.) < 0$ and $l_{xx}(.) > 0$ imply that,

$$(\forall x < x^*)[x + \bar{t}l(x, \bar{y}) > x^* + \bar{t}l(x^*, \bar{y})].$$

Also, $\bar{q} \in (0, 1) \Rightarrow x^* + \bar{t}l(x^*, \bar{y}) > x^* + \bar{q}\bar{t}l(x^*, \bar{y})$. Moreover, $\bar{x} > x^*$ solves $(A13)$, $l_x(.) < 0$ and $l_{xx}(.) > 0$ imply that $x^* + \bar{q}\bar{t}l(x^*, \bar{y}) > \bar{x} + \bar{q}\bar{t}l(\bar{x}, \bar{y})$. Therefore, we have

$$(\forall x < x^*)[x + \bar{t}l(x, \bar{y}) > \bar{x} + \bar{q}\bar{t}l(\bar{x}, \bar{y})].$$

That is, given that $(\bar{t}, \bar{y})$ is opted by the victim, regardless of the choice of s by the injurer, he is strictly worse off opting a $x < x^*$ rather than $\bar{x}$. Also, when $x \geq x^*$, by Lemma 4, the choice of s by the injurer is such that $s \leq s_p^*$. Therefore, $\mathrm{B}_i^{\Gamma_f}(\bar{t}, \bar{y}) \in \bar{S} \times \bar{X}$.

Similarly, we can show that $\mathrm{B}_v^{\Gamma_f}(\bar{s}, \bar{x}) \in \bar{T} \times \bar{Y}$.

Case 2: $\bar{x} > x^* \& \bar{y} = y^*$: In this case, the argument provided in Case 1 shows that $\mathrm{B}_i^{\Gamma_f}(\bar{t}, \bar{y}) \in \bar{S} \times \bar{X}$.

Moreover, $\bar{x} > x^* \& \bar{y} = y^*$ imply that either $q(\bar{x}, \bar{y}) = 1$ or is very close 1, i.e., at equilibrium $((\bar{s}, \bar{x}), (\bar{t}, \bar{y}))$ the victim's liability is either zero or close to zero, and therefore his activity level is close t_p^*. On the other hand, if he deviates to some $y < y^*$ his liability is full. Due to the arguments provided for Lemma 3, for the victim a choice of $y < y^*$ cannot be a better choice than that of y^*.[28] Also, when $y \geq y^*$, by Lemma 4, the choice of t by the victim is such that $t \leq t_p^*$. Therefore, $\mathrm{B}_v^{\Gamma_f}(\bar{s}, \bar{x}) \in \bar{T} \times \bar{Y}$.

[27] It is easy to see that in this case $\bar{q} \in (0, 1)$; since $\bar{q} = 0$ would mean that $\bar{x} = x^*$, and $\bar{q} = 1$ would mean that $\bar{y} = y^*$.

[28] It should be pointed out here that this argument depends on the mainstream understanding that the expected accident costs are high enough so as to deter a party from being solely negligent, especially when this party (the victim, in this instance) while non-negligent bears only a small fraction of the accident costs. This is a reasonable assumption. Since, when a party while non-negligent bears only a small fraction of accident loss, it enjoys the benefit of its excessive activity level, while the other party bears most of the costs. On the other hand, if it chooses to be negligent, it will bear the entire accident costs.

Case 3: $\bar{x} = x^* \& \bar{y} > y^*$: This case is analogous to Case 2.

Case 4: $\bar{x} = x^* \& \bar{y} = y^*$: In this case, it is easy to see that $\bar{q}(\bar{x}, \bar{y}) = \bar{q} \in (0, 1)$.[29] Moreover, $\bar{s} \leq s^* \& \bar{t} \leq t^*$ is not a possibility.[30] Therefore, the following subcases arise.

Subcase 1: $\bar{s} > s^* \& \bar{t} > t^*$:[31] In this subcase, in view of $\bar{y} = y^* \& \bar{t} > t^*$ and $(A8)$, no $x < x^*$ can be an optimal choice for the injurer. Indeed, arguing along the lines in Case 1, it can be shown that the injurer [the victim] is worse off opting $x < x^*$ $[y < y^*]$ rather than x^* $[y^*]$.

Subcase 2: $\bar{s} = s^* \& \bar{t} > t^*$: In this subcase, it is easy to see that the injurer is worse off opting $x < x^*$ rather than x^*. Also, the assumption that $((s^*, x^*), (t^*, y^*))$ is the unique social optimum implies that for any (t, y) where $y < y^*$,

$$v(t) - ty - s^* t l(x^*, y^*) < v(t^*) - t^* y^* - s^* t^* l(x^*, y^*) < v(t^*) - t^* y^* - (1 - \bar{q}) s^* t^* l(x^*, y^*).$$

That is, given that (s^*, x^*) is opted by the injurer, for the victim the choice of (t^*, y^*) is strictly better than that of any (t, y), where $y < y^*$.

Subcase 3: $\bar{s} > s^* \& \bar{t} = t^*$: This subcase is analogous to Subcase 2.

Subcase 4: $\bar{s} > s^* \& \bar{t} < t^*$: In view of $\bar{x} = x^* \& \bar{s} > s^*$, $(A10)$ implies that the victim is worse off opting $y < y^*$ rather than y^*. Also, $\bar{s} > s^* \& \bar{q} < 1$ imply that injurer is worse off opting $x < x^*$ rather than x^*.[32]

Subcase 5: $\bar{s} < s^* \& \bar{t} > t^*$: This subcase is analogous to Subcase 4.

Therefore, in all the cases $\mathrm{B}_i^{\Gamma_f}(\bar{t}, \bar{y}) \in \bar{S} \times \bar{X}$ and $\mathrm{B}_v^{\Gamma_f}(\bar{s}, \bar{x}) \in \bar{T} \times \bar{Y}$. Now, that $((\bar{s}, \bar{x}), (\bar{t}, \bar{y}))$ is a N.E. of $\bar{\Gamma}_f$ and that $\bar{\Gamma}_f$ is a restriction of Γ_f mean that $((\bar{s}, \bar{x}), (\bar{t}, \bar{y}))$ is a N.E. of Γ_f $\sharp$

Proof of Theorem 3: Take any liability rule f that satisfies Properties (P1) and (P2). Due to (P2), for all $x \geq x^*$ and $y \geq y^*$, $q(x, y) = q(x^*, y^*)$. In view of Theorem 2, there exists a profile $((s, x), (t, y))$ such that: $x \geq x^* \& y \geq y^*$ and $((s, x), (t, y))$ is a N.E. of f. As

[29]Note that from Lemma 3, $\bar{q} = 0 \Rightarrow \bar{y} > y^*$, and $\bar{q} = 1 \Rightarrow \bar{x} > x^*$.

[30]In this case, $\bar{s} = s^* \& \bar{t} = t^*$ is not possible due to Shavell (1987). Furthermore, $(A7)$ and $(A9)$ imply that $\bar{s} \leq s^* \& \bar{t} < t^*$, or $\bar{s} < s^* \& \bar{t} \leq t^*$ is not a possibility.

[31]This appears to be the most likely case, since $q \in (0, 1)$.

[32] See footnote 23.

in the text, we assume that $((s,x),(t,y))$ is a unique N.E. of f. From equations (7)-(10), the equilibrium $((s,x),(t,y))$ is a function of q, i.e., $((s(q),x(q)),(t(q),y(q)))$. Therefore, the equilibrium payoffs of the parties and, hence, the total social welfare, W, are functions of q. We can write W as a function of q:

$$W(q) = u(s(q)) + v(t(q)) - s(q)x(q) - t(q)y(q) - \phi(s(q),t(q))l(x(q),y(q)).$$

Let $q = 0$. When $q = 0$, from Lemma 4 and Theorem 1, equilibrium $((s(0),x(0)),(t(0),y(0)))$ is unique and is such that $s(0) = s_p^* > s^*, x(0) = x^*, y(0) > y^*$ & $t(0) < t^*$. Furthermore, from (7)-(10) it follows that $s(0),t(0),y(0)$ satisfy the following.

$$u'(s(0)) = x^* = x(0),$$

$$v'(t(0)) = y(0) + \phi_t(s(0),t(0))l(x(0),y(0)),$$

$$t(0) + \phi(s(0),t(0))l_y(x(0),y(0)) = 0.$$

Moreover, when q is positive but very small, regardless of $s,t,$& y, for all $x > x^*$, $\parallel \frac{q\phi(s,t)}{s}l_x(x,y) \parallel < 1$. Therefore, when q increases slightly from 0, for the injurer the optimal x remains at x^*, i.e., $\frac{dx(0)}{dq} = 0$. Also, given that $\frac{dx(0)}{dq} = 0$, the only implication of an increase in q from 0 is that s is determined by $u'(s) = x^* + q\phi_s(s,t)l(x,y)$ (see (7)), where $q\phi_s(s,t)l(x,y)$ is always positive. Therefore, $\frac{ds(0)}{dq} < 0$. That is, we have

$$\frac{dx(0)}{dq} = 0; \ \frac{ds(0)}{dq} < 0.$$

Similarly, in view of $y(0) > y^*$, from (11)-(13) we have

$$\frac{dt(0)}{dq} > 0, \ \frac{dy(0)}{dq} < 0.$$

Now,
$\frac{dW(0)}{dq} = u'(s(0))\frac{ds(0)}{dq} + v'(t(0))\frac{dt(0)}{dq} - x(0)\frac{ds(0)}{dq} - s(0)\frac{dx(0)}{dq} - y(0)\frac{dt(0)}{dq} - t(0)\frac{dy(0)}{dq} - \phi_s(s(0),t(0))l(x(0),y(0))\frac{ds(0)}{dq} - \phi_t(s(0),t(0))l(x(0),y(0))\frac{dt(0)}{dq} - \phi(s(0)t(0))l_x(x(0),y(0))\frac{dx(0)}{dq} - \phi(s(0)t(0))l_y(x(0),y(0))\frac{dy(0)}{dq}$.

In view of the above equalities, $\frac{dW(0)}{dq} = -\phi_s(s(0),t(0))l(x(0),y(0))\frac{ds(0)}{dq}$. Since $\phi_s > 0$ and $\frac{ds(0)}{dq} < 0$, therefore, $\frac{dW(0)}{dq} > 0$.

Similarly, it can be shown that $\frac{dW(1)}{dq} < 0$.

Furthermore, assuming that $s, x, t, \&y$ are continuous functions of q, W is a continuous function of q on a compact domain $[0, 1]$. Therefore, there exists $q \in (0, 1)$ that maximizes the social welfare. $\sharp$

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