MAKING THE PUNISHMENT FIT THE CRIME OR TALIBAN JUSTICE? OPTIMAL PENALTIES WITHOUT COMMITMENT

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Abstract

This paper argues that graduated penalties observed in most legal systems may be an attempt to direct law enforcement efforts towards crimes that are socially more harmful, thereby achieving better deterrence overall. The critical assumptions are: the state cannot commit to a monitoring strategy, and has mixed motives (objectives other than deterrence). However, graduated penalties arise only in the presence of secondary motives that value punishment in itself, such as retribution or fines collected from violators. Other motives that are unrelated to the size of punishment, such as prevention of criminal attempts, will also lead to distortions, but those cannot be corrected by restructuring penalties. The overall harshness of a criminal justice system and the retributive instincts of its designers may be related in counter intuitive ways, and law enforcement may be improved through strategic delegation.

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1 Introduction

In a well known paper, Becker (1968) made the point that efficient law enforcement policies should set fines at the highest feasible level. Fines and policing are substitutes for the purpose of deterrence, but only policing is costly in terms of resources. Therefore, it is optimal to rely on the costless instrument—fines—to the extent possible.

Legal systems in most societies, however, specify penalties that increase with the social harm caused by the proscribed activity. Murder carries considerably greater sanction than parking violations. This paper outlines one reason why penalties may be dovetailed to the magnitude of the crime. The argument rests on a couple of features that have received very little attention in the literature on optimal deterrence, but which, I will argue, are critical aspects of legal design and enforcement in the real world. First, legislation and policing strategies are often shaped by mixed motives—in addition to deterrence, other imperatives like redress, retribution and prevention of criminal attempts play a significant role. Second, it is unreasonable to assume that the state can commit beforehand to all relevant parameters of a legal system. While penalties are typically not subject to arbitrary revision in a system governed by the rule of law, decisions to allocate costly resources to the pursuit and prosecution of various criminal activities are more discretionary and responsive to changing behavior patterns. I capture this by introducing partial commitment in the standard deterrence model—the choice of penalties at the legislative stage is subject to commitment (in the Stackelberg sense), but monitoring strategies in the enforcement stage must be co-determined with citizens’ choices over legal and illegal actions (in the Cournot sense). This paper shows that mixed motives and partial commitment have some interesting implications regarding the optimal structure of penalties in a simple general equilibrium model of crime and punishment. In particular, they can give rise to the kind of graduated penalties commonly observed, but that conclusion rests on the exact mix of underlying motives considered.

Ex ante, society would like to concentrate scarce law enforcement resources on highly harmful activities, so as to discourage people from undertaking them. Ex post, however, it is optimal to adopt a policing strategy that maximizes other things—e.g., the collection of fines to reduce the damage caused by various crimes (redress), the pain and loss imposed on
criminals (retribution), the number of criminal attempts successfully stopped (prevention), or some combination thereof. While the deterrence motive demands the concentration of resources on crimes that are rare (as would be the case if serious crimes were successfully deterred), the redress, retribution or prevention motives provide a temptation to shift them to crimes that are common. This creates a time inconsistency problem which would tend to erode optimal deterrence, unless law enforcement had some way of committing to a policy that wasn’t \textit{ex post} optimal. In the framework considered in this paper, the presence of a full fledged commitment mechanism implies that optimal fines for all kinds of crime are maximal, i.e., the benchmark in our analysis will remove the known factors that can give rise to graduated penalties. However, if commitment is only partial, I show that lawmakers may want to reduce the penalty on petty crimes as a \textit{credible} way of announcing their intent to pursue serious crimes more vigorously.

The time inconsistency and the consequent departure from the first best (full commitment) solution arises for all different motives that may be assumed to work in conjunction with deterrence—redress, retribution or prevention. Moreover, the penalty structure chosen affects outcomes in a non-trivial manner in every case. However, the second best (partial commitment) solution requires a moderation of some punishments only if the secondary motive is redress or retribution, but not prevention. Put another way, if a society derives some \textit{direct} benefit from the punishments handed out, it may want to choose lower punishments for many crimes, usually the less harmful ones (punishment fits the crime). On the other hand, a society which has a neutral attitude towards the instrument, i.e., neither values nor suffers a cost on account of the punishments \textit{per se}, will want to punish all crimes to the maximum possible extent (Taliban justice). A taste for punishment makes one punish less!

The key to this counter-intuitive result lies in the observation that when the state values punishment for its own sake, the selective reduction of penalties will alter the incentives of not only criminals but also the law and order machinery itself.\footnote{Since all choices are inter-dependent in a strategic context, altering the penalty structure will change the state’s monitoring choices even when penalties do not directly enter its payoff function. The important distinction is that in the presence of a ‘taste for punishment’, choice of penalties shifts the state’s reaction function in the monitoring and enforcement stage, and not merely equilibrium choices.} If the state suffers from a commitment problem in the first place, denying itself some rewards affords a way of ex-
ercising strategic self manipulation (a phenomenon which also arises in models of strategic delegation, e.g., Fershtman and Judd(1987)) that can be exploited to its advantage.

The idea that excessively harsh penalties on relatively innocuous crimes can cause a misallocation of police resources is sometimes voiced in the popular press and in legislative debates. For example, an editorial in the Washington Times dated October 10, 2003 criticized a crackdown on drunk driving conducted by the Washington D.C police department as a distraction from more serious concerns like terrorism and violent crime. An opinion piece in The Hawaii Reporter on June 12, 2003 (Rowland (2003)) criticized a local seat belt enforcement drive for the same reason, going further to identify the negative incentive effects of flawed legislation as the principal cause:

[I]t was never clear what was really accomplished except for extracting $277,046 by force of law and roadblocks at 124 locations, and that compliance with the law was “up”... The seat belt compliance program is a great example of misallocation of resources by misplaced incentives fashioned by legislators at all levels. In other words, the police made money on the deal, so they did it [italics mine].

These same concerns were echoed by Minnesota State Representative Phyllis Kahn (see Kahn (2001)), while introducing a bill to legalize ticket scalping—at the time a misdemeanor in Minneapolis punishable by 90 days in prison, a $700 fine or both. In defending the bill, she stated

At a time [the eve of a major basketball game] when there are thousands of people visiting the Twin Cities, we should be making them feel as safe and welcome as possible. Police resources should be used to enforce pedestrian right-of-way or to make sure our guests aren’t getting stuck in traffic or getting their pockets picked. Assigning so many officers to a crack-down on ticket scalping is a waste of valuable police time, and I hope this legislation will be a first step in curbing that misallocation of public resources.

Criticism of the “war on drugs” or “tough on crime” legislation sometimes focuses on inefficiency and waste, as opposed to libertarian or compassionate arguments. At the same
time, advocates of harsh penal codes often cite their deterrence benefits rather than moral fitness. Our analysis suggests that both sides may have a point, depending on the exact motives that shape the behavior of law enforcement.

The rest of the paper is organized as follows. Section 2 discusses the related literature. In section 3, I lay out a general model which nests all possible motives discussed in the paper. Sections 4 and 5 analyze special cases where either a redress-retribution motive is present or a prevention motive, in conjunction with deterrence. The results are then compared. Section 6 solves an example where the distribution of benefits are uniform, and illustrates some additional interesting possibilities. Section 7 concludes.

2 Related Literature

The application of rational choice models to legislation and illegal activity, which started with Becker (1968), has, with a few exceptions, remained focused on cost benefit analysis with deterrence as the sole objective. Nevertheless, there is a thriving debate among philosophers and legal theorists regarding the justification of imposing suffering on offenders, both from a normative and a descriptive viewpoint. A fairly standard distinction often drawn is that between backward looking or retributivist justifications—those based on evaluation of an act already committed—and forward looking or consequentialist rationales—those based on evaluation of acts that could be committed in the future. Many writers have expressed the view that a proper account of existing criminal justice systems or our intuitions about how they should be designed is impossible without combining elements from both strands of thought (Hart (1958)). In a more empirical vein, response of experimental subjects to questionnaires about crime and punishment often reveal a strong retributive motive but a weak deterrence motive (Sunstein, Schkade and Kahneman (2000)). Analyzing actual sentencing data, Glaesar and Sacerdote (2000) find that random victim characteristics have a significant effect on sentencing decisions for crimes like vehicular manslaughter, suggesting the presence of moral and emotional factors apart from deterrence. These observations

\begin{footnote}{See the entry under “Punishment” in the Stanford Encyclopedia of Philosophy for a brief overview of the main themes and arguments. As quoted there, Nietzsche supposedly made the remark that the discourse supporting various legal systems is “overdetermined by utilities of every sort.”}

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suggest that the economic theory of crime and law enforcement could be enriched by going beyond deterrence and incorporating a multiplicity of objectives.

In economic models that deal with random monitoring, including the crime literature, the typical assumption is that the choice of probability is committed to beforehand, e.g., by specifying it in a binding contract. Some papers depart from this trend and model monitoring choices to be an outcome of equilibrium in some game rather than a commitment. Examples include Khalil (1997) in the context of a procurement problem, Graetz, Reinganum and Wilde (1986) in a model of tax auditing, and Tsebelis (1989) on law enforcement. The last paper is the one thematically closest to this, but it lacks several other modeling features analyzed here, such as multiple crimes or mixed motives, and the paper’s conclusion that raising penalties will leave the crime rate unchanged is an artifact of the simple $2 \times 2$ game considered. As for empirical evidence regarding the impact of monitoring on crime, Di Tella and Schargorsky (2004) and Levitt (1997) among others find a significant deterrent effect of police presence for many illegal activities.

A number of papers have derived graduated penalties in deterrence based models assuming commitment to a monitoring policy. There are two major factors identified in this literature as the reason for gradualism. One is the presence of general, as opposed to specific, monitoring—situations where different crimes must be monitored at a common rate since they cannot be targeted separately (Shavell (1991)). Another is marginal deterrence (Stigler (1970), Shavell (1992), Wilde (1992), Mookherjee and Png (1994)). Marginal deterrence is achieved if criminals can be induced to switch from more harmful to less harmful activities. When penalties are uniformly high, some people may be deterred from any crime whatsoever, but those who choose to break the law will tend to gravitate towards the most serious offenses. This paper considers scenarios which are the mirror image of general monitoring and marginal deterrence models. Instead of allowing criminals to switch between crimes while confining law enforcement to a common monitoring strategy, I assume enforcement resources to be mobile across crimes but not criminals’ efforts. These are complementary approaches that address plausible but distinct scenarios. For example, people may easily switch from exceeding the speed limit by 5 mph to 20 mph or from concealing a small part of their income to all of it, but it is hard to imagine jaywalkers becoming bank robbers because the difference in penalties is insufficient.
Another suggested reason for making the punishment fit the crime is jury aversion to the possibility of conviction errors (Andreoni (1991)), which makes conviction less likely when the penalty is increased and may have a negative effect on deterrence. Some papers show that maximal fines need not be optimal in specific situations, but do not demonstrate that there should be a positive relationship between damage and punishment. Suggested factors include risk aversion (Polinsky and Shavell (1979)), wealth constraints (Polinsky and Shavell (1991)), heterogeneity in apprehension probability (Bebchuk and Kaplow (1993)), corruption among law enforcers (Becker and Stigler (1974), Bowles and Garoupa (1997)) and criminals’ costly investment in avoidance (Malik(1990)). For more comprehensive discussions on the literature on optimal penalties, see the surveys by Ehrlich (1996), Garoupa (1997) and Polinsky and Shavell (2000).

Finally, Persico (2002) uses a very similar two sector model to address the question whether racial profiling promotes deterrence at the cost of fairness. However, in his model, fines are exogenous and while the sectors differ in the distribution of benefits from committing a crime, the social damage is symmetric (since Persico’s concern is profiling, his model essentially evaluates different monitoring strategies for two observationally distinct sub-populations who can commit the same crime but possibly at different rates). Persico also assumes that police aim to “maximize arrests”, which is compatible with all the different motivations considered here (retribution or redress versus prevention) because of the symmetry of his case, and hence cannot distinguish between the effects of these factors in a more general setting involving multiple crimes.

3 A General Model

There are two kinds of illegal activity, labeled 1 and 2. Corresponding to each, there is a population of potential offenders of measure one, who choose whether or not to commit the offense. The decision to commit each type of crime is an independent act, i.e., we disallow at the outset the kind of substitution possibilities between crimes that raise the issue of marginal deterrence.

The private benefit $b_i$ of committing crime $i$ is a random variable with distribution $F_i(b_i)$ in
the population. This benefit accrues conditional on the crime being successfully completed; in case the attempt is thwarted, the benefit is 0. Below, I discuss how the monitoring of an activity by law enforcement affects the success rate of criminal attempts. We assume that $F_i(.)$ is continuously differentiable over its domain.

The social harm from criminal act $i$ is $c_i$. Without loss of generality, I will assume $c_1 > c_2$, i.e., activity 1 is the more serious crime from a social viewpoint. The police choose how frequently to monitor the population for each type of infraction, which in turn affects the crime specific rate of conviction of criminals. Let $p_i$ denote the probability that a random person who has committed the type $i$ crime will be apprehended by the police. This number depends on the resources allocated towards fighting the crime in question, and is hence a choice variable for law enforcement. In the parlance of the law and economics literature, monitoring is assumed to be specific rather than general. However, the total monitoring resources is fixed, and only its allocation across different crimes is subject to choice. Mathematically, we impose the constraint that the average conviction rate must be some given number $p$, i.e.,\footnote{Two generalizations can easily be accommodated without any qualitative modification to the results. First, $p$ can be endogenized by assuming a convex cost function for total law enforcement resources, and allowing it to be chosen in the first stage. For any given $p$, its allocation across crimes will be subject to exactly the analysis done here. Second, the assumption that the two kinds of monitoring activity are perfect substitutes can easily be relaxed by assuming some downward sloping and possibly non-linear frontier of the form $\varphi(p_1, p_2) = 0$ from which $(p_1, p_2)$ must be chosen. Similar results will be obtained if $\varphi$ allows sufficient but not perfect substitutability.}

$$\frac{1}{2} (p_1 + p_2) = p$$

(1)

Lawmakers also choose penalties $s_1$ and $s_2$ for the two crimes, but cannot impose penalties exceeding some value $S$, which captures wealth or moral constraints on punishment. These penalties are best interpreted as fines, but can also be treated as prison sentences, if imprisoning offenders does not impose a net cost on society (which would be true, for example, if the incapacitation benefits exceed the cost of running prisons).

In formulating the complementary problems of legislation and law enforcement, I will allow a number of different motives to influence the choices of legislators and police. I assume no fundamental conflict of objectives between lawmakers and law enforcers, who
will jointly be referred to as “authorities”. First, there is the standard deterrence motive—the authorities want less people to commit each crime than more. Second there may be a redress or retributive motive—every dollar collected in fines from apprehended criminals (or every year of imprisonment, if penalties are interpreted as prison terms) contributes to social welfare by an amount \( \lambda \). One way to think about this is to view the revenue collected from criminals as a “sin tax” which finances public goods (including law enforcement itself) or compensates victims without the need for distortionary taxation on productive activity (redress). If the penalty is a prison term, \( \lambda \) may be thought of as benefits from incapacitation. Alternatively, it could be an ethical utility derived from imposing a burden on those who have committed harmful acts against society (retribution). Finally, suppose that out of all individuals trying to commit act \( i \) and apprehended by the police, a fraction \( \gamma_i \) are stopped before they can complete their crime (or, if they have already committed the act, the police can undo the damage). For example, a police patrol may stop a bank robbery in progress through timely response, or recover part or whole of the loot through follow-up investigation. The last factor constitutes what may be called a preventive motive. Note the distinction from the deterrence motive, which arises from the desire to discourage people from committing crimes in the first place, while prevention refers to stopping people who have decided to commit them nevertheless.

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However, due to the presence of mixed motives and the natural sequencing of legislation and enforcement, the authorities suffer from a time inconsistency problem, the nature of which will become clearer in the course of the analysis.

The purely retributive interpretation may be problematic, given the assumed linearity of the utility from punishment. The authorities may derive greater ethical satisfaction from punishing those who have committed more harmful acts, in which case \( \lambda \) will be crime specific. Further, one imagines there will be diminishing marginal utility from punishing, and the function may even become downward sloping (punishment becomes costly) after a peak is reached at some level of penalty. The notion of an \( \text{ex post} \) optimal punishment or “just desert” has been much discussed in the law and philosophy literature, and there is some evidence that most people form their punitive opinions guided by such a notion (Sunstein, Schkade and Kahneman (2000)). Note, however, that since my main results are that optimal punishments may not be maximal and may be designed to fit the crime, assuming a discriminatory taste for punishment (and distaste beyond a point) should reinforce the results. Nevertheless, exploring more realistic specifications of preferences seems like a promising avenue for future research.
of harms minus any benefits accruing from penalties. The objective function can then be written as\(^6\)

\[
W = \sum_{i=1}^{2} \theta_i [(1 - p_i \gamma_i) c_i - \lambda p_i s_i]
\]  

(2)

where \(\theta_i\) is the measure of people who choose to commit crime \(i\).

### 3.1 Equilibrium with Full Commitment

First, let us outline the full commitment case. This arises when the authorities can announce (and commit to) all aspects of law and policing, i.e., the entire policy vector \((s_1, s_2, p_1, p_2)\). People in the population take this as given and choose whether to commit a crime or not, based on their individual expected benefits and expected costs. The marginal individual is one for whom the expected benefit equals the expected cost, and who is consequently indifferent between committing the crime and being a law abiding citizen. Note that because of the possible prevention factor, the expected benefit is only \((1 - p_i \gamma_i)b\), while the expected cost is \(p_i s_i\). For an arbitrary policy vector, the measure of people who commit a crime of type \(i\) is therefore \(\theta_i = 1 - F_i \left( \frac{p_i s_i}{1 - p_i \gamma_i} \right)\). We make the following assumptions on the distributions and parameters, which will be maintained throughout:

**Assumption I:** (Incomplete deterrence) \(F_i \left( \frac{2p_i S}{1 - 2p_i \gamma_i} \right) < 1\) for \(i = 1, 2\).

**Assumption NE:** (No extortion) \(c_i > \frac{2p_i S}{1 - 2p_i \gamma_i}\) for \(i = 1, 2\).

Assumption I ensures that neither crime can be completely deterred. Even if all resources were concentrated on any one activity and maximal penalties imposed on it, there will always be some measure of people who will find it worthwhile to engage in it. Clearly, some kind of limited resource assumption has to be made to generate trade-offs and make the problem

\(^6\)Unlike much of the deterrence literature, I do not include the private benefits of criminals in the calculation of social welfare. The results do not depend on this as long as the activities are clearly socially harmful, i.e., the social cost of the activities outweigh the criminals’ private gain in all situations. At any rate, while the cost benefit approach is useful especially in those problems where the scope of legally permissible actions is itself an issue (e.g., determining speed limits or pollution quotas), its relevance to actions that are universally deemed immoral is questionable. Most people will find it rather odd if one insists that social policy towards rapists should take into account their enjoyment of rape.
interesting. I make a slightly stronger assumption than is necessary to get the general flavor of the results. This is done in order to avoid the inconvenience of corners (in terms of deterrence).

Assumption NE guarantees that even if all resources were concentrated on one crime, its net harm will remain positive. This ensures that deterrence remains the motive, as opposed to extortion. The analysis may be quite different if the latter is the case, but it is not the kind of issue this paper aims to address. Note that meeting these requirements is not a problem—Assumption I can be satisfied by “stretching” out the distributions, and Assumption NE simply requires the harms to be high enough.

Denote by asterisks (*) the magnitudes of a full commitment equilibrium. These are the values of the instruments that minimize net social damage given by (2), after incorporating appropriate expressions for $\theta_i$ to reflect citizens’ responses to the choice of these instruments. In other words, the optimal policy vector under full commitment is the one which solves

$$\min_{s_1, s_2, p_1, p_2} \sum_{i=1}^{2} \left[ 1 - F_i \left( \frac{p_i s_i}{1 - p_i \gamma_i} \right) \right] \left[ (1 - p_i \gamma_i) c_i - \lambda p_i s_i \right]$$  \hspace{1cm} (3)

subject to $s_i \leq S$, $p_i \geq 0$ and (1).

We postpone discussion of the salient properties of this equilibrium till the next section. Before proceeding any further, we discuss an alternative scenario where the authorities can only commit to the penalties for various crimes, but not the resources allocated towards pursuing them.

### 3.2 Equilibrium with Partial Commitment

The assumption that the authorities can publicly commit to all law enforcement parameters seems unrealistic. Since the penalties imposed on the guilty must be supported by law, these do involve a good deal of prior commitment, especially if laws allow little room for discretion in sentencing. How the police department allocates its time and effort is a day to day response to situations on the ground, and typically cannot be legislated. To address this more plausible scenario, we consider a partial commitment case, where the authorities first choose $s_1$ and $s_2$, and then play a simultaneous move game with the citizens in which $p_1, p_2$ and the decisions whether to commit each type of crime are taken simultaneously.
In any interior equilibrium of the subgame, where both types of crime are monitored to some degree, the expected marginal returns from targeting police resources at either activity must be equal. In the case of corner outcomes, where the police exclusively target one kind of activity, it must be that the marginal returns from monitoring that activity (weakly) exceeds that from the other. Note that in computing these marginal returns to monitoring, the measure of people $\theta_i$ engaged in each crime $i$ must be treated as exogenous, since policing strategy and criminal choices are simultaneous in the subgame. From (2), the marginal return to increasing $p_i$ is $\theta_i(\gamma_i c_i + \lambda s_i)$. Collecting together these observations and replacing $\theta_i = 1 - F_i \left( \frac{p_i s_i}{1 - p_i \gamma_i} \right)$ as the final step, we can write

\[
\begin{align*}
1 - F_1 \left( \frac{p_1 s_1}{1 - p_1 \gamma_1} \right) (\gamma_1 c_1 + \lambda s_1) &\geq 1 - F_2 \left( \frac{p_2 s_2}{1 - p_2 \gamma_2} \right) (\gamma_2 c_2 + \lambda s_2) \text{ if } p_1 \in (0, 2p) \\
&= 0
\end{align*}
\]  

We will focus on the subgame perfect equilibrium of this partial commitment case. Denote the relevant magnitudes by double asterisks (**). For arbitrary $(s_1, s_2)$, let $p_1(s_1, s_2)$ and $p_2(s_1, s_2)$ be the monitoring levels in the equilibrium of the subgame, as described in (4), together with the resource constraint (1). It can easily be checked that each subgame has a unique equilibrium, so that these are single valued functions. Finally, $s_1^{**}$ and $s_2^{**}$ are obtained by substituting these functional expressions into the objective function in (3), and maximizing it with respect to $s_1$ and $s_2$.

4 Preventive Motives

I first consider the extreme case where the penalties have no direct social utility ($\lambda = 0$), making redress or retribution irrelevant in the design of laws and their enforcement. However, prevention is allowed to play a role ($\gamma_i > 0$). I call this the prevention model.

In the prevention model with partial commitment, recalling the general objective function (3) and the constraint (4) implied by equilibrium in the subgame, and setting $\lambda = 0$, the problem reduces to

\[
\min_{s_1, s_2} \sum_{i=1}^{2} \left[ 1 - F_i \left( \frac{p_i s_i}{1 - p_i \gamma_i} \right) \right] (1 - p_i \gamma_i) c_i
\]  

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subject to

\[
\left[ 1 - F_1 \left( \frac{p_1 s_1}{1 - p_1 \gamma_1} \right) \right] \gamma_1 c_1 \geq \left[ 1 - F_2 \left( \frac{p_2 s_2}{1 - p_2 \gamma_2} \right) \right] \gamma_2 c_2 \quad \text{if } p_1 \in (0, 2p) = 2p \]

(6)

and (1), \( s_i \leq S \) and \( p_i \geq 0 \) for \( i = 1, 2 \).

It is straightforward that under full commitment, optimal punishments are maximal. For fixed \( p_1 \) and \( p_2 \), increased sanctions on any activity increases the expected cost of engaging in that activity and will therefore cause fewer people to make that choice. When the authorities cannot commit to an allocation of monitoring resources, however, the penalties will also affect the equilibrium choice of \( p_1 \) and \( p_2 \) in the monitoring subgame, and this must be taken into account while choosing penalties in the first stage. Our first result is that in the preventive model, optimal penalties are still maximal when this added complication is present.

**Proposition 1** In the prevention model, there is always an equilibrium in which optimal penalties are maximal under either full or partial commitment, i.e., \( s_i^* = s_i^{**} = S \) for \( i = 1, 2 \). Further, if \( p_i^* > 0 \) (\( p_i^* > 0 \)) for \( i = 1, 2 \), it must be that \( s_i^* = S \) \( s_i = S \).\(^7\)

See Appendix B for proof. The basic intuition can be understood by examining (6), the equilibrium condition in the subgame, in the case of an interior solution (the argument generalizes to corner solutions, as shown in the appendix). I will reproduce this condition below

\[
\frac{1 - F_1 \left( \frac{p_1 s_1}{1 - p_1 \gamma_1} \right)}{1 - F_2 \left( \frac{p_2 s_2}{1 - p_2 \gamma_2} \right)} = \frac{\gamma_2 c_2}{\gamma_1 c_1} \quad (7)
\]

That is, the ratio of the crime rates takes a value independent of the penalties. One implication of this is that the two crime rates must necessarily move together when some penalty is changed, and it is only one small further step to see that they must both go down when either \( s_1 \) or \( s_2 \) is increased. If crime rates went up, it must be the case that monitoring resources flowed away from the activity which has been subjected to higher sanctions, in

\(^7\)Strictly speaking, it is necessary that penalties be maximal only if both activities are allocated some monitoring effort. If \( p_2^2 = 0 \), for example, then nobody is deterred in activity 2 regardless of the penalty, and hence it can be set at any value, including \( S \).
which case the other activity is now being monitored more closely. Since the penalty on that activity has remained unchanged, it couldn’t be the case that it now produces more crime.

It may be tempting to think that introducing lack of commitment has no effect on the allocation of monitoring resources compared to the first best (the full commitment case), which is why restructuring penalties is not necessary. This is untrue. Absence of commitment generally gives rise to distortions, even when distributions and prevention rates are symmetric. To see this, return to the full commitment problem, and assuming an interior solution, write the first order condition:

$$
\left[1 - F_1\left(\frac{p_1s_1}{1-p_1\gamma_1}\right)\right] \gamma_1 c_1 = \left[1 - F_2\left(\frac{p_2s_2}{1-p_2\gamma_2}\right)\right] \gamma_2 c_2
\tag{8}
$$

This differs from the monitoring equilibrium condition in the partial commitment case by the extra term on the right hand side. This term captures the deterrence effect, the effect of changing the allocation of monitoring resources ex ante, when people can learn about it and respond accordingly, and precisely the effect which is absent when these reallocations are made concurrently with the decisions to engage in crimes. The deterrence effect will generally not be zero at the first best unless by an accidental configuration of the primitives, and hence the allocation will depart from the first best when commitment is absent. In fact under a mild assumption on the distribution of benefits, satisfied by both uniform and exponential distributions, an under policing of the more harmful activity arises.

**Proposition 2** In the prevention model, if prevention rates are the same ($\gamma_1 = \gamma_2 = \gamma$) across activities, and $F_1(.) = F_2(.) = F(.)$, more monitoring resources are allocated to the more harmful activity under full commitment, i.e., $p^*_1 > p > p^*_2$. Further, if $F(.)$ has the non-decreasing hazard rate property, then absence of commitment to monitoring leads to under monitoring of the more harmful activity, i.e., $p^{**}_1 \leq p^*_1$ with the inequality strict if $p^*_1 < 2p$.

For a formal proof, see Appendix B, but I will provide a sketch of the idea here. In the partial commitment equilibrium, any possible preventive gains from reallocating police resources have already been exploited, but deterrence gains (were these reallocations to be
announced beforehand in a credible way) have not. Given symmetric response elasticities and prevention rates, the more serious crime draws more resources in a partial commitment equilibrium and has a lower crime rate, because otherwise, switching the monitoring levels would reduce total harm. Under the hazard rate assumption, publicly shifting a small additional amount of monitoring resource from the lesser to the greater crime would deter a larger fraction of offenders in the more harmful activity than the fraction of new criminals it creates in the less harmful one. Since the crime rates are already in inverse proportion to their social harms, this would result in a decrease in net harm for society if the shift were publicly observable.

It needs emphasizing that the monitoring distortion identified in Proposition 2 is not a result of agency problems within the police department but the usual dynamic inconsistency that arises in many games. Unlike prevention of ongoing attempts, deterrence is an \textit{ex ante} motive which ceases to operate once people have already decided whether or not to engage in crimes.

5 Redress or Retributive Motives

Next, I will consider another special case where redress or retributive motives are present (\(\lambda > 0\)) but preventive motives are absent (\(\gamma_i = 0\)). This could arise if the kind of crimes under consideration are hard to prevent but possible to address after the fact (e.g., tax evasion). One could also think of this as a limiting case of scenarios where redress or retributive factors are large relative to preventive possibilities.

For convenience, the description of equilibrium under partial commitment is reproduced below by incorporating the parametric assumptions of this section in (3) and (4):

\[
\min_{s_1, s_2} \sum_{i=1}^{2} \left[ 1 - F_i(p_i s_i) \right] (c_i - \lambda p_i s_i) \tag{9}
\]

subject to

\[
\begin{align*}
[1 - F_1(p_1 s_1)] s_1 &\geq 2p \\
[1 - F_2(p_2 s_2)] s_2 &\geq 0 \quad \text{if } p_1 \in (0, 2p) \tag{10}
\end{align*}
\]
and (1), $s_i \leq S$ and $p_i \geq 0$ for $i = 1, 2$.

The following property is easily established:

**Proposition 3** Consider the redress-retribution model ($\gamma_i = 0$ for $i = 1, 2$). Under full commitment, there is always an equilibrium in which penalties are maximal, i.e., $s_1^* = s_2^* = S$. If $p_i^* > 0$, it must be that $s_i^* = S$ for $i = 1, 2$. Further, if $F_1(b) = F_2(b)$ for all $b$, the more harmful activity is monitored more intensively, i.e., $p_1^* > p > p_2^*$.

While the formal proof is in Appendix B, the basic argument is very simple. With fixed $p_1$ and $p_2$, penalties have both a positive deterrent effect as well as a positive redress-retribution effect—*ceteris paribus*, they reduce the number of people committing the relevant offense, and reduce the margin of net harm from each crime as well. Hence, the optimal fines are maximal if commitment were possible.

As for the comparison of the optimal monitoring choices, they depend not only on the relative harm, but also the elasticities of response to expected penalties. If there is symmetry in the second aspect, it is intuitive that the more harmful activity will draw greater scrutiny. Under symmetry, it can be shown that if more resources are allocated to the less harmful crime, reversing that allocation lowers net social harm.

As in the pure prevention case considered in the previous section, the partial commitment equilibrium cannot replicate the full commitment equilibrium, implying a social loss arising out of the inability to commit to a policing strategy. To see this, assume $F_1(.) = F_2(.)$ and suppose $s_1 = s_2 = S$. From (10), it follows that in the second stage of the game, $p_1 = p = p_2$. This allocation runs contrary to the characterization of the full commitment allocation in Proposition 1, demonstrating that relaxing the commitment power has an effect on allocations and payoffs, as in the prevention model. What is interesting here is the implication for the optimal choice of penalties.

The next result shows that when the authorities cannot commit to an allocation of monitoring resources across different crimes, it may be optimal to reduce the penalty on some crime below what is feasible. If the distribution of benefits is symmetric, then any reduced penalty will apply to the less serious crime. Graduated penalties will arise if the difference in harms is large enough, or the redress motive relatively weaker than the deterrence motive. This result on the restructuring of penalties in the presence of redress or retributive motives
stands in sharp contrast to what we obtained for purely preventive motives in Proposition 1.

**Proposition 4** Assume $F_1(b) = F_2(b) = F(b)$ for all $b$. In any partial commitment equilibrium of the redress-retribution model, the penalty on the more harmful activity is always maximal, i.e., $s_1^{**} = S$. The penalty on the less harmful activity is less than maximal ($s_2^{**} < S$) if

$$\alpha < \frac{c_1 F(pS)}{[F(2pS) - F(pS)] [c_1 - 2\lambda pS]}$$

where $\alpha = \frac{c_2}{c_1}$ (11)

Here, I will sketch the basic intuition behind the proof, leaving the more formal treatment to Appendix B. First, a “switching argument” as before establishes that it can never be optimal to impose a smaller penalty on the more harmful crime. If that were the case, then in equilibrium, more people would commit the lesser crime than the serious one. If the penalties were switched, the measures of offenders would also switch, due to the assumed symmetry of the distributions. This preserves the total revenue collected from penalties, but reduces the cumulative social damage by substituting a more harmful activity by a less harmful one. It is also easy to show that the penalties on both crimes cannot be less than maximal, because otherwise, there is a way of increasing these penalties such that both crime rates are proportionately reduced. The only question that remains is whether there may be circumstances where it is optimal to reduce the penalty on the lesser crime strictly below maximum. Now, reducing the penalty on the lesser crime will typically\(^8\), as an equilibrium response, shift policing resources to the more serious crime, deterring it to a greater degree but allowing more of the less harmful activity. This is an acceptable tradeoff if the difference in harms is large enough.

\(^8\)Though it sounds intuitive enough, for infinitesimal changes, this effect is not general, but depends on an elasticity condition. When the penalty on one activity is reduced, it is made less attractive as a revenue source, and this direct effect will tend to create a flight of policing resources towards more remunerative arrests. However, the disincentive effect of reduced penalties also encourages more people to commit that particular crime, which has the indirect effect of attracting police attention. Whether there will be a net reduction in monitoring depends on the relative strengths of these two effects. It can be shown that starting from arbitrary fines $(s_1, s_2)$ and associated equilibrium monitoring levels $(p_1, p_2)$, slightly reducing the penalty on crime $i$ will increase the equilibrium allocation of police resources to the other crime if and only if $\frac{xf(x)}{1-F(x)} < 1$ evaluated at $x = p_is_i$. 16
A major difference between the two variants of the model that have been considered is that in the latter, penalties produce a direct social benefit which is absent in the former. Nevertheless, the authorities will impose the most draconian punishment on a lesser crime when they do not care about the punishment per se, but may want to soften it when they find punishment to be useful in itself. The key, of course, lies in how the structure of penalties affects incentives in the subgame, and thereby overall deterrence.

Why are graduated penalties optimum in the redress-retribution case, but not in the prevention case? To understand this, consider the monitoring equilibrium in the redress-retribution model (again focusing on interior solution), which takes the form

\[
1 - F_1 \left( \frac{p_1 s_1}{1 - p_1 \gamma_1} \right) = \frac{s_2}{s_1}
\]

(12)

In this case, unlike (7), the ratio of crime rates is not a constant, but equal to the ratio of the penalties. When \( s_2 \) is lowered, it allows a potential substitution—lowering the crime rate in the more harmful activity (the numerator on the left hand side) at the cost of increasing the crime rate in the less harmful one (the denominator). The prevention model presented no such trade-off, since the two crime rates always moved together. Under the right conditions, the slope of this trade-off is high enough to make a reduction in penalties on the lesser crime worthwhile.

(11) is derived by comparing the total social cost that arises when the lesser crime is legalized, against what would be incurred if penalties were uniformly maximal. It is only sufficient, not necessary, for the optimal penalty on the lesser crime to be less than maximal. An alternative sufficient condition can be derived by examining the derivative of social cost with respect to \( s_2 \) when both penalties are set at \( S \), and ask under what conditions it is positive. Typically, this will depend on the elasticities of crime rates with respect to expected cost, i.e., the properties of the density function \( f(.) \), in addition to the value of \( \alpha \). Also, whenever (11) is satisfied, it is not necessarily true that completely legalizing activity 2 is optimal. There may be penalties that are positive but less than maximal which minimize net social cost.

The need for lowering punishment critically depends on the general equilibrium effects. Optimal penalties are always maximal in a single act model of crime with an exogenous cost.
of monitoring, even if similar assumptions are introduced (i.e., there are revenue motives in addition to deterrence motives, and the authorities cannot commit to a monitoring level). This is shown in Appendix A, but the basic intuition is not hard to see. In a single act model, commitment problems always lead to under deterrence, and fairly standard arguments establish that raising penalties will lower the crime rate. Only in a multi act model does the situation arise that some activity may be over deterred, and lowering the penalty on that activity may move the allocation in the right direction.

6 An Example and Further Observations

In this section, I consider a specific example and explicitly derive solutions for the various cases. The solutions illustrate some further interesting properties that may arise in models of this class.

Suppose the distribution of benefits for activity 1 is uniform on \([0, a]\), and that for activity 2 uniform on \([0, 1]\), where \(a \geq 1\). Further, let \(p = 0.5\) and \(S = 1\).

First, consider the redress-retribution model \((\gamma_i = 0)\) with \(\lambda = 1\). If the authorities can commit to a monitoring strategy, taking punishments to be maximal (Proposition 3) and noting that the resource constraint implies \(p_1 + p_2 = 1\), the problem can be written as

\[
\min_{p_1 \in [0, 1]} \frac{1}{a} (a - p_1)(c_1 - p_1) + p_1(c_2 - 1 + 1) \tag{13}
\]

This is a strictly convex function, since the second derivative is \(2 \left(1 + \frac{1}{a}\right) > 0\). There is either a unique interior solution, or the solution lies in one of the two corners. The solution is \(p^*_1 = 0\) when the following Kuhn-Tucker condition holds: the first derivative is non-negative at \(p_1 = 0\). This happens when

\[
\frac{a}{p_2} \geq \frac{c_1}{c_2 - 2}
\]

When \(a\) is very high, the serious crime is hard to deter, since the elasticity of response to higher expected penalties is low. In such situations, it is better to concentrate law enforcement resources in those areas where behavior is more responsive, even though the harm is not as high. Note that whenever \(p^*_1 = 0\), the same outcome can be reproduced even
in the absence of commitment by setting $s_1 = 0$, and that choosing $s_1 = S$ will lead to an interior (hence sub-optimal) allocation\(^9\). The preceding discussion can be summarized as

**Observation 1:** *If the crime rate in the more harmful act is sufficiently inelastic with respect to expected penalties and the state cannot commit to a monitoring strategy, optimal punishments may be inversely related to social harm.*

It seems unlikely this situation will arise often in reality, at least to any pronounced degree. In any case, the more general point remains—when punishment is valuable in itself, it may not be optimal to punish every type of harmful action maximally.

For the rest of this section, I will take $a = 1$, i.e., the distribution of benefits are identical. In this case, $p_1^* = 0$ is ruled out. The solution lies at the other corner ($p_1^* = 1$) if the following Kuhn-Tucker condition is satisfied: the derivative of the objective function in (13) is non-positive at $p_1 = 1$. Using $a = 1$, this yields

$$c_1 - c_2 \geq 2$$

(14) is the condition under which the less harmful activity will not be monitored at all and hence effectively legalized under commitment.

Turning to the case without commitment, at any interior allocation, the ratio of crime rates must equal the inverse ratio of penalties by (10). Since $s_1^{**} = S = 1$ by Proposition 4, this implies

$$\frac{1 - p_1}{p_1} = s_2 \Rightarrow p_1 = \frac{1}{1 + s_2}$$

Note that an interior allocation is indeed achieved for any $s_2 > 0$. Utilizing the fact that $s_1^{**} = S = 1$, social cost can be written as

$$W = (1 - p_1)(c_1 - p_1) + (1 - p_2)(c_2 - p_2s_2)$$

Using the resource constraint $p_1 + p_2 = 1$ and substituting the expression for $p_1$ obtained above into the objective function and simplifying, we get the following expression for social

---

\(^9\)If $p_1 = 0$, the measure of people committing crime 1 will be one, while the measure of people committing crime 2 is less than one. Since penalties are both maximal, the marginal revenue from allocating police resources to the first activity exceeds that in the second, so this situation cannot be an equilibrium.
cost as a function of the penalty chosen for activity 2:

\[ W(s_2) = \frac{(c_1 - 1)s_2 + c_2}{1 + s_2} \]

The first derivative can be computed as

\[ W'(s_2) = \frac{c_1 - c_2 - 1}{(1 + s_2)^2} \]

which is clearly decreasing in \( s_2 \), i.e., the function is strictly concave. Hence, the problem of minimizing \( W \) by choosing \( s_2 \) always yields a corner solution, with either \( s_2^* = 0 \) or \( s_2^* = 1 \). The former is obtained whenever the derivative is non-negative at \( s_2 = 0 \), i.e.,

\[ c_1 - c_2 \geq 1 \]  \hspace{1cm} (15)

(15) is the condition under which the less harmful activity will be legalized in the absence of commitment. Comparing (14) and (15), one can clearly see that legalization is optimal in the absence of commitment for a strictly larger set of parameter values than those for which it is optimal under commitment. Recall from previous discussion that whenever \( p_2^* = 0 \), the commitment outcome can be achieved by choosing \( s_2 = 0 \). This leads to the following observation

**Observation 2:** If it is optimal to legalize an activity when the state can commit to any allocation of monitoring resources, it is also optimal to do so in the absence of commitment. However, the converse is not true, i.e., there may be situations where an activity that would have attracted a positive amount of penalties and monitoring under commitment is optimally legalized in the absence of commitment.

I next turn to the total harm and collection of penalties that arise under the two different assumptions regarding secondary motives. The total harm \( c \) caused by the activities is given by

\[ c = (1 - p_1^{**})c_1 + p_1^{**}c_2 \]  \hspace{1cm} (16)

while the sum of revenues \( R \) collected from the penalties is

\[ R = (1 - p_1^{**})p_1^{**} + p_1^{**}(1 - p_1^{**})s_2^{**} = p_1^{**}(1 - p_1^{**})(1 + s_2^{**}) \]  \hspace{1cm} (17)
In the redress-retribution model, there are two different cases to consider: interior and corner allocations. First, suppose (15) is satisfied, so that \( s_2^* = 0 \) and \( p_1^* = 1 \). In this case, \( c = c_2 \) and \( R = 0 \). If (15) does not hold, then the optimum penalty lies at the other corner: \( s_2^* = 1 \). Since the crime rates will be in the ratio of penalties, we get \( p_1^* = \frac{1}{2} \). Using this in (16) and (17), we get \( c = \frac{1}{2}(c_1 + c_2) \) and \( R = \frac{1}{2} \). Collecting together these observations, we can write

\[
\begin{align*}
  c_{\text{retr}} &= \begin{cases} 
    c_2 & \text{if } c_1 - c_2 \geq 1 \\
    \frac{1}{2}(c_1 + c_2) & \text{if } c_1 - c_2 < 1
  \end{cases} \\
  R_{\text{retr}} &= \begin{cases} 
    0 & \text{if } c_1 - c_2 \geq 1 \\
    \frac{1}{2} & \text{if } c_1 - c_2 < 1
  \end{cases}
\end{align*}
\]

where the subscript “retr” represents the values obtained in the redress-retribution model.

Next, consider the prevention model (\( \lambda = 0 \)), with the common prevention rate being positive but negligibly small (\( \gamma_1 = \gamma_2 = \gamma \approx 0 \)). In the absence of commitment, punishments are maximal (Proposition 1) and the allocation of monitoring resources will be such that the ratio of crime rates is equal to the inverse ratio of the harms (it is easy to see that an interior solution always obtains in this example), i.e.,

\[
\frac{1 - p_1^*}{p_1^*} = \frac{c_2}{c_1} \Rightarrow p_1^* = \frac{c_1}{c_1 + c_2}
\]

Using this in (16) and (17) above, and using the subscript “prev” to distinguish magnitudes obtained in the prevention case, we obtain expressions for the total harm and total revenue arising in the prevention model

\[
\begin{align*}
  c_{\text{prev}} &= \frac{2c_1c_2}{c_1 + c_2} \\
  R_{\text{prev}} &= \frac{2c_1c_2}{(c_1 + c_2)^2}
\end{align*}
\]

Suppose \( c_1 - c_2 \geq 1 \). In this case, comparison is straightforward, and we obtain: \( c_{\text{retr}} < c_{\text{prev}} \) and \( R_{\text{retr}} < R_{\text{prev}} \). Crime is higher when there is no distraction posed by revenues, and revenues are lower when collecting them is indeed one objective! To state it formally:

**Observation 3:** The total harm from crime may be higher when the state’s only objective is to deter and prevent crime, and revenues may be lower when the state has the secondary objective of raising revenues in addition to deterring crime.
Comparing welfare across the two polar cases is problematic, since they arise from different sets of preferences. Nevertheless, one can ask if a “no-envy” condition holds: i.e., will a state motivated only by deterrence and prevention prefer the outcome that arises when punishment is valued? Conversely, will a state that values punishment wish that it could convince people otherwise? The answer to this question depends on parameters and distributions, but the example solved here illustrates that no-envy may fail in both directions.

Under the assumption that the prevention rate $\gamma$ is negligible, the authorities in the prevention model care only about the cumulative harm, $c$. Observe that when $c_1 - c_2 \geq 1$, $c_{\text{retr}} < c_{\text{prev}}$. In this case, a state focused only on reducing crime would have been more effective vis-a-vis that objective if it had some retribution or revenue motive. On the other hand, the authorities in the redress-retribution case envy the outcome in the prevention model if the additional revenue outweighs the increased crime in the other outcome, i.e., if $R_{\text{prev}} - R_{\text{retr}} < c_{\text{prev}} - c_{\text{retr}}$. Again, considering the case where $c_1 - c_2 \geq 1$, this is satisfied if

$$\frac{2c_1c_2}{(c_1 + c_2)^2} - 0 > \frac{2c_1c_2}{c_1 + c_2} - c_2$$

or

$$c_1 - c_2 < \frac{2c_1}{c_1 + c_2}$$

Since the right hand side above is greater than 1, we have a non-empty region of the parameter space, described by

$$1 < c_1 - c_2 < \frac{2c_1}{c_1 + c_2}$$

where no-envy fails both ways. To summarize:

**Observation 4:** It is possible that a state whose only objective is to reduce the harm from crime would be better off delegating authority to someone who also values retribution or revenues. Also, a state that values retribution or revenues in addition to reducing crime may be better off delegating authority to someone whose only objective is to reduce crime. There are parameters and distributions for which these are simultaneously true.

The last observation points to several interesting possibilities that will not be pursued in detail here. First, and most directly, it shows the possible benefits of delegation at various levels—administrative, political or otherwise. For example, an electorate concerned solely
about improving law and order as a practical matter may want to vote for political parties with a strongly retributive view of justice. Second, the presence of police corruption may actually solve some of the problems raised by the absence of commitment. Presumably, bribes will be positively related to the official penalties chosen by the legislature, which gives it some leverage over the ultimate allocation of monitoring resources. Last, if there is incomplete information regarding the actual objectives of the lawmakers and law enforcers, it opens up interesting possibilities of signaling through the penalty structure chosen. As we have seen, there may be scenarios where each type wants to mimic the other.

7 Conclusion

This paper argues that graduated penalties observed in most legal systems may be an attempt to direct law enforcement efforts towards crimes that are socially more harmful, thereby achieving better deterrence overall. The critical assumptions are: the state cannot commit to a monitoring strategy, and has mixed motives (objectives other than deterrence). However, graduated penalties arise only in the presence of secondary motives that value punishment in itself, such as retribution or fines collected from violators. Other motives that are unrelated to the size of punishment, such as prevention of criminal attempts, will also lead to distortions, but those cannot be corrected by restructuring penalties. The overall harshness of a criminal justice system and the retributive instincts of its designers may be related in counter intuitive ways, and law enforcement may be improved through strategic delegation. Other than reaching these specific conclusions, the paper also tries to extend the framework of analysis in the study of optimal deterrence, by incorporating plausible secondary objectives, dropping strong assumptions regarding policy commitment and exploring general equilibrium effects.
8 Appendix A: The One Act Model

In this section, I show that optimal penalties are always maximal in a one act model of crime, even in the presence of mixed motives and commitment problems. This demonstrates that the general equilibrium effects which arise when there are competing demands on police resources are crucial to the result that penalties may need to be moderated or clearly harmful acts legalized.

Consider a single harmful act whose benefits are distributed according to $F(b)$, social harm of each act being $c$, the penalty $s \leq S$ and the intensity of monitoring $p$, where $p$ can be generated subject to an increasing, convex cost function $\phi(p)$. To guarantee interior solutions, assume that $\phi(.)$ satisfies the Inada conditions: $\phi'(0) = 0$ and $\phi'(1) = \infty$, and further, $F(S) < 1$ (not everyone can be deterred). Each unit of penalty generates a benefit of $\lambda$ to the authorities who impose sanctions and monitor the population for infractions. Since prevention failed to produce less than maximal penalties even in the multi-act model, we ignore it here.

I will only analyze the case of partial commitment, since full commitment will yield maximal penalties for the usual reasons. Fix some penalty $s$. In the subgame, the choice of monitoring is optimal given the crime rate, which yields the first order condition

$$\lambda s [1 - F(ps)] = \phi'(p)$$

(18)

This yields the monitoring choice $p(s)$ as a function of the penalty chosen in the first stage. The left hand side is the marginal revenue from increasing the monitoring rate, while the right hand side is the marginal cost. At the legislative stage, the problem is

$$\min_s [1 - F(ps)] (c - \lambda ps) + \phi(p)$$

(19)

subject to rational anticipation of the outcome in the monitoring stage, i.e., $p = p(s)$.

(18) has the immediate implication that the crime rate $1 - F(ps)$ must be decreasing in the choice of $s$. Suppose not, i.e., suppose $1 - F(ps)$ goes up when $s$ is increased. Then, from (18), it follows that $p$ must be higher, since the marginal returns from monitoring (the left hand side) is higher. However, this means expected sanctions $ps$ must be higher too, which contradicts the assumption that the crime rate $1 - F(ps)$ has gone up.
Now consider two penalties $s'$ and $s''$, with $s'' > s'$. Let $p'$ and $p''$ denote the corresponding monitoring choices that arise in the subgame, and let $\theta' = 1 - F(p's')$ and $\theta'' = 1 - F(p''s'')$ be the resultant crime rates. Then

$$
[1 - F(p''s'')](c - \lambda p''s'') + \phi(p'')
= \theta''(c - \lambda p''s'') + \phi(p'')
< \theta''(c - \lambda p's'') + \phi(p') \text{ since } p'' \text{ is optimal, given } s'', \theta''
< \theta'(c - \lambda p's') + \phi(p') \text{ since } \theta'' < \theta' \text{ and } s'' > s'
= [1 - F(p's')](c - \lambda p's') + \phi(p')
$$

This establishes that the objective function in (19) is strictly decreasing in $s$. Hence, the optimal penalty is the maximal penalty.
Appendix B: Proofs

Proof of Proposition 1: In the full commitment case, the proof is trivial, since the objective function is increasing in \( s_i \), and strictly so if \( p_1, p_2 > 0 \).

Turning to the partial commitment case, the proof is divided into three parts.

Case 1: \( p_i^* > 0 \) for \( i = 1, 2 \). In this case, the solution to the problem coincides with the solution to the more restrictive problem where \((s_1, s_2)\) must be such that (6) holds with equality. Substituting (6) into the objective function (5), it can be rewritten as

\[
1 - F_1 \left( \frac{p_1 s_1}{1 - p_1 \gamma_1} \right) \left[ (1 - p_1 \gamma_1) c_1 + \frac{\gamma_1 c_1}{\gamma_2 c_2} (1 - p_2 \gamma_2) c_2 \right]
\]

\[
= c_1 \left[ 1 - F_1 \left( \frac{p_1 s_1}{1 - p_1 \gamma_1} \right) \right] \left[ 1 + \frac{\gamma_1}{\gamma_2} - (p_1 + p_2) \gamma_1 \right]
\]

\[
= \frac{c_1}{\gamma_2} (\gamma_1 + \gamma_2 - 2p\gamma_1\gamma_2) \left[ 1 - F_1 \left( \frac{p_1 s_1}{1 - p_1 \gamma_1} \right) \right]
\]

where the last line follows from the resource constraint (1). Minimizing this objective function amounts to minimizing the value of the expression \( 1 - F_1 \left( \frac{p_1 s_1}{1 - p_1 \gamma_1} \right) \).

Continuity of \( F_i(\cdot) \) implies continuity of \( p_i(s_1, s_2) \), which in turn implies that if \( p_i(s_1, s_2) > 0 \) at some \((s_1', s_2')\), then the inequality continues to hold in a small enough neighborhood of \((s_1', s_2')\). Note that (6) together with (1) implies that when some penalty \( s_i \) is changed infinitesimally, the measure of people committing each kind of crime, i.e., \( 1 - F_i \left( \frac{p_i s_i}{1 - p_i \gamma_i} \right) \) and \( 1 - F_2 \left( \frac{p_2 s_2}{1 - p_2 \gamma_2} \right) \), must both move in the same direction, since their ratio is a constant. I claim that these magnitudes are strictly decreasing functions of \( s_1 \) and \( s_2 \). Consider a pair of penalties \((s_1', s_2')\) and another pair \((s_1'', s_2'')\) such that (without loss of generality) \( s_1'' = s_1' \) and \( s_2'' > s_2' \), and let \((p_1', p_2') \gg 0\) and \((p_1'', p_2'') \gg 0\) be the pair of monitoring choices respectively which solve (6) and (1). Suppose, contrary to claim

\[
1 - F_1 \left( \frac{p_1'' s_1''}{1 - p_1'' \gamma_1} \right) \geq 1 - F_1 \left( \frac{p_1' s_1'}{1 - p_1' \gamma_1} \right)
\]

for \( i = 1, 2 \).

Since \( s_1'' \geq s_1' \), the above inequality implies \( p_1'' \leq p_1' \), with the inequality strict whenever \( s_1'' > s_1' \). Since \( s_2'' > s_2' \), we have \( p_2'' < p_2' \) and \( p_1'' \leq p_1' \). However, this violates the resource
constraint, generating a contradiction. Hence, contrary to supposition, \(1 - F_1 \left( \frac{p_1 s_1}{1 - p_1 \gamma_1} \right)\) (and hence also the objective function) is strictly decreasing in the penalties \(s_1\) and \(s_2\). Therefore, if \(p_1^{**}, p_2^{**} > 0\), optimal penalties must be maximal, i.e., \(s_i^{**} = S\) for \(i = 1, 2\).

**Case 2:** \(p_2^{**} = 0\). In this case, (6) implies
\[
1 - F_1 \left( \frac{2p s_1^{**}}{1 - 2p \gamma_1} \right) \geq \frac{\gamma_2 c_2}{\gamma_1 c_1}
\]
and the objective function (5) assumes the value
\[
\left[ 1 - F_1 \left( \frac{2p s_1^{**}}{1 - 2p \gamma_1} \right) \right] (1 - 2p \gamma_1)c_1 + c_2
\]
If the inequality above is weak, then an argument exactly as above establishes that the objective function must be strictly decreasing in the penalties in any small neighborhood, and hence they must be maximal. Suppose the inequality is strict. Since the objective function is decreasing strictly in \(s_1\) and weakly in \(s_2\), \(s_1^{**}\) must be maximal and setting \(s_2^{**}\) to be maximal yields the same outcome as any other value.

**Case 3:** \(p_1^{**} = 0\). The argument in this case exactly mirrors that in case 2, and is hence omitted.

**Proof of Proposition 2:** Let \(\gamma_1 = \gamma_2 = \gamma\) and \(F_1(.) = F_2(.) = F(.)\). In what follows, the fact that optimal penalties are maximal both with and without commitment (Proposition 1) will be used throughout. We first show that \(p_1^* > p > p_2^*\). Suppose not, i.e., let \(p_1^* \leq p\). Then, if the monitoring levels are switched, i.e., choosing \(p_1 = p_2^*\) and \(p_2 = p_1^*\) instead, the change in net social cost is given by
\[
\Delta W = (c_1 - c_2) \left[ \left\{ 1 - F \left( \frac{p_2^* S}{1 - p_2^* \gamma} \right) \right\} (1 - p_2^* \gamma) - \left\{ 1 - F \left( \frac{p_1^* S}{1 - p_1^* \gamma} \right) \right\} (1 - p_1^* \gamma) \right]
\]
If \(p_1^* < p < p_2^*\), \(\Delta W < 0\), because the term inside square brackets is negative. This contradicts the fact that \((p_1^*, p_2^*)\) minimizes \(W\), hence \(p_1^* \geq p\). It remains to show that the inequality must be strict.

Suppose \(p_1^* = p\). Then, using the symmetry assumptions in the first order condition (8) for interior solutions, we get
\[
(c_1 - c_2) \left[ \gamma \left\{ 1 - F \left( \frac{p S}{1 - p \gamma} \right) \right\} + \frac{S}{(1 - p S)} f \left( \frac{p S}{1 - p \gamma} \right) \right] = 0
\]

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However, the expression on the left hand side is strictly positive, generating a contradiction. Hence, (8) is not satisfied at \( p_1^* = p \) and it can be ruled out as the solution.

For the next part of the proposition, assume the hazard rate \( h(.) = \frac{f(.)}{1-F(.)} \) is non-decreasing, and define

\[
\psi(p_1) = \left[ 1 - F \left( \frac{p_1 S}{1 - p_1 \gamma} \right) \right] \gamma c_1 - \left[ 1 - F \left( \frac{(2p - p_1) S}{1 - (2p - p_1) \gamma} \right) \right] \gamma c_2
\]

Suppose \( p_1^* \) is interior but contrary to claim, \( p_1^{**} \geq p_1^* \). (8), the first order condition for an interior optimum, yields

\[
\frac{c_1 S}{(1 - p_1^* \gamma)} f \left( p_1^* S \frac{1}{1 - p_1^* \gamma} \right) = \frac{c_2 S}{(1 - p_2^* \gamma)} f \left( p_2^* S \frac{1}{1 - p_2^* \gamma} \right) - \psi(p_1^*)
\]

By assumption, \( p_1^{**} \geq p_1^* > 0 \). Hence, (6) implies \( \psi(p_1^{**}) \geq 0 \). Noting that \( \psi(.) \) is a decreasing function, we have \( \psi(p_1^*) \geq 0 \). Hence

\[
\frac{c_1 S}{(1 - p_1^* \gamma)} f \left( p_1^* S \frac{1}{1 - p_1^* \gamma} \right) \leq \frac{c_2 S}{(1 - p_2^* \gamma)} f \left( p_2^* S \frac{1}{1 - p_2^* \gamma} \right)
\]

or

\[
\frac{f \left( \frac{p_1^* S}{1 - p_1^* \gamma} \right)}{f \left( \frac{p_2^* S}{1 - p_2^* \gamma} \right)} < \frac{c_2}{c_1} \quad \text{since} \quad (1 - p_1^* \gamma) < (1 - p_2^* \gamma)
\]

Utilizing (6) once more, we can write

\[
\frac{c_2}{c_1} \leq \frac{1 - F \left( \frac{p_1^{**} S}{1 - p_1^{**} \gamma} \right)}{1 - F \left( \frac{p_2^{**} S}{1 - p_2^{**} \gamma} \right)}
\]

\[
\leq \frac{1 - F \left( \frac{p_1^* S}{1 - p_1^* \gamma} \right)}{1 - F \left( \frac{p_2^* S}{1 - p_2^* \gamma} \right)} \quad \text{since} \quad p_1^{**} \geq p_1^* \quad \text{by assumption}.
\]

Combining the last two inequalities, we get

\[
h \left( \frac{p_1^* S}{1 - p_1^* \gamma} \right) < h \left( \frac{p_2^* S}{1 - p_2^* \gamma} \right)
\]

But this is impossible if the hazard rate is non-decreasing, since it was already established that \( p_1^* > p_2^* \). Hence, contrary to assumption, \( p_1^{**} < p_1^* \).
Finally, if $p_1^*$ is not interior, i.e., $p_1^* = 2p$, it is trivially true that $p_1^{**} \leq p_1^*$.

**Proof of Proposition 3:** If $p_i^* > 0$, it is easily verified that under Assumption I, the objective function is strictly decreasing in $s_i$, and therefore, each penalty is optimally chosen to be at its maximal level. Basically, since $p_1$ and $p_2$ can be chosen independent of the fines, increasing $s_i$ reduces both the number of people committing the crime, $1 - F_i(p_is_i)$, as well as the net social loss from each criminal action, $c_i - \lambda p_is_i$. If $p_i^* = 0$, varying $s_i$ has no effect on social cost $W$, and hence any choice (including the maximal penalty $S$) is optimum.

For the second part, let $F_1(.) = F_2(.) = F(.)$. Setting the fines at the maximal level, the first-order condition for the choice of $p_1$ and $p_2$ (if the solution is interior) boils down to

$$ F(p_1^*S) - F(p_2^*S) = \frac{1}{\lambda} [f(p_1^*S)(c_1 - \lambda p_1^*S) - f(p_2^*S)(c_2 - \lambda p_2^*S)] $$

(20)

The case $p_1^* = p = p_2^*$ can then be ruled out because in that case, the left hand side of (20) takes the value 0, while the right hand side is $\frac{1}{\lambda}(c_1 - c_2)f(pS) > 0$. Suppose, then, $p_1^* < p < p_2^*$. We can show that in this case, the value of the objective function can be lowered further by switching around these probabilities, i.e., by choosing $p_1 = p_2^*$ and $p_2 = p_1^*$ instead. The change in social cost from doing so is given by

$$ \Delta W = [1 - F(p_2^*S)](c_1 - \lambda p_2^*S) + [1 - F(p_1^*S)](c_2 - \lambda p_2^*S) $$

$$ - [1 - F(p_1^*S)](c_1 - \lambda p_1^*S) - [1 - F(p_2^*S)](c_2 - \lambda p_2^*S) $$

$$ = (c_1 - c_2) [F(p_1^*S) - F(p_2^*S)] < 0 $$

which contradicts the fact that $p_1^*$ and $p_2^*$ constitute an optimum.

**Proof of Proposition 4:** We first show that $s_1^{**} \geq s_2^{**}$. Suppose not, i.e., $s_1^{**} < s_2^{**}$. From (10) and assuming symmetric distribution of benefits, it follows that

$$ F(p_1^{**}s_1^{**}) < F(p_2^{**}s_2^{**}) $$

Now consider the penalties being switched, i.e., $s_1 = s_2^{**}$ and $s_2 = s_1^{**}$. From (10), we conclude that the equilibrium probabilities will be switched too, i.e., $p_1 = p_2^{**}$ and $p_2 = p_1^{**}$. The change in social cost due to this switch is given by

$$ \Delta W = [1 - F(p_2^{**}s_2^{**})](c_1 - \lambda p_2^{**}s_2^{**}) + [1 - F(p_1^{**}s_1^{**})](c_2 - \lambda p_1^{**}s_1^{**}) $$

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\[-[1 - F(p_{1}^{**} s_{1}^{**})](c_{1} - \lambda p_{1}^{**} s_{1}^{**}) - [1 - F(p_{2}^{**} s_{2}^{**})](c_{2} - \lambda p_{2}^{**} s_{2}^{**})
= (c_{1} - c_{2}) [F(p_{1}^{**} s_{1}^{**}) - F(p_{2}^{**} s_{2}^{**})] < 0\]

which contradicts the fact that $s_{1}^{**}, s_{2}^{**}$ are optimum choices. Hence, contrary to supposition, $s_{1}^{**} \geq s_{2}^{**}$. 

Next, suppose $s_{1}^{**} < S$. Consider a new pair of penalties, $s'_{1} = s_{1}^{**} + \epsilon$ and $s'_{2} = s_{2}^{**} + \theta \frac{s_{2}^{**}}{s_{1}^{**}}$, where $\epsilon$ is chosen small enough such that $s'_{1} \leq S$ and $s'_{2} \leq S$. Let $(p'_{1}, p'_{2})$ be the equilibrium in the subgame induced by $(s'_{1}, s'_{2})$. By construction, \( \frac{s'_{1}}{s'_{2}} = \frac{s_{1}^{**}}{s_{2}^{**}} \), and hence using (10), we get

\[
\frac{1 - F(p'_{1} s'_{1})}{1 - F(p'_{2} s'_{2})} = \frac{1 - F(p_{1}^{**} s_{1}^{**})}{1 - F(p_{2}^{**} s_{2}^{**})}
\]

I claim that $p'_{i} s'_{i} > p_{i}^{**} s_{i}^{**}$ for $i = 1, 2$. Suppose not. Then it must be that $p'_{i} s'_{i} \leq p_{i}^{**} s_{i}^{**}$ for $i = 1, 2$ (in order to satisfy the equality above), which in turn implies

\[
p'_{i} \leq p_{i}^{**} \left( \frac{s_{i}^{**}}{s'_{i}} \right) \Rightarrow p'_{i} < p_{i}^{**} \text{ since } \frac{s_{i}^{**}}{s'_{i}} < 1
\]

However, it cannot be that both $p'_{1} < p_{1}^{**}$ and $p'_{2} < p_{2}^{**}$, because it violates the resource constraint (1). Hence, we have established that $p'_{i} s'_{i} > p_{i}^{**} s_{i}^{**}$ for $i = 1, 2$. Now, inspection of the objective function in (9) makes it clear that it is decreasing in the values of the expected penalties $p_{1} s_{1}$ and $p_{2} s_{2}$. We conclude that the net social cost $W$ is lower at $(s'_{1}, s'_{2})$ compared to $(s_{1}^{*}, s_{2}^{*})$, which contradicts optimality. Hence, it cannot be that $s_{1}^{**} < S$, as supposed.

Thus far, we have established that $s_{1}^{**} = S \geq s_{2}^{**}$. It remains to show the last part of the proposition, that $s_{2}^{**}$ is strictly lower than the maximal penalty $S$ when $\alpha$ is low enough. For this to be true, it is sufficient that legalizing activity 2 ($s_{2} = 0$) creates lower social cost than having maximal penalties ($s_{1} = s_{2} = S$). When $s_{2} = 0$, the police will devote all resources to monitoring activity 1, and hence $W$ is given by

\[
[1 - F(2pS)](c_{1} - 2\lambda pS) + \alpha c_{1} \quad (21)
\]

If $s_{1} = s_{2} = S$, then upon using (10), we get $p_{1} = p_{2} = p$ in the subgame. Social cost is given by

\[
[1 - F(pS)] [(1 + \alpha) c_{1} - 2\lambda pS] \quad (22)
\]
Subtracting (22) from (21), we get

\[ [F(pS) - F(2pS)] [c_1 - 2\lambda pS] + \alpha c_1 F(pS)] \]

The first term is negative, while the second is positive, so clearly the net difference is negative if \( \alpha \) is small enough, in which case the optimal penalty on activity 2 must be less than maximal. The exact condition, (11), is obtained by requiring the above expression to be negative. ■
10 References


   URL: http://plato.stanford.edu/entries/punishment/


