FIXED COSTS, THE BALANCED-BUDGET MULTIPLIER AND WELFARE

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by

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Abstract

In a two-sector model, where one of the sectors is monopolistically competitive and subject to increasing returns to scale but without love for variety, we analyze the effects of a balanced budget fiscal expansion. Such an expansion could increase the welfare of the representative individual, if elasticities of substitution in production and consumption are low. A reorganization of production takes place--increasing returns enabling a rise in real income.

JEL Classifications: E1, E2, L1.
Key Words: Fiscal Policy, New-Keynesian Models, Monopolistic Competition

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1. INTRODUCTION

In the last quarter of a century, or thereabouts, there has been an attempt to provide microeconomic foundations for Keynesian macroeconomics.\(^1\) While this endeavour has not thrown up a universally accepted alternative model, it is the view of practitioners that perfect competition and constant returns to scale—which were the cornerstones of the Walrasian system—need to be jettisoned. Accordingly, monopolistic competition—because, of the non-competitive market structures, it lends itself to general equilibrium analysis most readily—with increasing returns to scale have been accorded pride of place in the new set-up. Monopolistic competition allows firms to be price setters and increasing returns makes the size of the firm important.

In such a setting, it is easy to get Keynesian-type multipliers. There is a co-ordination problem among agents, together with the prices set by firms exceeding their marginal cost. An increase in government expenditure—even if it gives no utility i.e., is wasteful—can solve this co-ordination problem. But to get the multipliers, we require that “the range of products in the economy is fixed, and hence rents are not dissipated away by the process of entry. If unrestricted entry is possible, rents would disappear and so would the complimentarity through the multiplier process. Startz (1989), for example, argues that the multiplier should be much smaller in the long run than in the short run. However, free entry brings another source of complimentarity if the entry of new firms expands the variety of products supplied in the market.” (Matsuyama (1995) p. 709).\(^2\)

New-Keynesian models have often discussed the balanced-budget multiplier. In Mankiw (1988), Startz (1989), Pagano (1990), Matsuyama (1995), Heijdra (1998), Devereux, Head and Lapham (2000) and Coto-Martínez and Dixon (2003), a balanced-budget fiscal expansion reduces welfare (or equivalently the multiplier is less than unity in real terms), except possibly when factors of production are elastically supplied and there is increasing returns to specialization.\(^3\)

I examine the effects of a balanced-budget fiscal expansion in a two-sector non-monetary model, where one of the sectors in monopolistically competitive. In this static model,


\(^2\) With a fixed number of firms, the multiplier is obtained from the fact that price exceeds marginal cost, and, therefore, firms may be happy to provide any additional output that is demanded and would increase their profits. With free entry, this channel is blocked off and the returns to specialization argument is invoked i.e., an increased number of brands causes the price index to fall, and, depending on the exact details of the model, this either causes costs to fall (returns to specialization or the “Ethier” effect) or raises real wages because the consumption price index falls with increased variety (love for variety). Fatas (1997) shows that it is increasing returns that matters for the multiplier, and not monopolistic competition.

\(^3\) For the multiplier without love of variety see e.g., Startz (1989, p.749); Pagano (1990, section IV); Heijdra (1998, Proposition 3); Coto-Martínez and Dixon (2003, Proposition 3). Heijdra (1998) and Devereux, Head and Lapham (2000) look at welfare when there is love of variety plus elastic factor supplies (capital accumulation and elastic labour supply).
technology and (upper-tier) preferences are assumed Leontief and the monopolistically competitive sector has free entry and increasing returns to scale, which are internal to the firm.\(^4\) Factors of production are inelastically supplied and there is no love for variety (or returns to specialization). I show that in such a set up, a balanced-budget expansion could lead to a fall in the number of firms active in the monopolistically competitive sector and yet could make every individual better off. In the model, since there is no love for variety, there are “too many” firms in the initial equilibrium, and the size of the differentiated goods sector is “too small”. The fiscal expansion causes a reorganization of production, with exit taking place--this reorganization acts as a substitute for elastic labour supplies. In the new equilibrium, the welfare of a representative individual is higher.

In this paper, I seek to provide an example of a welfare-improving balanced-budget multiplier. In the process, I have assumed extreme functional forms. These can be relaxed (somewhat) and the results will still survive (see footnotes 12 and 13 for discussion of these).

2. THE MODEL

The economy has a representative consumer, firms and the government. The consumer consumes two goods—a homogeneous good (which is the numeraire) and a differentiated good--in fixed proportions. The latter is produced using an increasing-returns-to-scale technology, due to the presence of a fixed cost component. Free entry in this monopolistically competitive sector ensures zero profits. The government spends the lump-sum taxes it raises. We assume that government expenditure gives no utility to the consumer.

The representative household consumes the two goods. It maximizes the utility function in (1a) subject to the budget constraint in (1b)

\[ u \equiv \left[ V^{(\varepsilon^{-1})/\varepsilon} + y^{(\varepsilon^{-1})/\varepsilon} \right]^{\varepsilon/(\varepsilon-1)} \quad (1a) \]

\[ PV + y = Z \quad (1b) \]

where \( V \) is the (aggregate of) consumption of the differentiated good, \( P \) is the associated price index--these are defined in equations (4) and (5) below—\( y \) is the consumption of the homogeneous good, \( \varepsilon \) is the elasticity of substitution between \( V \) and \( y \), and \( Z \) is the disposable income.

We have the following demand functions

\[ V = Z \left\{ P^{\varepsilon^{-1}} / (1 + P^{1-\varepsilon}) \right\} \]

\[ y = Z / (1 + P^{1-\varepsilon}) \]

\(^4\) Returns-to-specialization renders the increasing returns external to the firm. See also Weil (1989).
We assume the elasticity of substitution between the two goods, $\varepsilon$, is zero i.e., the upper-tier utility function (in equation (1 a)) is Leontief\textsuperscript{5}, so that

\begin{align*}
V &= Z/(1 + P) \\
y &= Z/(1 + P)
\end{align*} \tag{2a, 2b}

The indirect utility function is given by

\begin{equation}
I = Z/(1 + P) \tag{3}
\end{equation}

Note that $I = y = V = Z/(1 + P)$--something that will be exploited below.

The indices V and P of the differentiated good are given by

\begin{align*}
V &= n^{1/(1-\sigma)}(\sum_{i}v_{i}^{\sigma-1}/\sigma)^{\sigma/(\sigma-1)} \tag{4} \\
P &= (n^{-1}\sum_{i}p_{i}^{1-\sigma})^{1/(1-\sigma)} \tag{5}
\end{align*}

where $v_{i}$ is the amount of the $i$\textsuperscript{th} brand consumed (whose price is $p_{i}$) and $\sigma$ (assumed to be greater than one) is the elasticity of substitution between the various brands of V--it will also be the elasticity of demand facing each firm (in equation (6) below). The number of brands, $n$, is large enough to treat it as a continuous variable. Note that in equations (4) and (5) we have ruled out any love-for-variety.\textsuperscript{6}

Given $V$ from equation (2a), the consumer allocates this over the various brands. We thus have the demand for the $i$\textsuperscript{th} brand

\begin{equation}
v_{i} = \left(p_{i}/P\right)^{-\sigma}\left(1 + P\right)^{-1}Z \quad i = 1, \ldots, n \tag{6}
\end{equation}

We shall drop the subscripts for the brands, since we shall be considering a symmetric equilibrium where all brands will be priced equally and the demand for all brands will be the same. In such an equilibrium $V=nv$ (from equation (4)), and $P=p$ (from equation (5)).

The differentiated good is produced under increasing returns to scale due to the presence of fixed costs. The factor and the homogeneous goods markets are competitive while the market

\textsuperscript{5} See footnote 12 for the general case.

\textsuperscript{6} We are following the “macroeconomic tradition” here of e.g., Kiyotaki (1988), Startz (1989), and, more recently, Sen (2002) and Coto-Martinez and Dixon (2002). Heijdra and van der Ploeg (1996), Benassy (1996) and Heijdra (1998) discuss alternative specifications involving “love for variety” or the “Ethier” effect.
for the differentiated good is monopolistically competitive with free entry—the Chamberlinian “large group” case. There are two factors of production—call these, labour and land.\(^7\)

Let us turn to the pricing decision of the firms. The homogeneous good and the variable cost component of the differentiated good are produced using constant-returns-to-scale technologies. The fixed cost is like an overhead—the input required is fixed but as the factor prices change, so do the overhead costs. We have:

\[
\begin{align*}
\alpha_{my}s + \alpha_{ly}w &= 1 \\
(a_{my}s + a_{ly}w)\sigma (\sigma - 1)^{-1} &= p \\
a_{mf}s + a_{lf}w &= \sigma^{-1} (px)
\end{align*}
\]

where \(a_{ij}\) is the amount of the input \(i\) used in “line” \(j\) (\(i = \text{land and labour}, \text{and } j = y, x \text{ and } F\)), \(w\) is the wage rate, \(s\) is the return to land and the output of a brand is given by \(x\).

Equation (7) is the price equal to cost (average and marginal) in the homogeneous good (the numeraire) production.\(^8\) The price of a brand of the differentiated good is a mark-up on variable costs (equation (8)). Free entry implies, in equation (9), \((1/\sigma)\) of total revenue must cover fixed costs (since \((1-(1/\sigma))\) goes to cover variable costs). Note that the fixed cost requires each firm to hire \(a_{lf}\) of labour and \(a_{mf}\) of land before it can start production. Therefore, the overhead costs vary with factor prices.

Equations (7), (8) and (9) are three equations in four unknowns. Hence we can solve for \(w, s\) and \(p\) as functions of \(x\)—the details are given in the Appendix.

Technology is assumed to be Leontief in all sectors i.e., the \(a_{ij}\)’s are constants (see footnote 13 for a discussion of the general case). We assume the following factor intensities: \(a_{lx}/a_{mx} > a_{lf}/a_{mf} > a_{ly}/a_{my}\) i.e., the homogeneous good is the most land-intensive and the variable cost component is the most labour-intensive—think of the homogeneous (resp. differentiated) good as an agricultural (resp. “manufactured”—literally “made by hand”) product.

National income is equal to factor earnings—\(Z \equiv w+s\). We will look at an experiment where, starting from an initial level of zero, the government adopts a balanced budget policy with lump-sum taxes equaling its expenditure (represented by \(G\)). The government expenditure is “wasteful” i.e. ‘affects neither utility nor production.

\(^7\) This is a static model so not much hinges on which input is called what. As a matter of fact, if we reverse the factor intensities assumed below, we could still obtain the multipliers that are derived in this paper. In a dynamic model, it does matter which factor can be accumulated and which cannot be.

\(^8\) Hornstein (1993) and Heijdra (1998) allow for variable returns-to-scale in the marginal cost component. In this paper, the fixed cost is the only element giving rise to increasing returns (which are internal to the firm)
There are four markets--two factor markets and two goods markets. By Walras’ Law we need to consider only three market-clearing conditions. Equations (10), (11) and (12) below give the market-clearing condition for the homogeneous good, labour and land (factors are supplied inelastically and these supplies are normalized to unity). Equation (10) incorporates the balanced budget.

\[ Y = \frac{w + s - G}{1 + P} \]  

(10)  

\[ a_{my} Y + a_{nx} nx + a_{mf} n = 1 \]  

(11)  

\[ a_{iy} Y + a_{ix} nx + a_{if} n = 1 \]  

(12)  

where \( Y \) (resp. \( nx \)) is the output of the homogeneous (resp. differentiated) good. From equations (10) to (12) we can solve for \( Y, x \) and \( n \) (after substituting for \( w, s \) and \( p \) from (7) to (9)) in terms of \( G \) (the details are given in the Appendix). Note here we can solve for these variables and have full employment of factors, even though technologies in the various sectors are Leontief.

3. A BALANCED-BUDGET INCREASE IN GOVERNMENT EXPENDITURE

Suppose now the government increases its expenditure--which does not give utility--by \( G \), from an initial level of zero, financed by lump-sum taxes. Let all of the increased government demand (which is fixed at \( G \) in terms of the numeraire) be directed towards the differentiated good and also let the elasticity of substitution in government consumption be \( \sigma \) (so that its demand is given by a relation like equation (6)).

\[ \Omega = \begin{pmatrix} \lambda_{ly} - \lambda_{my} \\ \lambda_{ix} - \lambda_{y} \\ \lambda_{my} - \lambda_{mx} \end{pmatrix} \]

To derive the welfare effects, we recall from equations (2) and (3) that \( V = y = I = \frac{Z}{1+P} \) i.e., if \( y \) increases, so does \( V \). We have from equations (10), (11) and (12) (a hat above a variable denotes a percentage change)

\[ \frac{\hat{I}}{dG} = \frac{\hat{Y}}{dG} = \left( \begin{pmatrix} \hat{n} + \hat{v} \end{pmatrix} \right) / dG = \begin{pmatrix} \Omega Z \end{pmatrix}^{-1} \begin{pmatrix} \lambda_{ly} \lambda_{mf} - \lambda_{mx} \lambda_{if} \end{pmatrix} > 0 \]  

(13)  

\[ \frac{\hat{x}}{dG} = \left\{ 1 / (Z \Omega) \right\} \left[ \lambda_{ly} - \lambda_{my} \right] > 0 \]  

(14)  

\[ \frac{\hat{n}}{dG} = -\left\{ 1 / (Z \Omega) \right\} \left[ \lambda_{ix} \lambda_{mx} - \lambda_{my} \lambda_{tx} \right] < 0 \]  

(15)  

In signing the expressions in (13) and (14), \( \Omega \equiv \begin{pmatrix} \lambda_{ly} \lambda_{mf} - \lambda_{mx} \lambda_{if} \end{pmatrix} + \begin{pmatrix} \alpha (\lambda_{ly} - \lambda_{my}) / \sigma \end{pmatrix} \) is assumed negative--\( \alpha \) is the share of the differentiated good in total expenditure and \( \lambda_{iy} \) is the

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9 In equilibrium the output per brand, \( x \), is equal to the demand per brand, \( v \).
10 See Heijdra and van der Ploeg (1996) for a model with the provision of a public good by the government in a model with monopolistic competition.
11 Otherwise a change in the composition of demand changes the elasticity of demand facing a firm. See Solow (1986) and, especially, Gali (1994) for interesting applications.
share of the sector j in the employment of input i (in the initial steady state). For example, \( \Omega < 0 \) is true for the following parameter values: \( \lambda_{ix} = 0.6, \lambda_{IF} = 0.3, \lambda_{mx} = 0.08, \lambda_{mF} = 0.12, \alpha = 0.6, \sigma = 4. \) It is interesting to note that the rise in welfare occurs, under our factor intensity assumption, when: (a) the share of the differentiated good is high; and (b) the elasticity of substitution between brands (or, of demand) is low.

Thus, in real terms, following the balanced budget increase in G (which is directed towards the differentiated good), we have a multiplier that exceeds unity. This because the right hand side of equation (13) is the change in “real income” i.e.,

\[ \hat{Z} - \alpha \hat{P} > 0 \]

To get some intuition, let us look at this diagrammatically. Equation (10) (or its log-linearized version (A.8)) tells us that for the homogeneous goods market to clear, an increase in Y will cause real income \((w + s)/(1 + P)\) to rise (for a given level of G). Now real income is increasing in x, so output per brand must rise. This is shown by the YY curve in figure 1, whose equation is

\[ YY: \quad d \log Y = [\alpha \sigma^{-1}] d \log x - Z^{-1} dG \quad (16) \]

Equations (11) and (12) can be solved for Y in terms of x (i.e., by eliminating n)--these are the combinations of Y and x which are consistent with full employment in both the factor markets--the FF curve in figure 1. Under our assumption about factor-intensities (and zero elasticities of substitution in production), the FF curve is upward sloping and, if \( \Omega < 0, \) is flatter than the YY curve.

\[ FF: \quad d \log Y = [(\lambda_{ix} \lambda_{mF} - \lambda_{mx} \lambda_{IF})/(\lambda_{my} - \lambda_{iy})] d \log x \quad (17) \]

A rise in G shifts the YY line down, and in the new equilibrium (at point B--the initial equilibrium was at A) raises both Y and x. The mechanism at work is as follows: the fiscal expansion raises demand for the differentiated good. This is achieved by a rise in the output per firm--the resources for this are found by a contraction in the number of firms i.e., exit takes place. Now, given our factor-intensity assumption, the fixed cost component has a land-labour intensity that lies between the variable cost component and the homogeneous good. Thus, as the fixed cost component has a land-labour intensity that lies between the variable cost component and the homogeneous good. Thus, as the

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12 For the general CES upper-tier utility function equation (10) would become:

\[ d \log Y = \left(\alpha \sigma^{-1} + \alpha \varepsilon (\theta_{iy} - \theta_{ix}) \lambda^{-1} \right) d \log x - Z^{-1} dG. \]

As \( \varepsilon \) rises, from its assumed value of zero, the YY curve becomes flatter, finally becoming negatively sloped.

13 In the general case, the FF curve is given by:

\[ d \log Y = \left(\lambda_{ix} \lambda_{mF} - \lambda_{mx} \lambda_{IF}\right) + \Delta \left(1 - \lambda_{my} \sum_{i} \lambda_{my} \theta_{mi} + \sum \lambda_{mF} \theta_{m} \right) d \log x \]

This becomes steeper, for our assumed factor intensities, as the elasticities in production, the \( \theta_{i}^{-1} \)’s increase.

14 Contrast this with e.g., Devereux, Head and Lapham (1996), where all changes in the output of the differentiated goods sector is achieved by entry and exit, with output per firm remaining constant.
demand for overhead inputs fall, the excess land goes to increase the supply of the homogeneous good. Since both the factors of production in our model are inelastically supplied, if a “sector” has to expand, another has to contract. Here that is achieved through an exit—nx rises as n falls. Further with increasing returns, as x (and nx) rises, p falls. The “small” elasticities of substitution ensure that there are “larger” changes in prices (including those of the factors of production).  

Note that with a fixed number of firms there is no multiplier—equations (11) and (12) determine Y and x (and hence nx) independent of demand. Thus in our model the free entry multiplier is greater than the multiplier with a fixed number of firms, the latter being zero.  

Note this is in spite of our having switched off the “love-of-variety” channel.

In the industrial organization literature there has been some discussion of the issue of production versus allocative efficiency (see e.g., Mankiw and Whinston (1986), Suzumura and Kiyono (1987). In particular, suppose in an oligopolistic set-up, each firm has to incur significant fixed costs to produce, then from a social perspective, the fewer the number of firms the better. But this could make each incumbent firm large in relation to the market and this aspect is welfare-reducing. This result is obtained in partial equilibrium models. In my model, which is a general equilibrium one, the second channel is missing (by assuming a very large number of firms) and thus fiscal policy operating through the first channel increases welfare.

Thus an expansionary fiscal policy by expanding the size of the differentiated goods sector and exploiting the scale economies (or equivalently reducing the fixed cost from society’s point of view), raises welfare. Indeed, through the reduction of resources required for fixed costs, it is able to increase the supply of both the goods—remember that the goods are consumed in fixed proportions.

4. CONCLUSIONS

I have shown that in a monopolistically competitive economy with increasing returns to scale, welfare rises following a balanced-budget fiscal expansion. The assumed low elasticities of substitution in production and consumption require large changes in prices to equilibrate markets following a shock. Increasing returns ensure that costs in the differentiated goods sector fall as output rises. Exit occurs from the imperfectly competitive sector but this does not affect

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15 Note that we do not literally require either $\epsilon$ or the $\omega_i$'s to be zero—there are a range of “low” values of these which will give us YY steeper than FF. See Solow (1986, p. 312) for a discussion of elasticity of demand and the value of the multiplier in his model.

16 Note that since the price of the differentiated good falls, the multiplier in terms of the homogeneous good may not be positive.

17 This is contrary to received wisdom. See the quote from Matsuyama (1995) in the Introduction; also see Startz (1989), and, more recently, Coto-Martinez and Dixon (2003, Proposition 4). Note that in that case, in my model the assumed zero elasticity-of-substitution in demand would leave some homogeneous good unsold.

18 In particular, see Mankiw and Whinston (1986, section 4) for a discussion of a set-up with love-for-variety and excess (or insufficient) entry.
welfare directly because there is no love-for-variety per se.\textsuperscript{19} The multiplier is obtained from a reorganization of production i.e., there are fewer firms with more output per firm, saving on wasteful overhead costs while allowing for an increase in the homogeneous goods production as well.

The purpose of this paper was to provide an example-- the first as far as I am aware--of a new-Keynesian model where the balanced-budget multiplier exceeding unity without relying on a love-for-variety (or returns to specialization) effect.\textsuperscript{20} In so doing I had to make some very strong assumptions, which will need to be addressed in future work. A prime example of that is the fact exit occurs in the process of generating the multiplier.\textsuperscript{21} This could be “fixed” e.g., by introducing factors whose supplies are elastic.

\textsuperscript{19} A love-for-variety would dampen the process at work but would not necessarily overturn the results.

\textsuperscript{20} See Thomas (1995), for an incomplete markets model, where a similar multiplier can be obtained. His model, though, is dynamic, has labour-leisure choice and positive profits.

\textsuperscript{21} See Aloi and Dixon (2002) for a discussion of entry in the expansionary phase of the cycle, especially the empirical regularities reported on pp. 4 and 5 there.


APPENDIX

Logarithmically differentiating equations, (7), (8) and (9), we have (a “hat” over a variable denotes a percentage change e.g., \( \hat{x} = dx / x \))

\[
\begin{align*}
\theta_{ly} \cdot \hat{w} + \theta_{my} \cdot \hat{s} &= 0 \quad (A.1) \\
\theta_{lx} \cdot \hat{w} + \theta_{mx} \cdot \hat{s} &= \hat{p} \quad (A.2) \\
\theta_{lf} \cdot \hat{w} + \theta_{mf} \cdot \hat{s} &= \hat{p} + \hat{x} \quad (A.3)
\end{align*}
\]

where \( \theta_{ij} \) is the share of the \( i^{th} \) input in the \( j^{th} \) “cost” equation (e.g., \( \theta_{lf} \equiv a_{lf} w/(\sigma^{-1}(p.x.)) \)).

We can solve the above three equations for \( \hat{w}, \hat{s} \) and \( \hat{p} \) in terms of \( \hat{x} \). These are given in equations (A.4), (A.5) and (A.6) below

\[
\begin{align*}
\hat{w} / \hat{x} &= -\theta_{my} / \Delta \quad (A.4) \\
\hat{s} / \hat{x} &= \theta_{ly} / \Delta \quad (A.5) \\
\hat{p} / \hat{x} &= (\theta_{ly} - \theta_{lx}) / \Delta \quad (A.6)
\end{align*}
\]

where \( \Delta \equiv \theta_{lx} - \theta_{lf} \) (A.7)

Then logarithmically differentiating the goods market equilibrium and the two factor market-clearing equation ((10), (11) and (12)) (and substituting from (A.4) to (A.7)), we can solve for \( \hat{Y}, \hat{n} \) and \( \hat{x} \) in terms of dG.

\[
\begin{align*}
\hat{Y} &= \Delta^{-1}[(\eta_{m} \theta_{ly} - \eta_{l} \theta_{my}) - \alpha(\theta_{ly} - \theta_{lx})] \hat{x} - Z^{-1}.dG \quad (A.8) \\
\lambda_{ly} \hat{Y} + \lambda_{lx} \hat{x} + (1 - \lambda_{ly}) \hat{n} &= 0 \quad (A.9) \\
\lambda_{my} \hat{Y} + \lambda_{mx} \hat{x} + (1 - \lambda_{my}) \hat{n} &= 0 \quad (A.10)
\end{align*}
\]

Here \( \eta_{i} \) is the share of the \( i^{th} \) input in national income and \( \lambda_{ij} \) is the share of the \( j^{th} \) sector in the employment of the \( i^{th} \) input. The term multiplying \( \hat{x} \) (call it H) in (A.8) is equal to \( \alpha / \sigma \). To see this

\[
H \equiv \Delta^{-1}[(\eta_{m} \theta_{ly} - \eta_{l} \theta_{my}) - \alpha(\theta_{ly} - \theta_{lx})] = \Delta^{-1}[\alpha \theta_{lx} - \eta_{l} + (1 - \alpha) \theta_{ly}]
\]
Now this is \( \eta_i \equiv w(a_{ij} Y + a_{ik} n x + a_{if} n) = (1 - \alpha) \theta_{j} + \{ \alpha \theta_{ix} (\sigma - 1) / \sigma \} + \alpha \theta_{if} / \sigma \)

Substituting in the expression for \( H \), and remembering that \( \Delta \equiv \theta_{ix} - \theta_{if} \), we get \( H = \alpha / \sigma \) (because, \( \theta_{ix} \equiv wa_{ix} / (wa_{ix} + sa_{mi}) \)) and \( \theta_{is} \equiv wa_{if} / (wa_{if} + sa_{mi}) \).