CDE
November, 2008

COMPARATIVE VIGILANCE

Allan M. Feldman
Email: Allan_Feldman@Brown.edu
Department of Economics, Brown University
Providence, RI 02906 USA

Ram Singh
Email: ramsingh@econdse.org
Delhi School of Economics
University of Delhi

Working Paper No. 173

Centre for Development Economics
Department of Economics, Delhi School of Economics

Comparative Vigilance

Allan M. Feldman
Department of Economics, Brown University
Providence, RI 02906 USA
Allan_Feldman@Brown.edu
http://www.econ.brown.edu/fac/allan_feldman

Ram Singh
Department of Economics
Delhi School of Economics
University of Delhi
Delhi 110007, India
ramsingh@econdse.org
http:www.econdse.org/faculty-frame.htm

August 8, 2008

Abstract

A growing body of literature suggests that courts and juries are inclined toward division of liability between two strictly non-negligent or "vigilant" parties. However, standard models of liability rules do not provide for vigilance-based sharing of liability. In this paper, we explore the economic efficiency of liability rules based on comparative vigilance. We devise rules that are efficient and that reward vigilance. It is commonly believed that discontinuous liability shares are necessary for efficiency. However we develop a liability rule, which we call the "super-symmetric rule," that is both efficient and continuous, that is based on comparative negligence when both parties are negligent and on comparative vigilance when both parties are vigilant, and that is always responsive to increased care. Moreover, our super-symmetric rule divides accident losses into two parts: one part creates incentives for efficiency; the other part provides equity.

Keywords: Comparative vigilance, equity, economic efficiency, tort liability rules, Nash equilibrium, social costs, pure comparative vigilance, super-symmetric rule

JEL classification: K13, D61.

1 INTRODUCTION

Tort liability rules determine how accident losses are distributed among the parties involved. The economic analysis of liability rules has typically considered accidents resulting from the risky activities of two parties. This analysis has led to characterizing some rules as efficient and others as inefficient. Models used in economic analysis have imposed certain conditions on the structure of liability rules, some of which vary from rule to rule, some of which do not. The rules that have been proven to be efficient generally satisfy the following three conditions:

(A) When one party is negligent and the other is not, the negligent party bears all the accident loss;

(B) When both parties are non-negligent, the liability shares do not depend on the degrees of "vigilance" shown by the parties, that is, the care levels above and beyond what is efficient.

(C) When both parties are non-negligent, all the accident loss falls on just one party.[1]

While mainstream economic analysis continues to rely on these three conditions, they have been criticized by legal scholars. In this paper, we analyze the implications of relaxing these conditions.

Various scholars have criticized conditions B and C above.[2] Condition C says that when both parties are non-negligent, one and only one party is liable for the entire loss; which party it is depends on the rule in force.[3] Calabresi and Cooper (1996) and Parisi and Fon (2004) have argued that when parties are either both negligent, or both non-negligent, equity considerations demand sharing of liability - making only one party bear all the loss is not justified. Calabresi and Cooper (1996) and Honoré (1997) have recommended proportionate or

[1] See the modeling of liability rules in Brown (1973), Diamond (1974), Polinsky (1989), Landes and Posner (1987), Shavell (1987), Barnes and Stout (1992), Posner (1992), Levmore (1994), Kaplow (1995), Biggar (1995), Miceli (1997), Cooter and Ulen (1997), Feldman and Frost (1998), Jain and Singh (2002), Kim (2004), Kim and Feldman (2006), and Singh (2007), among others.

[2] For criticism of the modeling of liability rules on various grounds including these two assumptions, see Grady (1989), Kahan (1989), Marks (1994), Burrow (1999) and Wright (2002).

[3] This party is the victim under the rules of negligence, negligence with a defense of contributory negligence, negligence with a defense of comparative negligence. Under the rule of strict liability with a defense of contributory negligence, however, the injurer bears the entire loss.

comparative apportionment of accident losses in such instances.[4] These scholars have argued that judicial decision making is influenced by equity considerations. Moreover, some studies have shown that courts and juries are inclined toward comparative apportionment of losses when both parties are negligent *and* when both are non-negligent. This phenomenon has been observed in several countries, including France, Germany, Japan and U.S. (See Calabresi and Cooper (1996), Yoshihsa (1999), Grimley (2000), Yu (2000) and Parisi and Fon (2004).) In India, special legislation requires sharing of certain types of accident losses when both injurer and victim are non-negligent.[5]

Condition A requires that all the loss fall on the negligent party when one party is negligent and the other is not. Kahan (1989) and Honoré (1997) have argued that this is not consistent with the doctrine of "causation." (Also see Singh (2007).). Under a standard liability rule like simple negligence, condition A implies that for at least one party, the liability share jumps abruptly from zero to one hundred percent, as that party goes from just meeting his standard of care, to just falling short of meeting that standard.[6] Kahan (1989) and Grady (1989), though on different grounds, have argued that this striking discontinuity is not part of the functioning of the law of torts.[7]

In short, there is a body of literature arguing that some of the basic assumptions of economic models are neither desirable nor in keeping with the reality of judicial decision making. Nonetheless, the usual economic modeling of liability rules continues to incorporate conditions A, B and C. The widespread use of these conditions is not surprising, because they have helped us to make precise and robust statements about the characteristics of various liability rules, including their efficiency properties. Parisi and Fon (2004) have remarked that the discontinuity in liability shares (implied by condition A) provides strong incentives for the parties to opt for efficient care levels. Jain and Singh (2002), Kim (2004), and Singh (2006)

[4] The only rule that allows loss sharing is the rule of comparative negligence. However, under this rule, loss sharing takes place only when both parties are negligent; not when both parties are non-negligent. For an analysis of this rule see Schwartz, (1978), Landes and Posner (1980), Cooter and Ulen (1986), Haddock and Curran (1985), Rubinfeld (1987), and Rea (1987). For a critical review of some of these works see Liao and White (2002), and Bar-Gill and Ben-Shahar (2003).

[5] Sections 140 and 163 A of Motor Vehicle Act (1988) require that in instances of death and physical injuries, the victim be partially compensated if both parties were non-negligent.

[6] Note that if the victim is non-negligent, under the rules of negligence, negligence with a defense of contributory negligence and negligence with a defense of comparative negligence, the injurer's liability jumps from nil (when he is non-negligent) to full liability (when he is negligent). Similarly, under the rule of strict liability with a defense of contributory negligence, there is drastic jump in the victim's liability.

[7] See Mark (1994) and Miceli (1996) for commentary on Grady (1989).

have shown that these 3 conditions play a large role in efficiency analysis. But, as we shall see below, the conditions can be relaxed along the lines suggested by legal scholars without sacrificing efficiency.

In this paper we will develop liability rules that are 'equitable' as well as efficient. When both parties are negligent (that is, taking less than the efficient amount of care), one party's degree of negligence is defined by her shortfall in care divided by the combined shortfalls in care of the two parties. We think that when both parties are negligent, an equitable liability rule should have liability shares that are *generally increasing with degrees of negligence*. When both parties are non-negligent (taking at least the efficient amount of care) and at least one is vigilant (taking more than the efficient amount of care), each party's degree of vigilance can be defined as her excess of care divided by the combined excesses of care of the two parties. We think that when both parties are vigilant, an equitable liability rule should have liability shares that are *generally decreasing with degrees of vigilance*. We also believe liability shares should vary continuously over all possible combinations of care levels. And, of course, we want our rules to result in efficient outcomes. That is, social-cost-minimizing care levels for the two parties should always be Nash equilibria, and there shouldn't be other (inefficient) Nash equilibria.

In section 2 of this paper we lay out our model and most of our definitions.

In section 3 we briefly discuss pure comparative negligence, which, when both parties are negligent, makes each party's liability share equal to her degree of negligence. As is well known, under pure comparative negligence the efficient combination of care levels of the two parties is a Nash equilibrium (theorem 1) and there are no other Nash equilibria (theorem 2). We then turn to a rule which closely parallels pure comparative negligence, but which is defined when both parties are non-negligent and at least one is vigilant. This is pure comparative vigilance; it makes each party's liability share equal to 1 minus her degree of vigilance. Pure comparative vigilance and its relatives discussed in section 3 depart from conditions B and C above. We show that the efficient pair of care levels is *not* a Nash equilibrium under pure comparative vigilance (theorem 3), and we construct an example to show that under pure comparative vigilance, there may be inefficient Nash equilibria which involve both parties taking too much care.

The negative results for pure comparative vigilance are interesting when compared with the efficiency results (theorems 1 and 2) for pure comparative negligence. Under both pure comparative vigilance and pure comparative negligence, by increasing her care a party unilaterally reduces her liability share. Why does this result in efficiency for pure comparative negligence but inefficiency for pure comparative vigilance? The explanation is quite simple: when both parties are negligent, the "equity" consideration (reward more care) and the efficiency consideration (efficiency requires more care by both parties) are aligned. In contrast, when both parties are taking too much care, under pure comparative vigilance, the "equity" consideration (reward more care) and the efficiency consideration (the parties are already taking more care than they should) are opposed.

We then turn to modified versions of comparative vigilance. We construct a rule that is analogous to comparative negligence with fixed division, that is, with fixed liability shares, and we show (theorems 4 and 5) that our rule of comparative *vigilance* with fixed division has the desired efficiency properties. Comparative vigilance with fixed division departs from condition C above.

However, comparative vigilance with fixed division shows undesirable discontinuities as the care levels of the parties fall just below their efficient levels. (In this respect it is like the traditional rules of negligence, negligence with comparative negligence, and so on.) Moreover we find the fixed liability shares unappealing; why should the division of damages be the same when the first party is extremely careful and the second party barely meets her standard of care, as it is when the first party barely meets her standard and the second party is extremely careful?

In section 4 of the paper we develop a new rule which (1) has liability shares that are continuous over all combinations of care levels, (2) is based on the idea of comparative negligence when both parties are negligent and on the idea of comparative vigilance when both parties are vigilant, (3) treats comparative negligence and comparative vigilance symmetrically, and (4) allows increased care to reduce liability shares even when one party is negligent and the other party is non-negligent. This rule departs from all three of the traditional conditions A, B and C. And yet, the rule is efficient: the efficient care combination is a Nash equilibrium (theorem 7), and there is no Nash equilibrium in the region where both parties take too much care (theorem 6) or too little care (theorem 8). We call the new rule the

"super-symmetric comparative negligence and vigilance rule," or the "super-symmetric" rule for short.

Mainstream economic models of liability rules require sharp discontinuities in liability share; our super-symmetric rule does not. Mainstream models lean on conditions A, B and C; our super-symmetric rule drops all three. Finally, our super-symmetric rule has one more feature that sets it off: Standard tort liability rules allocate *total* expected accident costs between the two parties. Our super-symmetric rule divides expected accident costs into two parts: a *fixed* component equal to the party's expected costs at the efficient care point, and a *variable* component equal to the rest. Our liability rule apportions only the *variable* component according to comparative negligence, comparative vigilance, and so on. This is the mathematical key to several of our results.

.

2 DICHOTOMOUS AND MIXED LIABILITY RULES

Model Preliminaries

We assume there are two risk-neutral people, *X* and *Y*, who engage in some activity that creates a risk of accidents. An accident is an unintended and unforeseen bad outcome. If an accident occurs, there is one victim, who sustains a monetary loss $L > 0$, and one injurer, who sustains no loss. We assume that *X* is the injurer and *Y* is the victim. For simplicity, the loss L is assumed to be constant.

Let x and y denote person *X*'s and person *Y*'s care levels, respectively, measured by their care expenditures. Following the standard modeling in tort liability literature since Brown (1973), we assume that each person can choose any level of care between 0 and ∞, and that the probability of an accident, $p(x, y)$, is a continuous and differentiable function of x and y. We assume that increasing care levels reduce the probability of an accident, and thus expected accident costs ($p_x < 0$, and $p_y < 0$). We also assume that, for all *x* and *y*, $p(x, y) > 0$, $p_{xx} > 0$, $p_{yy} > 0$, and $p_{xy} > 0$.

Total social cost (*TSC*) is defined as the sum of care-taking costs of both parties and expected accident costs. That is, $TSC = x + y + p(x, y)L$. The social goal is, as usual, to minimize total social cost. Let (x^*, y^*) denote the solution to the *TSC* minimization problem.

We will assume for the sake of clarity in this paper that (x^*, y^*) is unique. We call (x^*, y^*) the *efficient* care combination.

Negligence-Based Liability Rules

In our model, when an accident occurs the entire loss L is initially born by the victim *Y*. The court then enforces a *liability rule*, which determines where L ultimately falls: on the victim, on the injurer, or on both according to some mixing rule. A negligence-based liability rule is defined in terms of which parties are negligent. If both are negligent, it may be defined in terms of the degree to which they are negligent; if both are non-negligent, it may be defined in terms of the degree to which they are vigilant. A party is *negligent* if her care expenditure is *less* than the court-enforced *standard of care*. A party is *non-negligent* if her care expenditure is *greater than or equal to* the standard of care. A party is *strictly non-negligent*, or *vigilant*, if her care expenditure is *greater* than the standard of care. We assume that everyone, including the court, knows the expected loss function and the governing liability rule, that the court can solve the *TSC* minimization problem, and that everyone, including the court, can observe each party's care level accurately.

Finally, we assume that the standard of care for each party is set at the efficient level. That is, for instance, party *X* is found *negligent* by the court if and only if she spends $x < x^*$. This is an especially basic assumption, and *we will use it throughout* this paper. We call it:

> *Axiom 1*: The court sets the standards of care at the efficient levels (x^*, y^*). That is, party *X* (or *Y*) is *negligent* if and only if $x < x^*$ (or $y < y^*$).

For the related terminology: Party *X* (or *Y*) is *non-negligent* if $x^* \le x$ (or $y^* \le y$). Party *X* (or *Y*) is *strictly non-negligent*, or *vigilant*, if $x^* < x$ (or $y^* < y$).

Typically, legal rules only allow a court to distribute the loss *L* between the two parties, that is, to assign each party a fraction of the loss, greater than or equal to zero, with the two fractions summing to one. Such rules will be called normal. With a normal rule, a party in a lawsuit ends up with some loss between 0 and *L*, and the sum of the losses falling on the two parties equals *L*. Non-normal rules allow the fractions falling on the two parties to be negative

or greater than one (e.g., punitive damages), or to sum to numbers other than one (e.g., with the rest of society picking up some of the losses; or with both parties paying *L*, the "both pay full costs" rule).

Formally, a *normal negligence-based liability rule* examines the actual behavior of both parties, plus the efficient behavior of both parties, plus *L*, plus the probability function $p(x, y)$, and then enforces a normal loss allocation of the accident loss *L* between the parties. A *normal loss allocation* is a vector of weights (w_X, w_Y), where $0 \le w_X, w_Y \le 1$, $w_X + w_Y = 1$, w_X = the fraction of the loss the liability rule places on party *X*, and w_Y = the fraction of the loss the liability rule places on party *Y*. In this paper we focus on normal liability rules.

We will use $(\overline{w}_X, \overline{w}_Y)$ to represent liability weights at the efficient point (x^*, y^*), and we assume $0 \le \overline{w}_X, \overline{w}_Y \le 1$ and $\overline{w}_X + \overline{w}_Y = 1$.

Negligence-based liability rules are usually defined by specifying how they operate in 4 different domains: *Domain 1*, where both parties are non-negligent; *domain 2*, where party *X* is negligent and party *Y* is non-negligent; *domain 3*, where both parties are negligent; and *domain 4*, where party *Y* is negligent and party *X* is non-negligent. (See figure 1 below.) Because *X* is non-negligent if and only if $x^* \le x$, and similarly for *Y*, domain 1 is the set $\{(x, y) \,|\, x^* \le x \,\&\, y^* \le y\}$. It will often be necessary to partition this set into the point (x^*, y^*), and the rest (where at least one person is strictly non-negligent, or vigilant). Domain 2 is the set $\{(x, y) \,|\, x < x^* \,\&\, y^* \le y\}$ (that is, *X* negligent and *Y* non-negligent), and domain 4 is $\{(x, y) \,|\, x^* \le x \,\&\, y < y^*\}$ (that is, *X* non-negligent and *Y* negligent). Domain 3 is the set $\{(x, y) \,|\, x < x^* \,\&\, y < y^*\}$. We call a liability rule *dichotomous in a domain* if it places all costs on just one party in that domain. We call a liability rule *dichotomous* if it is dichotomous in all 4 domains. For a normal dichotomous liability rule in a domain, (w_X, w_Y) must equal either $(0,1)$ or $(1,0)$. For example, the familiar real world negligence-based liability rules are dichotomous in domains 2 and 4, where one and only one party is negligent; and they place all the losses on the negligent party in those domains. (This is condition A from section 1.) Such rules include simple negligence, negligence with a defense of contributory negligence,

negligence with a defense of comparative negligence, strict liability with a defense of contributory negligence, and so on.

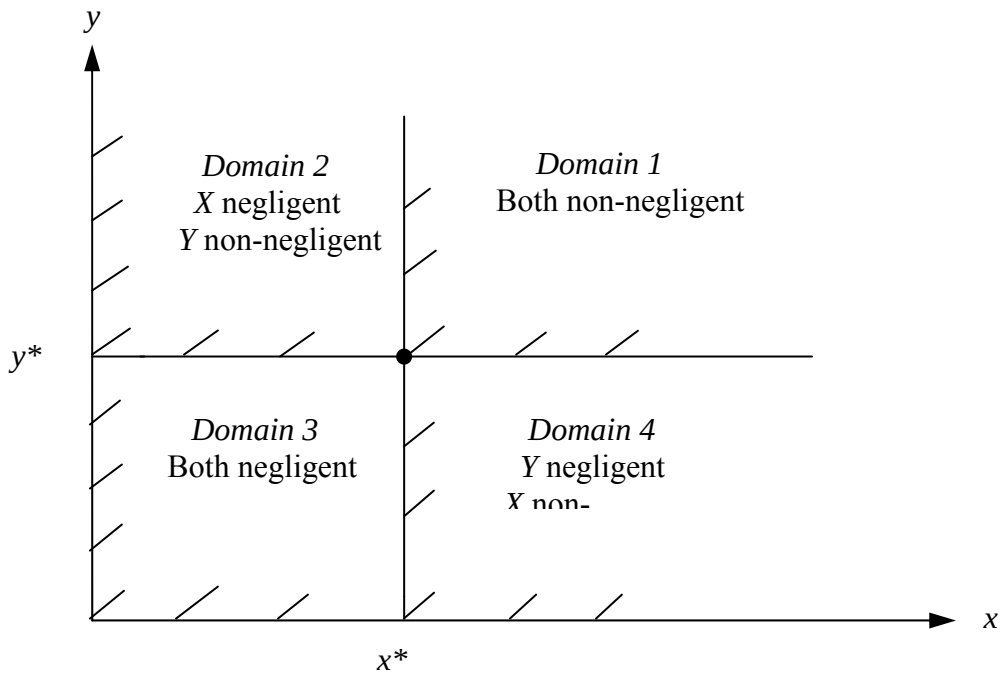

Figure 1. The Four Domains. Note the slanted lines show which domains contain which domain boundaries.

We now restate condition A from section 1, and we will call it an axiom here because of its importance in the real world. We use this assumption in section 3 of this paper, but we *will drop it* in section 4, when we develop our super-symmetric rule.

Axiom 2: In domains 2 and 4, all losses fall on the negligent party. That is, if $x < x^* \;\&\; y^* \le y$, then $(w_X, w_Y) = (1,0)$, and if $x^* \le x \;\&\; y < y^*$, then $(w_X, w_Y) = (0,1)$.

In domain 1, where both parties are non-negligent, the real world liability rules are dichotomous; that is they place all the losses on just one party. Moreover, that party remains fixed, no matter what the degrees of vigilance of the two parties might be. The following axiom formalizes this idea, and is both simpler and logically stronger than the related conditions B and C from section 1 above. (That is, it implies both.) Although axiom 3 (in

combination with axioms 1 and 2) has played an important role in efficiency characterizations of liability rules in the literature (see Jain and Singh (2002)), *we will drop it throughout* this paper.

Axiom 3: In domain 1, either $(w_X, w_Y) = (1,0)$ for all $x^* \le x$ & $y^* \le y$, or $(w_X, w_Y) = (0,1)$ for all $x^* \le x$ & $y^* \le y$.

Mixed Rules (or Fuzzy Rules)

We call a liability rule *mixed in a domain* if it places non-zero costs on both parties at some points in that domain. For example, pure comparative negligence is mixed in the both parties negligent domain, i.e., domain 3. We call a liability rule *mixed* if it is mixed in at least one domain.

We use the following convenient notation throughout this paper: For any x and any y, let $\Delta x = x - x^*$ and $\Delta y = y - y^*$. Note that Δx (or Δy) can be positive, zero, or negative, depending on whether *X* (or *Y*) is vigilant, is at the optimum care level, or is negligent, respectively.

First, suppose both parties are negligent (domain 3). If both parties are negligent, then $\Delta x = x - x^* < 0$ and $\Delta y = y - y^* < 0$. Note that $-\Delta x = x^* - x$ is the dollar measure of party *X*'s shortfall from her proper (efficient) degree of care, and similarly with $-\Delta y$ and party *Y*. The most natural way to define party *X*'s relative degree of negligence, or degree of fault, is set it equal to the ratio of *X*'s shortfall in care to the sum of the shortfalls of the two parties. That is, *X*'s *degree of negligence (or degree of fault)* is $\frac{-\Delta x}{-\Delta x - \Delta y} = \frac{\Delta x}{\Delta x + \Delta y}$. Similarly, party *Y*'s degree of negligence (or degree of fault) is $\frac{\Delta y}{\Delta x + \Delta y}$. Under a *pure comparative negligence* liability rule, the fractional parts of losses that fall on the respective parties are set equal to these respective factors. That is, $w_X = \frac{\Delta x}{\Delta x + \Delta y}$ and $w_Y = \frac{\Delta y}{\Delta x + \Delta y}$. Note that $w_X + w_Y = 1$.

Second, suppose both parties are non-negligent (domain 1). Then $\Delta x = x - x^* \ge 0$ and $\Delta y = y - y^* \ge 0$. Can we define a mixed liability rule in this domain, with the same logic

as is used for the rule of pure comparative negligence in the both parties negligent domain? The answer is: Yes, easily.

For this purpose we will require that at least one of the inequalities be strong, in order to avoid having a zero in the denominator of a fraction. That is, we are assuming at least one party is strictly non-negligent, or vigilant.

In the analysis of pure comparative negligence, party *X*'s relative degree of negligence is defined as her shortfall in care divided by the sum of the two parties' shortfalls in care. Following Calabresi and Cooper (1996) and Parisi and Fon (2004), in the both parties *non-negligent* domain, we define party *X*'s relative degree of vigilance as her excess of care divided by the sum of the two parties' excesses of care. (Parisi and Fon use the term "diligence" instead of "vigilance.") That is, party *X*'s *degree of vigilance* is $\frac{\Delta x}{\Delta x + \Delta y}$. Similarly, party *Y*'s degree of vigilance is $\frac{\Delta y}{\Delta x + \Delta y}$. Note that the algebraic expressions for degrees of vigilance in domain 1 are identical to the expressions for degree of negligence in domain 3.

In the both parties negligent domain, for the pure comparative negligence liability rule, we let party *X*'s liability weight w_X equal her degree of negligence. In the both parties non-negligent domain, we obviously don't want party *X*'s liability weight to be equal to her degree of *vigilance*; the idea is to *reward* vigilance rather than punish it. What makes sense is for *party X*'s vigilance to add to *party Y*'s liability weight. The straightforward way to do this is to assume that the fractional part of losses falling on party *X* equals 1 minus her degree of vigilance, and similarly for party *Y*.

That is, when both parties are non-negligent (with at least one party strictly non-negligent), *X*'s share of the loss equals $w_X = 1 - \frac{\Delta x}{\Delta x + \Delta y} = \frac{\Delta y}{\Delta x + \Delta y}$, and *Y*'s share of the loss equals $w_Y = \frac{\Delta x}{\Delta x + \Delta y}$. Note that $w_X + w_Y = 1$. We call a liability rule that uses these weights in domain 1 a *pure comparative vigilance rule*.

It may be helpful to comment on the reasonableness of rewarding vigilance. Is it wrong to reward *X*'s care when her care is already wasteful, in the sense of being more than efficient? The answer is: Not really. First, even in domain 1, when both parties are non-negligent, care

is a good rather than a bad, because it decreases accident probabilities and expected losses. Second, if the liability rule puts positive shares on both parties, extra care on *X*'s part is beneficial for both *X* and *Y*. That is, it creates a beneficial externality. Third, there is certainly no necessity for society to penalize excessive care on *X*'s part, since *X* pays for that extra care herself, out of her own pocket. Fourth, in any case, in our model rewarding vigilance does not simply mean rewarding excess care, rather it means rewarding *relative* degrees of excess care; we are comparing *X*'s extra care to *Y*'s extra care. In other words, if both *X* and *Y* increase their extra care by the same proportion, neither is rewarded for it. If both decrease their excess care by the same proportion, neither is punished. But suppose, for example, that between the two of them their excess care is a constant hundred dollars: $\Delta x + \Delta y = 100$. Should they then be treated the same no matter who is very careful and who is marginally careful? We think it is more equitable if they are not; we think the party with the higher excess care should be rewarded with a lower share of the losses than the party with the lower excess care. Fifth and finally, in section 4 below we will analyze a new rule, our "super-symmetric rule," which rewards vigilance, but does so in a way that induces both parties to choose efficient care levels – with zero excess care - in equilibrium.

3 EFFICIENCY AND NASH EQUILIBRIUM

In this section of the paper, but not in section 4 which follows, we assume axiom 2: When one party is negligent and the other is not, all losses fall on the negligent party.

Preliminaries – Pure Comparative Negligence

In the standard theory of negligence-based liability rules, a rule is characterized by specifying how the accident costs are born in the 4 domains. For instance, the simple negligence rule sets $w_X = 1$ if and only if *X* is negligent. (Remember, *X* is the injurer.) Therefore $w_X = 0$ if both are non-negligent; $w_X = 1$ when *X* is negligent and *Y* is not; $w_X = 0$ when *Y* is negligent and *X* is not; and $w_X = 1$ when both are negligent. The rule of negligence with comparative negligence as a defense sets $w_X = 0$ if both are non-negligent; $w_X = 1$ when *X* is negligent and

Y is not; $w_X = 0$ when *Y* is negligent and *X* is not; and $w_X = \frac{\Delta x}{\Delta x + \Delta y}$ ("pure comparative negligence") when both are negligent.

Once a liability rule is specified, the economist asks whether the rule would induce rational *X* and *Y* to end up at the efficient, social-cost-minimizing point (x^*, y^*). There are really two parts to this question: First, is (x^*, y^*) a Nash equilibrium under the rule? Second, is it the only Nash equilibrium? Here are the questions and answers for the rule of negligence with a defense of comparative negligence. Since these are well-known results, we omit the proofs:

> **Theorem 1:** (x^*, y^*) is a Nash equilibrium under negligence with a defense of comparative negligence.

> **Theorem 2**: (x^*, y^*) is the only Nash equilibrium under negligence with a defense of comparative negligence. In particular, there is no Nash equilibrium in the both parties negligent domain.

Pure Comparative Vigilance

Next we turn to pure comparative vigilance. The initial focus is on domain 1, where both are non-negligent. This rule sets $w_X = \frac{\Delta y}{\Delta x + \Delta y}$ and $w_Y = \frac{\Delta x}{\Delta x + \Delta y}$ when both parties are non-negligent and at least one is strictly non-negligent, or vigilant. Recall that at the efficient point (x^*, y^*) the weights are $w_X = \overline{w}_X$ and $w_Y = \overline{w}_Y$. Based on axiom 2, $w_X = 1$ when *X* is negligent and *Y* is not; and $w_X = 0$ when *Y* is negligent and *X* is not.

If we wanted to complete the specification of our liability rule at this point, we would have to indicate what the weights are in domain 3, where both are negligent. But we need not do so at this time. We can get interesting results without completely specifying the rule.

We start out by asking: Do efficiency theorems like theorems 1 and 2 hold for pure comparative vigilance? The answer, it turns out, is No. (In what follows, references in proofs or text to "right," "left," "up" or "down" all refer to figure 1.)

> **Theorem 3:** (x^*, y^*) is *not* a Nash equilibrium under pure comparative vigilance.

Proof: Suppose $y = y^*$. If *X* chooses $x < x^*$ she is negligent and *Y* is not; therefore *X* is liable and pays all the accident costs. Therefore she will want to move to the right, since she is attempting to minimize $x + 1p(x, y^*)L$. Once she reaches $x = x^*$, w_X suddenly takes on the value $\overline{w}_X$. There are two relevant possibilities: $0 < \overline{w}_X \leq 1$, and $0 = \overline{w}_X$.

In the former case, *X* bears costs $x^* + \overline{w}_X\, p(x^*, y^*)L$ at x^*. But farther to the right, at $x^* + \varepsilon$, say, w_X drops to zero under the pure comparative vigilance rule. So *X* does not stop at x^*, she moves instead to $x^* + \varepsilon$. Since $x^* + \varepsilon/2$ is always better than $x^* + \varepsilon$, for any $\varepsilon > 0$, there is no Nash equilibrium.

In the latter case, party *X* has no incentive to move. But now consider party *Y*. At (x^*, y^*), $\overline{w}_X = 0$, and therefore $\overline{w}_Y = 1$. Party *Y* bears costs $y^* + 1p(x^*, y^*)L$. If she increases y by ε, w_Y suddenly drops to zero under the pure comparative vigilance rule. Therefore she moves to $y^* + \varepsilon$, and once again there is no Nash equilibrium. QED.

The next question is this: is there something like theorem 2 for pure comparative vigilance? Can we rule out inefficient Nash equilibria in the both parties non-negligent domain? We now focus on the part of domain 1 where both are non-negligent and at least one is strictly non-negligent, that is, vigilant. The first thing to note is that no Nash equilibrium could occur where only one party is vigilant; they must both be. For instance, if *X* is at $x > x^*$ and *Y* is at y^*, then $w_X = \dfrac{\Delta y}{\Delta x + \Delta y} = 0$. Then *X* will want to move to the left, so they could not be at a Nash equilibrium. Therefore we can confine our search for inefficient Nash equilibria in domain 1 to its interior, where both parties are vigilant. That is, $\Delta x > 0$ and $\Delta y > 0$.

Under pure comparative vigilance, can we show there is *no* both-parties-vigilant Nash equilibrium? We have the following conjecture:

Conjecture: Let (x, y) be any point in the interior of domain 1, with both parties vigilant. Then (x, y) is *not* a Nash equilibrium under pure comparative vigilance.

In fact the conjecture is false, which we establish with the following:

Counterexample to conjecture: Let the accident probability function be $p(x, y) = (1 + x)^{-1}(1 + y)^{-1}$, and let $L = 216 = 6^3$. It is easy to show that the efficient point is at $x^* = y^* = L^{1/3} - 1 = 6 - 1 = 5$. Consider the both-parties vigilant point $(\overline{x}, \overline{y}) = (6.3646, 6.3646)$. Note that $(\Delta x, \Delta y) = (1.3646, 1.3646)$. With some effort it is

possible to show that $\bar{x}$ minimizes the burden on *X*, given the choice by *Y* of $\bar{y}$, and vice versa. (This requires the following steps: First show that $\bar{x}$ satisfies first and second order minimization conditions within the interior of domain 1. Second show that *X* cannot gain by moving into domain 2. The process is similar for *Y*.)

Therefore $(\bar{x}, \bar{y})$ is an inefficient, both parties vigilant, Nash equilibrium under pure comparative vigilance.

Theorem 3 and the false conjecture establish that pure comparative vigilance is fatally flawed; it does not lead to efficiency. We will now turn to modified versions of comparative vigilance which respond to these disappointing results.[8]

Comparative Vigilance With Fixed Division

The simplest modification of the comparative vigilance rule is to make it similar to the rule of comparative negligence with *fixed division*. In that rule, if both parties are negligent, their weights w_X and w_Y are set equal to constant values, no matter what the degrees of negligence. The values might be $w_X = w_Y = 1/2$, in which case we would have the traditional equal division rule.

Suppose then we are in the both parties non-negligent domain, with at least one party strictly non-negligent, or vigilant. We assume $w_X = \hat{w}_X$ and $w_Y = \hat{w}_Y$, for some constant $0 \le \hat{w}_X, \hat{w}_Y \le 1$, with $\hat{w}_X + \hat{w}_Y = 1$. We also assume these are the same weights as at the efficient point (x^*, y^*). That is we require that $(\hat{w}_X, \hat{w}_Y) = (\overline{w}_X, \overline{w}_Y)$. Following are the results that correspond to theorems 1 and 2.

Before proceeding, we need to introduce notation for the part of total social cost that falls on *X* and the part that falls on *Y*. We call *X*'s burden $F(x, y)$, which we abbreviate *F* when appropriate. We call *Y*'s burden $G(x, y)$, which we abbreviate *G*. Therefore, for any pair of weights (w_X, w_Y), $F = x + w_X\, p(x, y)L$, $G = y + w_Y\, p(x, y)L$, and total social cost is $TSC = F + G$.

[8] In this paper we provide two modified versions of comparative vigilance that give efficiency: comparative vigilance with fixed division and the super-symmetric rule. In we provide two more efficient modified versions of comparative vigilance. One of these shifts liability weights in all of domain 1, and the second only shifts liability weights only along the boundaries of domain 1. Both produce the desired results: the efficient points are unique Nash equilibria. However both create discontinuities in liability weights along the domain 1 boundaries. This is something that our super-symmetric rule, described in section 4 below, does not do.

Theorem 4: Assume constant liability weights of $(\hat{w}_X, \hat{w}_Y) = (\overline{w}_X, \overline{w}_Y)$ in domain 1. Then (x^*, y^*) is a Nash equilibrium under comparative vigilance with fixed division.

Proof: Suppose $y = y^*$, and consider whether party *X* wants to reduce or increase x. *X*'s burden at x^* is $F(x^*, y^*) = x^* + \overline{w}_X\, p(x^*, y^*)L$. To the left of x^* it is $F(x, y^*) = x + p(x, y^*)L$. Since the latter function is minimized at x^* and since $\overline{w}_X \le 1$, *X* certainly doesn't want to move to the left. Alternatively, consider a move to the right. An incremental move to the right changes her costs by $\frac{\partial F}{\partial x} = 1 + \overline{w}_X\, p_x(x^*, y^*)L$.

However, since (x^*, y^*) minimizes *TSC*, $1 + p_x(x^*, y^*)L = 0$. Therefore

$$\frac{\partial F}{\partial x} = 1 + \overline{w}_X\, p_x(x^*, y^*)L = -p_x(x^*, y^*)L + \overline{w}_X\, p_x(x^*, y^*)L = -(1 - \overline{w}_X)\, p_x(x^*, y^*)L \ge 0 \ .$$

Because of the regularity assumed for the $p(x, y)$ function, any move to the right causes the burden on *X* to increase (or stay the same). Similar arguments apply to party *Y*. QED.

Theorem 5: Assume constant liability weights of $(\hat{w}_X, \hat{w}_Y) = (\overline{w}_X, \overline{w}_Y)$ in domain 1. Let (x, y) be any point in domain 1 with at least one party strictly non-negligent (i.e., vigilant). Then (x, y) is *not* a Nash equilibrium under comparative vigilance with fixed division.

Proof: The proof is somewhat like the proofs of theorems 7 and 8 below, and is therefore omitted.

Although the fixed division version of comparative vigilance gives us theorems 4 and 5, it presents us with those constant weights in the both non-negligent domain. The constant weights are crude and unfair, and provide no extra reward to the party who spends an extra amount on care, once both parties are non-negligent. This brings us back to consideration of the degree of vigilance factors, $\frac{\Delta y}{\Delta x + \Delta y}$ and $\frac{\Delta x}{\Delta x + \Delta y}$. We know we cannot follow the pure comparative vigilance path without some modifications to guarantee efficiency. Our solution below is to modify comparative negligence and comparative vigilance; to abandon the usual discontinuities of traditional liability rules; and to abandon the all-or-nothing axiom 2, which requires $(w_X, w_Y) = (1, 0)$ in domain 2, where X is negligent and Y is not, and $(w_X, w_Y) = (0, 1)$ in domain 4.

4 A NEW RULE; THE BEST OF BOTH WORLDS

Preliminaries

We drop axiom 2 in this section. We now construct a liability rule that is mixed, rather than dichotomous, in domains 2 and 4, that uses a modified version of comparative negligence when both parties are negligent, in domain 3, and that uses a modified version of comparative vigilance when the parties are vigilant, in domain 1. This rule is continuous over the union of all 4 domains, and it is efficient. Because the rule is symmetric in its treatment of the parties in domains 1 and 3, as well as domains 2 and 4, we call it the *super-symmetric comparative negligence and vigilance rule*, or the *super-symmetric rule*, for short.

We start with the following definitions for w_X and w_Y in domain 1. In that domain both parties are non-negligent. For these definitions we also assume at least one is vigilant:

$$w_X = \overline{w}_X \frac{p(x^*, y^*)}{p(x, y)} - \frac{\Delta x}{\Delta x + \Delta y}\left(\frac{p(x^*, y^*)}{p(x, y)} - 1\right)$$

$$w_Y = \overline{w}_Y \frac{p(x^*, y^*)}{p(x, y)} - \frac{\Delta y}{\Delta x + \Delta y}\left(\frac{p(x^*, y^*)}{p(x, y)} - 1\right).$$

Note the important probability ratio $\frac{p(x^*, y^*)}{p(x, y)}$. Given that both parties are non-negligent and at least one is vigilant, it must be the case that $p(x, y) < p(x^*, y^*)$. Therefore the probability ratio is greater than 1, and the probability ratio minus 1 is positive. So the formula for w_X, for example, makes w_X equal to *X*'s liability share at the efficient point, times a number greater than 1, minus X's relative degree of vigilance, times a positive number.

Also note that the definitions imply $w_X + w_Y = \overline{w}_X + \overline{w}_Y = 1$. So the weights add up to 1, as they should. However, the definitions do not guarantee that the weights are bounded between 0 and 1, a requirement for what we have called normal liability rules. Therefore what follows in this section can be read in either of two ways: (1) That we are analyzing normal liability rules, and we are placing bounds on the weights, $0 \le w_X, w_Y \le 1$, but we are simply not writing out in full the (bounded) definitions of w_X, w_Y, $F(x, y)$ and $G(x, y)$. (2) Alternatively, that we are analyzing possibly non-normal liability rules, and we therefore need

not worry about the bounds on the weights. Either reading is mathematically permissible. We will remind the reader of this occasionally by noting that definitions would be more complex under the $0 \le w_X, w_Y \le 1$ constraint.

Next we will form the corresponding $F(x,y)$ and $G(x,y)$ functions, representing the burdens on parties *X* and *Y*. (The definitions would be more complex under the $0 \le w_X, w_Y \le 1$ constraint.)

$$F(x,y) = x + w_X\, p(x,y)L = x + \overline{w}_X\, p(x^*,y^*)L - \frac{\Delta x}{\Delta x + \Delta y}\Big(p(x^*,y^*)L - p(x,y)L\Big)$$

$$G(x,y) = y + w_Y\, p(x,y)L = y + \overline{w}_Y\, p(x^*,y^*)L - \frac{\Delta y}{\Delta x + \Delta y}\Big(p(x^*,y^*)L - p(x,y)L\Big).$$

Now consider a move from a point in the interior of domain 1 to the left, toward the $x = x^*$ boundary. The limit of $F(x,y)$ is $x^* + \overline{w}_X\, p(x^*,y^*)L$.

Next, consider domain 2, located to the left of domain 1, the region where $x < x^*$ and $y \ge y^*$. Axiom 2 would have $w_X = 1$ in this region, since party X is negligent and party Y is non-negligent. But this would create a discontinuity on the $x = x^*$ boundary between the two regions. And we are abandoning axiom 2. We now define w_X and w_Y in domain 2 as follows (with the observation that definitions would be more complex under the $0 \le w_X, w_Y \le 1$ constraint):

$$w_X = \overline{w}_X\, \frac{p(x^*,y^*)}{p(x,y)} + \frac{p(x,y) - p(x^*,y)}{p(x,y)}$$

$$w_Y = \overline{w}_Y\, \frac{p(x^*,y^*)}{p(x,y)} + \frac{p(x^*,y) - p(x^*,y^*)}{p(x,y)}.$$

Note that $w_X + w_Y = 1$. It follows that the corresponding $F(x,y)$ and $G(x,y)$ functions are:

$$F(x,y) = x + w_X\, p(x,y)L = x + \overline{w}_X\, p(x^*,y^*)L + \Big(p(x,y)L - p(x^*,y)L\Big)$$

$$G(x,y) = y + w_Y\, p(x,y)L = y + \overline{w}_Y\, p(x^*,y^*)L + \Big(p(x^*,y)L - p(x^*,y^*)L\Big)$$

Note that $F(x,y) + G(x,y) = x + y + p(x,y)L = TSC$.

Observe that in domain 2, where $x < x^*$ and $y \ge y^*$, the last term in $F(x,y)$, namely $\Big(p(x,y)L - p(x^*,y)L\Big)$, is *positive*, whereas the last term in $G(x,y)$, namely

$\left(p(x^*, y)L - p(x^*, y^*)L\right)$, is *negative* (or zero if $y = y^*$). Similarly for the second terms in the weights w_X and w_Y. That positive term in $F(x, y)$ is the *increase* in expected accident costs resulting from *X*'s negligence, given *Y*'s actual choice of y. The negative term in $G(x, y)$ is the *reduction* in expected accident costs resulting from *Y*'s vigilance, if *X* were choosing her efficient level of care x^*.

Now suppose we are at a point in domain 2, and move directly right, toward the $x = x^*$ boundary. The limit of $F(x, y)$ is $x^* + \overline{w}_X\, p(x^*, y^*)L$, the same limit if we started at a point in domain 1 and moved directly left. It follows that there is now no discontinuity of $F(x, y)$ at the $x = x^*$ boundary. This is true with or without the $0 \le w_X, w_Y \le 1$ constraint.

Similar definitions are made in domain 4, where *X* is non-negligent and *Y* is negligent or $x \ge x^*$ and $y < y^*$. To avoid a discontinuity along the $y = y^*$ boundary between domain 1 and domain 4, we define the domain 4 weights as follows (with usual understanding about the $0 \le w_X, w_Y \le 1$ constraint):

$$w_X = \overline{w}_X \frac{p(x^*, y^*)}{p(x, y)} + \frac{p(x, y^*) - p(x^*, y^*)}{p(x, y)}$$

$$w_Y = \overline{w}_Y \frac{p(x^*, y^*)}{p(x, y)} + \frac{p(x, y) - p(x, y^*)}{p(x, y)}.$$

Note that $w_X + w_Y = 1$. Now the corresponding $F(x, y)$ and $G(x, y)$ functions are:

$$F(x, y) = x + w_X\, p(x, y)L = x + \overline{w}_X\, p(x^*, y^*)L + \left(p(x, y^*)L - p(x^*, y^*)L\right)$$

$$G(x, y) = y + w_Y\, p(x, y)L = y + \overline{w}_Y\, p(x^*, y^*)L + \left(p(x, y)L - p(x, y^*)L\right)$$

Note that $F(x, y) + G(x, y) = x + y + p(x, y)L = TSC$. Also note that the last term in $F(x, y)$, namely $\left(p(x, y^*)L - p(x^*, y^*)L\right)$, is *negative* (or zero if $x = x^*$), whereas the last term in $G(x, y)$, namely $\left(p(x, y)L - p(x, y^*)L\right)$, is *positive.* Similar comments apply to the weights. Now suppose we are at a point in domain 4, and move directly up, toward the $y = y^*$ boundary. The limit of $G(x, y)$ is $y^* + \overline{w}_Y\, p(x^*, y^*)L$. If we are in the interior of domain 1, on the other hand, and move straight down, the limit of $G(x, y)$ is also $y^* + \overline{w}_Y\, p(x^*, y^*)L$. It

follows that there is no discontinuity of $G(x, y)$ at the $y = y^*$ boundary between domains 1 and 4. This is true with or without the $0 \leq w_X, w_Y \leq 1$ constraint.

We have now defined w_X, w_Y, $F(x, y)$ and $G(x, y)$ in 3 of the 4 domains. It may be useful to describe $F(x, y)$ and $G(x, y)$ verbally in each of these 3 domains:

In domain 1, where both parties are non-negligent, each party's burden ($F(x, y)$ or $G(x, y)$) equals:

(a) her own precaution expenditure, plus (b) her own part of expected accident costs at the efficient point (x^*, y^*), minus (c) her own degree of vigilance, times the reduction in expected accident costs resulting from the two parties' vigilance.

In domains 2 and 4, where one party is non-negligent and the other party is negligent, the burdens are:[9]

For the negligent party:

(a) her own precaution expenditure, plus (b) her own part of expected accident costs at the efficient point (x^*, y^*), plus (c) the *increase* in expected accident costs resulting from her negligence.

And for the non-negligent party:

(a) her own precaution expenditure, plus (b) her own part of expected accident costs at the efficient point (x^*, y^*), minus (c) the *reduction* in expected accident costs resulting from her vigilance.

It remains to describe what we want to happen in domain 3, where both parties are negligent. If we used pure comparative negligence, then each party's burden would be (a) her own precaution expenditure, plus (b) her degree of negligence times expected accident costs.

[9] In a sense, in domains 2 and 4, liability shares under the super-symmetric rule are consistent with the causation requirement in the law of torts. See Singh (2007). Also see Honoré (1997, p. 372), Keeton, Dobbs, Keeton, and Owen (1984), Hart and Honoré (1985), Kahan (1989), and Schroeder (1997).

Using that rule would create discontinuities along the borders between domain 3, and domains 2 and 4. We now modify comparative negligence, to avoid those discontinuities, and to create a concept that is exactly symmetric to our shifted comparative vigilance rule. What we want, in words, is the following in domain 3:

In domain 3, where both parties are negligent, each party's burden is:

(a) her own precaution expenditure, plus (b) her own part of expected accident costs at the efficient point (x^*, y^*), plus (c) her own degree of negligence times the increase in expected accident costs resulting from the two parties' negligence.

The required definitions for w_X and w_Y are as follows, with the usual understanding about the $0 \le w_X, w_Y \le 1$ constraint:

$$w_X = \overline{w}_X \frac{p(x^*, y^*)}{p(x, y)} + \frac{\Delta x}{\Delta x + \Delta y}\left(1 - \frac{p(x^*, y^*)}{p(x, y)}\right)$$

$$w_Y = \overline{w}_Y \frac{p(x^*, y^*)}{p(x, y)} + \frac{\Delta y}{\Delta x + \Delta y}\left(1 - \frac{p(x^*, y^*)}{p(x, y)}\right).$$

The corresponding $F(x, y)$ and $G(x, y)$ functions are now:

$$F(x, y) = x + w_X\, p(x, y)L = x + \overline{w}_X\, p(x^*, y^*)L + \frac{\Delta x}{\Delta x + \Delta y}\Big(p(x, y)L - p(x^*, y^*)L\Big)$$

$$G(x, y) = y + w_Y\, p(x, y)L = y + \overline{w}_Y\, p(x^*, y^*)L + \frac{\Delta y}{\Delta x + \Delta y}\Big(p(x, y)L - p(x^*, y^*)L\Big).$$

Note that these equations are formally identical to the domain 1 equations above! Because of the domain 1/domain 3 symmetry (and, less important, the domain 2/domain 4 symmetry), we call our new rule, defined over domains 1, 2, 3 and 4, the *super-symmetric comparative negligence and vigilance rule*, or the *super-symmetric rule*, for short.

We now have the following results. The first, theorem 6, establishes that the efficient point (x^*, y^*) is a Nash equilibrium under the super-symmetric rule. This parallels theorem 1 for negligence with comparative negligence as a defense, and theorem 4 for comparative vigilance with fixed division. The second, theorem 7, establishes that there are no other Nash equilibria in domain 1, the both non-negligent domain. This parallels theorem 5 for

comparative vigilance with fixed division. The third, theorem 8, establishes that there are no Nash equilibria in domain 3, the both negligent domain. This parallels theorem 2 for negligence with comparative negligence as a defense. The fourth, theorem 9, establishes that there are no Nash equilibria in domains 2 or 4, where one party is negligent and the other is non-negligent. This provides the desired result in those two domains, without recourse to axiom 2, which places all liability on the negligent party and none on the non-negligent party.

Theorem 6: (x^*, y^*) *is* a Nash equilibrium under the super-symmetric rule.

Proof: Suppose $y = y^*$. *X*'s burden at x^* is $F(x^*, y^*) = x^* + \overline{w}_X\, p(x^*, y^*)L$. To the right of x^* it is

$F(x, y^*) = x + \overline{w}_X\, p(x^*, y^*)L - \dfrac{\Delta x}{\Delta x + 0}\Big(\, p(x^*, y^*)L - p(x, y^*)L\Big) = x + p(x, y^*)L$ + a constant,

which is uniquely minimized at x^*. To the left of x^* it is

$F(x, y^*) = x + \overline{w}_X\, p(x^*, y^*)L + (\, p(x, y^*)L - p(x^*, y^*)L) = x + p(x, y^*)L +$ a constant, which is again uniquely minimized at x^*. Hence, *X* doesn't want to move to the right or to the left. Similar arguments apply to party *Y*.

Theorem 7: Let (x, y) be any point in the both parties non-negligent domain, domain 1, with at least one party strictly non-negligent (i.e., vigilant). Then (x, y) *is not* a Nash equilibrium under the super-symmetric rule.

Proof: Let (x, y) be a candidate Nash equilibrium in domain 1, with at least one party vigilant. The reader can confirm that both parties must be vigilant. *X*'s burden at (x, y) is $F(x, y) = x + w_X\, p(x, y)L = x + \overline{w}_X\, p(x^*, y^*)L - \dfrac{\Delta x}{\Delta x + \Delta y}\Big(\, p(x^*, y^*)L - p(x, y)L\Big)$.

On the other hand, if she moved to the left, to the domain 1/domain 2 boundary, Δx would drop to 0, and her burden would be $F(x^*, y) = x^* + \overline{w}_X\, p(x^*, y^*)L$. Now if (x, y) is a N.E., it must be the case that $F(x, y) \le F(x^*, y)$. Similarly, *Y*'s burden at (x, y) is

$G(x, y) = y + w_Y\, p(x, y)L = y + \overline{w}_Y\, p(x^*, y^*)L - \dfrac{\Delta y}{\Delta x + \Delta y}\Big(\, p(x^*, y^*)L - p(x, y)L\Big)$. On the other hand, if she moved straight down, to the domain 1/domain 4 boundary, her burden would be $G(x, y^*) = y^* + \overline{w}_Y\, p(x^*, y^*)L$. If (x, y) is a N.E., it must be the case that $G(x, y) \le G(x, y^*)$. Adding the two inequalities together gives

$x + y + p(x, y)L \le x^* + y^* + (\overline{w}_X + \overline{w}_Y)\, p(x^*, y^*)L = x^* + y^* + p(x^*, y^*)L$. But this is impossible, since (x^*, y^*) is a unique social cost minimizing point. QED.

Theorem 8: Let (x, y) be any point in the both parties negligent domain, that is, domain 3. Then (x, y) *is not* a Nash equilibrium under the super-symmetric rule.

Proof: Suppose $x < x^*$ and $y < y^*$. Assume (x, y) is a Nash equilibrium. Person *X*'s burden is $F(x, y) = x + w_X\, p(x, y)L = x + \overline{w}_X\, p(x^*, y^*)L + \frac{\Delta x}{\Delta x + \Delta y}\Big(p(x, y)L - p(x^*, y^*)L\Big)$, and person *Y*'s burden is $G(x, y) = y + w_Y\, p(x, y)L = y + \overline{w}_Y\, p(x^*, y^*)L + \frac{\Delta y}{\Delta x + \Delta y}\Big(p(x, y)L - p(x^*, y^*)L\Big)$. If party *X* moved to the right, to the domain 3/domain 4 boundary, her burden would become $F(x^*, y^*) = x^* + \overline{w}_X\, p(x^*, y^*)L$. If party *Y* moved straight up, to the domain 3/domain 2 boundary, her burden would become $G(x^*, y^*) = y^* + \overline{w}_Y\, p(x^*, y^*)L$. If (x, y) is a Nash equilibrium, neither can gain by making those moves. Therefore $F(x, y) \le F(x^*, y^*)$ and $G(x, y) \le G(x^*, y^*)$. Adding the two inequalities together gives $F(x, y) + G(x, y) \le F(x^*, y^*) + G(x^*, y^*)$. But this is impossible, since (x^*, y^*) is a unique social cost minimizing point. QED.

Theorem 9: Let (x, y) be any point in domain 2 or domain 4. Then (x, y) *is not* a Nash equilibrium under the super-symmetric rule.

Proof: The proof is similar to the proofs of theorem 6 and 8 above, and is therefore omitted.

Important Insights

Theorems 6-9 provide us with some important insights into the relationship between liability allocation and the incentives it creates for the parties. First, the common impression is that the discontinuity of liability shares required by axiom 2 is important for providing efficient incentives to the parties. (See the large literature cited in section 1.) In contrast, under the super-symmetric rule, as we move within *and across* domains, liability shares change continuously. Therefore, the super-symmetric rule demonstrates that discontinuity in liability shares is not required for efficiency.

Second, mainstream analysis holds that the economic efficiency or inefficiency of a liability rules turns on how the rule allocates *total* expected accident costs between the parties. Recall axioms 2 and 3. Our analysis shows the contrary: that apportionment of only *a part* of

expected costs - not of the entirety – can be used to construct an efficient rule. It may be worth expanding on this point.

Suppose party *X* has spent *x* on care and party *Y* has spent *y*. For these care levels, total expected accident costs are $p(x,y)L$. For the purpose of determining liability of the parties, the super-symmetric rule divides this loss into two parts; $p(x^*,y^*)L$ and $p(x,y)L - p(x^*,y^*)L$. Note that the first part, $p(x^*,y^*)L$, represents expected accident costs when the parties are at the efficient point (x^*,y^*), and is a fixed amount independent of the actual care choices made by the parties. Moreover, the super-symmetric rule puts no restrictions on the division of $p(x^*,y^*)L$ between the parties, other than that the shares $\bar{w}_X$ and $\bar{w}_Y$ are given and fixed over the union of four domains. The rule puts restrictions only on the division of $p(x,y)L - p(x^*,y^*)L$ between parties. This term is clearly a function of actual care levels opted for by the parties, and it is clearly zero at the efficient point. The two parties' shares of this component of loss differ across domains. The proofs of theorems 6-9 are sensitive only to these shares.[10]

In short, we divide total expected accident costs $p(x,y)L$ into two components, $p(x^*,y^*)L$ and $p(x,y)L - p(x^*,y^*)L$, which play very distinct roles. The division of the second component is carefully designed to ensure efficiency. The division of the first component, although it must be decided on and fixed in advance, can be freely made to address equity concerns, without any loss of efficiency.

5 CONCLUSIONS

Several legal scholars have advocated for a division of liability between non-negligent parties based on comparative vigilance. In this paper, we have explored the compatibility of vigilance-based liability rules with the requirements for economic efficiency. We have provided a super-symmetric rule that assigns liability shares based on the comparative negligence of the parties, when both parties are negligent, that assigns liability shares based on the comparative vigilance of the parties, when both parties are vigilant, and that is also

[10] Clearly, depending on the choice of care levels, $p(x,y)L - p(x^*,y^*)L$ is positive, negative or zero.

efficient. In short, we have shown that there exist efficient liability rules that reward vigilance. The reward scheme, however, is somewhat nuanced.

We have made two additional contributions to the existing literature on liability rules. We have shown that it is possible to achieve economic efficiency with continuous liability shares. Moreover, our analysis has revealed that the correct apportionment of only *a part* of expected accident losses - not of the entirety of those losses – can be used to construct an efficient rule. These contributions contrast with the mainstream beliefs about efficient liability rules.

Our super-symmetric rule divides expected accident costs into two components. Division of the first component can be determined (and fixed in advance) in a way that addresses concerns about equity. Proper division of the second part can ensure both equity and efficiency. The resulting rule is equitable and efficient, has continuous liability shares, punishes negligence, and rewards vigilance.

REFERENCES

[1] Bar-Gill, Oren, and Ben-Shahar, Omri (2003): "The Uneasy Case for Comparative Negligence." *American Law and Economics Review* 5: 433-469.

[2] Barnes, David W., and Stout, Lynn A. (1992): *The Economic Analysis of Tort Law*. St. Paul: West Publishing Company.

[3] Biggar, Darryl (1995): "A Model of Punitive Damages in Torts." *International Review of Law and Economics* 15: 1–24.

[4] Burrows, Paul (1999): "A Deferential Role for Efficiency Theory in Analyzing Causation-Based Tort Law." *European Journal of Law and Economics* 8: 29–49.

[5] Brown, John P. (1973): "Toward an Economic Theory of Liability." *Journal of Legal Studies* 2: 323-350.

[6] Calabrisi, Guido, and Cooper, Jeffrey (1996): 'New Directions in Tort Law." *Valparaiso University Law Review* 30: 859-884.

[7] Cooter, Robert D., and Ulen, Thomas S. (1986): "An Economic Case for Comparative Negligence." *New York University Law Review* 61: 1067-1110.

[8] Cooter, Robert D., and Ulen, Thomas S. (1997): *Law and Economics*, 2nd ed. New York: Addison-Wesley.

[9] Diamond, Peter (1974): "Single Activity Accidents." *Journal of Legal Studies* 3: 107-164.

[10] Feldman, Allan M., and Frost, John M. (1998): "A Simple Model of Efficient Tort Liability Rules." *Int'l Review of Law and Economics* 18: 201-215.

[11] Feldman, Allan, and Kim, Jeonghyun (2005): "The Hand Rule and *United States v. Carroll Towing Co.* Reconsidered." *American Law and Economics Review* 7: 523-543.

[12] Feldman, Allan, and Singh, Ram (2008): "Comparative Vigilance." Brown University Department of Economics Working Paper 2008-9.

[13] Grady, Mark F. (1989): "Untaken Precautions." *Journal of Legal Studies* 18: 139–156.

[14] Grimley, William (2000): "Comparative Fault Solidarity Delictual Obligations: On Further Consideration." Louisiana Law Review 60: 513-562.

[15] Haddock, David, and Curran, Christopher (1985): "An Economic Theory of Comparative Negligence." *Journal of Legal Studies* 14: 49-72.

[16] Hart, H. L. A., and Honoré, Tony (1985): *Causation in the Law* 2nd ed. Oxford: Oxford University Press.

[17] Honoré, Tony (1997): "The Morality of Tort Law." In David G. Owen (Ed.), *Philosophical Foundations of Tort Law*. Oxford: Clarendon Press.

[18] Jain, Satish and Singh, Ram (2002): "Efficient Liability Rules: Complete Characterization." *Journal of Economics* 75: 105-124.

[19] Kahan, Marcel. (1989): "Causation and Incentives to Take Care Under the Negligence Rule." *Journal of Legal Studies* 18: 427–447.

[20] Kaplow, Louis (1995): "A Model of the Optimal Complexity of Legal Rules." *Journal of Law, Economics, and Organization* 11: 150–163.

[21] Keeton, W. Page, Dobbs, Dan B., Keeton, Robert E., and Owen, David G. (1984): *Prosser and Keeton on Torts* 5th ed. St. Paul, Minnesota: West Publishing Co.

[22] Kim, Jeonghyun (2004): "A Complete Characterization of Efficient Liability Rules: Comment." *Journal of Economics* 81: 61-75.

[23] Kim, Jeonghyun, and Feldman, Allan (2006): "Victim or Injurer, Small Car or SUV: Tort Liability Rules Under Role-Type Uncertainty." *International Review of Law and Economics,* 26: 455-477.

[24] Landes, William M., and Posner, Richard A. (1980): "Multiple Tortfeasors: An Economic Analysis." *Journal of Legal Studies* 9: 517-555.

[25] Landes, William M., and Posner, Richard A. (1987): *The Economic Structure of Tort Law*. Cambridge, Massachusetts: Harvard University Press.

[26] Levmore, Saul (1994). *Foundations of Tort Law*. Oxford: Oxford University Press.

[27] Liao,Yu-Ping, and White, Michelle J. (2002): "No-Fault for Motor Vehicles: An Economic Analysis." *American Law and Economics Review* 4: 258–294.

[28] Marks, Stephen (1994): "Discontinuities, Causation, and Grady's Uncertainty Theorem." *Journal of Legal Studies* 23: 287–301.

[29] Miceli, Thomas J. (1996): "Cause in Fact, Proximate Cause, and the Hand Rule: Extending Grady's Positive Economic Theory of Negligence." *International Review of Law and Economics* 16: 473-482.

[30] Miceli, Thomas J. (1997): *Economics of the Law: Torts, Contracts, Property, Litigation*. Oxford: Oxford University Press.

[31] Parisi, Francesco, and Fon, Vincy (2004): "Comparative Causation." *American Law and Economics Review* 6: 345-368.

[32] Polinsky, A. Mitchell (1989): *An Introduction to Law and Economics* 2nd ed. Boston: Little, Brown and Co.

[33] Posner, Richard A. (1992): *Economic Analysis of Law* 4th ed. Boston: Little, Brown and Co.

[34] Rea, Samuel A., Jr. (1987): "The Economics of Comparative Negligence." *International Review of Law and Economics* 7: 149–162.

[35] Rubinfeld, Daniel L. (1987): "The Efficiency of Comparative Negligence." *Journal of Legal Studies* 16: 375-394.

[36] Schroeder, Christopher H. (1997): "Causation, Compensation, and Moral Responsibility." In David G. Owen (Ed.), *Philosophical Foundations of Tort Law*. Oxford: Clarendon Press.

[37] Schwartz, Gary T. (1978): "Contributory and Comparative Negligence: A Reappraisal." *Yale Law Journal* 87: 697–727.

[38] Shavell, Steven (1987): *Economic Analysis of Accident Law*. Cambridge, Massachusetts: Harvard University Press.

[39] Singh, Ram (2006): "On the Existence and Efficiency of Equilibria Under Liability Rules." NBER Working Paper 12625.

[40] Singh, Ram (2007): "'Causation-Consistent' Liability, Economic Efficiency and the Law of Torts." *International Review of Law and Economics* 27: 179-203.

[41] Wright, Richard W. (2002): "Negligence in the Courts: Introduction and Commentary, in Symposium, Negligence in the Courts: The Actual Practice." *Chicago-Kent Law Review* 77: 425-288.

[42] Yoshihsa, Nomi (1999): "Environmental Liability in Japan." In Ewoud Hondius (Ed.), *Modern Trends in Tort Law: Dutch and Japanese Law Compared*. Dordrecht, The Netherlands: Kluwer Academic Publishers.

[43] Yu, Li (2000): "Book Review: Modern Trends in Tort Law: Dutch and Japanese Law Compared." *Columbia Journal of European Law* 6: 147-148.