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Do Auction Bids Betray Expectations-based Reference Dependent Preferences? A test, experimental evidence, and estimates of loss aversion

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Abstract

In this paper, we provide a novel experimental auction design that exploits an exogenous variation in the probability of winning to test for the presence of expectations-based reference dependent preferences. We prove that (i) in this design, (which is a one parameter modification of a Becker-de Groote-Marschak (BDM) auction), a lower probability of winning will cause a loss averse agent to bid lower, for a large range of intrinsic values for the object. Data from an experiment demonstrate the existence of this effect. The effect would be absent if preferences were 'standard', or if the status quo was the reference point. Thus we contribute to the nascent literature that empirically documents the importance of expectations as a source of reference points. (ii) We further prove that the experimental design enables unique identification of participants' value distribution and loss aversion parameter. Our estimates of loss aversion suggest that women are more loss averse than men. Finally, as a contribution to experimental methodology, we prove that the BDM mechanism will underestimate loss averse participants' values, we quantify the underestimation, and we suggest methods to bound this bias.

JEL Codes: C91, D03, D44

Keywords: Auctions, Expectations, Loss Aversion, Reference dependence

1 Introduction

In this paper, we provide a novel experimental auction design that exploits an exogenous variation in the probability of winning to test for the presence of expectations-based reference dependent preferences. We prove that in this design, a lower probability of winning will cause a loss averse agent to bid lower, for a large range of intrinsic values for the object. The effect would be absent if preferences were 'standard'[1], or if the status quo was the reference point. Thus we contribute to the nascent literature that empirically documents the importance of expectations as a source of reference points. We provide evidence in favor of expectations-based reference dependence, by replicating an experiment based on this design at 4 educational institutions.

The auction underlying the experiment is the Becker-de Groote-Marschak (BDM) mechanism. In a BDM auction, a single bidder competes against a random draw or bid from a known distribution; if her bid beats the random draw, she wins the object and pays a price equal to the random draw; if her bid is lower, on the other hand, she loses. It is easy to see that if preferences are standard, this is an incentive-compatible mechanism (i.e., it is optimal to bid one's value for the object). Due to this property, the BDM auction is popular as a mechanism to 'elicit' preferences; however, our use of it is *not* as an elicitation mechanism.

If a bidder has reference dependent preferences, it is in general *not* optimal to bid one's value in a BDM auction. In a recent paper on reference dependent preferences, Koszegi and Rabin, 2006, provide a descriptive model in which an agent's utility is the sum of consumption utility (the utility from actual consumption: this is conventionally all there is to utility in many applications), and gain-loss utility with respect to a reference point.[2] With standard preferences, there is only consumption utility; in this case, in a BDM auction, bidding one's value for the object is a dominant strategy. However, if the outcome of a bid also gives rise to gain or loss sensations, when the outcome is compared to some reference point, this must be traded off as well in computing the optimal bid, and the 'bid your value' result does not in general hold.

[1]By standard preferences, we mean that the bidder has an intrinsic value v for the object; and winning it at a price p gives a payoff equal to $v-p$, whereas not winning gives a payoff equal to 0.

[2]In early formulations of reference dependence gain or loss with respect to a reference point was alone the carrier of utility (Kahneman and Tversky, 1979).

While earlier work modeled the reference point as exogenous, (often equating it to some 'status quo'), Koszegi and Rabin, 2006, (henceforth, K-R) marks an important departure: it introduces and models the notion of the reference point as endogenous expectations, and further, as a useful benchmark, assumes these expectations are rational. In this framework, gain-loss utility is forward looking: realized consumption is compared, and gain or loss assessed, with the immediately prior expectations about consumption. Lange and Ratan, 2010, adapt and develop this framework to analyze standard auctions, and use their framework to compare bidding behavior in induced values vs. homegrown values auctions, in first and second-price auctions. Our experiment has an auction environment and so the K-R type theory for it follows the Lange-Ratan modeling. The reference point in this formulation is a key factor in our experiment.

In the context of an auction, an agent's bid induces a distribution over outcomes (given the distribution of values of other agents and their bidding strategies).[3] Since this is the distribution of outcomes that is anticipated, in their Koszegi-Rabin type of model, gain or loss sensations arise when comparing the actual realized outcome with the possible outcomes in this distribution; thus this anticipated distribution of outcomes is the reference point. (See Section 2 for elaboration of this point).

We employ the Lange-Ratan modeling to analyze optimal bidding by a loss averse agent, in a BDM auction. Our experiment design manipulates the rational expectations-based reference point by assigning individuals to one of two treatments: individuals bid against uniform distributions with supports $[0, K_1]$ and $[0, K_2]$ respectively in the two treatments, with $K_2 > K_1$. Thus a typical bid b induces a higher probability of winning in Treatment 1 (i.e. against the distribution on $[0, K_1]$) than in Treatment 2; more generally, the distribution of outcomes induced differs across the two treatments. Suppose b is the optimal bid of a loss averse agent who has an intrinsic value v for the object, when she bids against a uniform distribution on $[0, K]$. We prove (Section 2, Proposition 3) that then for a wide range of possible values of v, expanding the support of the competing uniform distribution from some interval $[0, K]$ by increasing K, reduces the marginal benefit from bidding b; so, it is optimal to reduce the bid if the competing uniform distribution has an expanded support. Thus in this case of Koszegi-Rabin kind of reference point, we expect bids to be lower on average in Treatment 2, than in Treatment 1

[3] An outcome in this context is a pair of numbers, the first element of which signifies whether the agent won or lost, and the second element is the price she pays conditional on winning.

(see Proposition 3 for a precise statement).

The treatment effect embedded in the experiment design is absent if the reference point is the status quo. It is easy to see that a status quo reference point does not change when K is changed. The treatment effect is also absent if preferences are standard. In that case, if an agent's value for the object is v, it is optimal to bid v in both treatments. Our statistical comparison of bids in the two treatments shows that bids are indeed significantly lower in Treatment 2 (Sections 3-5), rejecting a null of zero treatment effect and affirming expectations-based, reference dependent preferences.

As the evidence favors reference dependent preferences, we then exploit our auction design to estimate loss aversion parameters, as well as agents' distribution of intrinsic values (since these are not equal to the observed bids, due to reference dependence). The methodological framework is that of the empirical auctions literature, which attempts to identify and structurally estimate models of auctions (the literature is recent but too numerous to cite: see for instance Athey and Haile, 2002, Guerre, Perrigne and Vuong, 2000, 2009); but to our knowledge, this is the first identification result in an auction setting with loss aversion; it is also the first paper to estimate loss aversion parameters in a *commodity* auction setting[4]. We prove (Section 4, Proposition 4) that our experimental design enables unique identification of participants' value distribution and loss aversion parameter. Our parametric likelihood estimates of loss aversion suggest that women are more loss averse than men.

We use the estimation exercise to accomplish two further objectives. First, we re-estimate the treatment effect more accurately, (i.e., by how much is the average bid lower in Treatment 2, in the relevant range of values?), using knowledge of loss aversion and value distribution parameters (Section 5.1). Finally, as a contribution to experimental methodology, we evaluate the bias in the BDM auction, if it is used for eliciting preferences when agents are actually loss averse. We show that the BDM mechanism will underestimate loss averse participants' values, we quantify the underestimation, and we suggest methods to bound this bias.

The theory of reference dependence with expectations-based reference points is recent, active, and has potentially important applications in various areas. Koszegi and Rabin, 2006, introduce applications to consumer behavior

[4]Ratan, 2010 is the only other paper that undertakes estimation of loss aversion in an auction; but there, the setting is one of induced values, making the estimation less onerous and identification superfluous.

and labor supply. Heidhues and Koszegi, 2008, study optimal pricing strategies for firms facing loss averse consumers. Lange and Ratan, 2010, develop the Koszegi-Rabin framework in the context of first and second price auctions, and show that if agents are loss averse, one can explain the aggressive bidding in induced value first price auction experiments, but this does not translate to similar bidding behavior in commodity auctions. Herweg, Muller and Weinschenk, 2009, and Macera, 2010 use the Koszegi-Rabin framework to develop theory in the context of optimal incentive contracts.

While the above papers develop and apply the theory, there is at this point a dearth of empirical evidence to demonstrate that reference points are indeed based on expectations. As far as we know, this paper is the only one so far to manipulate expectations-based reference points and study the effect in *auctions*. The only other paper that directly manipulates reference points based on expectations is Ericson and Fuster, 2010; they have two trading experiments[5]. Clearly, more experiments and experimental designs are necessary to build some stylized facts.

In the context of designing such experiments, we feel moreover that in a K-R setting, an auction is a simpler environment in which to set up a reference dependence experiment. In the Koszegi-Rabin paper, the agent's decision-making environment is as follows. Prior to the decision period, she knows that one of several choice sets can occur (with known probabilities). At this point, she makes a plan of what to choose, in each of these choice sets. This plan results in expectations, or a distribution (because the actual choice set is not known yet), over how she will consume, which becomes her reference point. Then uncertainty resolves and she faces one of the choice sets. In the K-R equilibrium notion ('personal equilibrium'), the agent's optimal choice from this set, subject to her reference point, must equal the choice she had planned from this choice set. In contrast to this setting, an auction has a simpler decision-making environment, since the only action performed is to choose a bid; thus there is no temporal separation between a plan and its execution (see also Section 2). We feel this property makes for simpler manipulations of reference points.

While the literature on loss aversion with exogenously given reference points (often, the status quo is the reference point used, but there are other sources of reference points as well) is large (e.g. Genesove and Mayer, 2001 (housing), Barberis, Huang and Santos, 2001 (finance), Camerer et al., 1997

[5]Other interesting papers which discuss implications of Koszegi and Rabin include Knetsch and Wong, 2009, Smith, 2008, Abeler, Falk, Gotte and Huffman, 2009, Crawford and Meng, 2011 and Spears, 2011.

and Fehr and Gotte, 2007 (labor supply), Sydnor, 2006 (insurance), Benartzi and Thaler, 1995 (trading)), it is not directly relevant to the objectives attempted in this paper.

In what follows, Section 2 contains a discussion of the reference dependent preferences model and the treatment effect. Section 3 describes the experiment, the data, and the first analysis of the treatment effect and the rejection of the hypothesis of standard preferences. Section 4 contains the identification result and estimation strategy. Section 5 presents the estimation results; it also uses these to estimate a more sophisticated, model based treatment effect; it further presents some simulations using our estimates. Section 6 concludes.

2 The Models

Since optimal bidding in a BDM auction when preferences are standard is well known, we discuss it briefly at a later point in this section; we begin with optimal bidding when preferences are reference dependent. The model is an application of the K-R framework to auctions, as developed by Lange and Ratan, 2010; we try to keep the notation similar. We now give a brief preface to the general model (see K-R and Lange-Ratan for details).

In our application, there are 2 commodities, money (commodity 0) and chocolate (commodity 1). The utility from commodity t, $u_t(c_t|r_t)$, depends on the consumption level c_t and reference point r_t. Consumption utility is $v_t(c_t) \equiv u_t(c_t|c_t)$ (K-R). Utility is a sum of consumption utility and gain-loss utility; following the simplification in Lange and Ratan, we normalize gains to zero. Then utility from commodity t is assumed to be given by

$$u_t(c_t|r_t) = v_t(c_t) - \theta \max\{0, v_t(r_t) - v_t(c_t)\}$$

with $\theta > 0$ implying that the agent is loss-averse. Total utility $u(c|r)$ (where c, r are vectors) is the sum of these utilities over the two commodities. Lange and Ratan show that with this specification, the WTA/WTP ratio (willingness to accept vs. pay) equals $(1+\theta)^2$ at the margin. If (vector) consumption and reference levels are stochastic with distributions F_c and F_r respectively, then the agent gets an expected utility $U(F_c|F_r) = \int\int u(c|r)dF_r(r)dF_c(c)$. That is, the expectation involves a point by point comparison of all possible consumption and reference vector configurations.

As explained in Lange and Ratan, the auction structure is simpler than the general K-R framework, when one formulates the rational expectations reference point. The reason is that the choice of a bid induces a distribution of possible consumption outcomes (in terms of obtaining the object and quantum of payment); with rational expectations, this is also the reference distribution. So, $F_c = F_r$. We now apply this framework to the experiment at hand.

In the experiment, which is more fully described in Section 3, each subject participates in a Becker-de Groote-Marschak (BDM) auction. The subject must submit a sealed bid for the object, knowing that there will be a competing bid that is randomly generated by a computer from a uniform distribution on $[0, K]$, where $K > 0$ is a given real number. If the subject's bid is at least as large as the competing bid, the subject wins the object and pays the *competing* bid; if her bid is lower, she loses.

To fix notation, suppose the random competing bid is from a distribution F with density f on $[0, K]$. In the reference dependent preferences model, if an agent has intrinsic value v for the object and bids b, her payoff is defined by

$$\begin{aligned} U(v, b, \theta) = & \textstyle\int_0^b (v-p)f(p)dp - \theta(1-F(b)) \int_0^b pf(p)dp \\ & -\theta \textstyle\int_0^b \left(\int_0^p (p-s)f(s)ds\right) f(p)dp - \theta v F(b)(1-F(b)) \end{aligned} \tag{1}$$

Here, θ is a measure of the degree of loss aversion. (i). The first term corresponds to 'consumption utility',(a weighted average of utilities of the type $(v-p)$ of winning at a price p)[6]. The other terms arise out of reference dependence and loss aversion. (ii). The agent expects to lose the auction, and keep her money, with probability $(1 - F(b))$. In each case of winning with price $p \leq b$, a money loss is experienced relative to this event. The second term gives the expectation of these money losses, multiplied by θ. (iii) The third term captures money losses when the agent wins the auction, pays a price p, but expects to pay $s < p$; the term is θ times the expectation of $(p - s)$ given that she wins the auction, over all $(p - s) > 0$. (iv). The fourth term comes from losses incurred from not winning the object (which happens with probability $(1-F(b))$), relative to reference outcomes in which the object is won (which happens with probability $F(b)$).

Substituting the uniform distribution ($f(s) = 1/K, F(s) = s/K$, for all $0 \leq s \leq K$) in the payoff function, and assuming that the bid $b \in [0, K]$ gives

$$U(v, b, \theta, K) = \frac{v}{K}(1-\theta)b + \frac{1}{K}\left(\theta(\frac{v}{K} - \frac{1}{2}) - \frac{1}{2}\right) b^2 + \frac{\theta}{3K^2} b^3 \tag{2}$$

[6]This also corresponds to the utility specification for standard preferences.

2.1 Optimal Bid Function

The first two Propositions and Corollary 1 describe various aspects of the optimal bid function. A brief preview: for a loss aversion parameter that is not very large (i.e., corresponding to marginal WTA/WTP ratio of less than 2), the optimal bid function is continuous. It is strictly increasing on an interval, until the optimal bid hits K; for larger values than this critical one, the optimal bid stays equal to K. The function is strictly convex on the stretch on which it is increasing. For values less than $(2/3)K$, the optimal bid is shaded below value; for an interval of values higher than (2/3)K, bids exceed values. See Figure 1.

Proposition 1 below characterizes the optimal bid function.

Proposition 1 *Let $\theta \in (0,1)$. Then there exists $\bar{v} > 0$ such that the optimal bid function is given by*

$$b(v) = \begin{cases} \frac{(1+\theta-(2\theta v/K))-\sqrt{(2\theta v/K-(1+\theta))^2-4\theta(1-\theta)v/K}}{2\theta/K} & \text{if } v < \bar{v} \\ K & \text{otherwise} \end{cases} \quad (3)$$

Proof. See Appendix.

A part of the proof sets up the problem of maximizing the utility (Eq.(2)) subject to the bid being in the interval $[0, K]$. (From Eq.(1), we can see that a bid greater than K fetches the same utility as bidding K (the smallest bid at which the agent wins the object with probability 1); so we can restrict bids to this interval). The first order condition for an interior optimum is a quadratic in bid b. The optimal bid function in Proposition 1 has 2 segments. On the first interval $[0, \bar{v}]$, the optimal bid is given by the lower root $lr(v, \theta, K)$ (or $lr(v)$ for short) of the first order quadratic equation. If $0 < \theta < 1$, $lr(v)$ is increasing and convex. For $v \geq \bar{v}$, the optimal bid equals K.

Proposition 1 leaves open the possibility of a jump in the optimal bid function, at $v = \bar{v}$. Whether the optimal bid function is continuous, or has a jump, depends on the magnitude of the loss aversion parameter θ; a jump discontinuity is introduced if θ is greater than a critical level, as the following Proposition shows.

Proposition 2 *Let $\theta_c = \sqrt{2} - 1$. If $\theta \leq \theta_c$, the optimal bid function is continuous, and $\bar{v}$ in Proposition 1 solves $lr(\bar{v}, \theta, K) = K$. If $\theta > \theta_c$, the* *optimal bid function has a jump at some value $\bar{v}$ at which the utility from bidding K equals the utility from bidding $lr(\bar{v}, \theta, K) < K$.*

Proof. Appendix.

The critical value θ_c of the loss aversion parameter corresponds to a marginal WTA/WTP ratio equal to 2. A value of θ significantly larger than this should show up in bid data as a significant gap between bids equal to K and lower bids. The bid data that we have does not exhibit such a gap, so for simplicity we will restrict the discussion of the treatment effect that follows to the case where the optimal bid function is continuous.[7]

We close this discussion with a corollary that is used later. It shows that for a large stretch of values $v \in (0, \frac{2}{3}K)$, the optimal bid involves shading the bid below one's value; whereas, for values beyond $\frac{2}{3}K$, up to a certain point, it is optimal to submit a bid greater than one's value.

Corollary 1 *Let $\theta \in (0, \theta_c]$. In the interval of values $[0, \bar{v}]$, the optimal bid function $b(v)$ is strictly convex; moreover, $b(v)$ satisfies*

$$b(v) \begin{cases} < v & \forall\ v \in (0, \frac{2}{3}K) \\ = v & for\ v = \frac{2}{3}K \\ > v & \forall\ v \in (\frac{2}{3}K, \bar{v}) \end{cases}$$

It is straightforward to verify convexity by differentiating the bid function in Proposition 1 twice: in fact, convexity is satisfied provided $0 < \theta < 1$. When $v \in [0, \bar{v}]$, Proposition 1 establishes that $b(v)$ equals the lower root $lr(v, \theta, K)$ of the quadratic first order condition. It is easy to show that the equation $lr(v, \theta, K) = v$ has a solution at $v = \frac{2}{3}K$. The Corollary thus follows, since $b(v)$ is convex, continuous and strictly increasing in the relevant interval. Figure 1 illustrates.

2.2 Theoretical Treatment Effect

We test whether agents' bidding behavior exhibits reference dependent preferences, as opposed to standard preferences, using a comparative static property of the reference dependence model. The experiment is designed to

[7] Strictly speaking, of course, this statement is true provided the bid data correspond to a single loss aversion parameter value, common to all bidders.

compare optimal bid functions in 2 alternative scenarios: bidding against a draw from uniform $U[0, K_1]$, and bidding against a draw from $U[0, K_2]$, with $K_2 > K_1$. These scenarios correspond to Treatments 1 and 2 respectively, of the experimental design (Section 3).

Consider first an agent with standard preferences, and intrinsic value v for the object. Winning the object at a price p (when the draw p is less than her bid) gives the agent a payoff equal to $v - p$ (so her expected payoff when bidding against a draw from the uniform distribution on $[0, K]$ is given by simply the first term in Eq.(1)). It is well known that in this case, it is a weakly dominant strategy for the agent to bid v. This agent's bid should therefore be identical across the 2 treatments.

There is a caveat. Bidding against a random draw from $U[0, K]$, in the current setup an agent with value $v \geq K$ is equally well off bidding K instead of v, and winning for sure. Thus the optimal bid function comparable to that for Proposition (1) for reference dependent preferences, is the function $\beta(v)$ defined by: $\beta(v) = v$ for all $v < K$, $\beta(v) = K$ for $v \geq K$. For an agent using this bid function, her bids would be identical across the 2 treatments if $v \leq K_1$. If $v > K_1$, her bid in treatment 2 (being equal to v) is higher than her bid of K_1 in treatment 1. We would therefore expect the average bid for Treatment 2 to be at least as large as the average bid for Treatment 1, if agents had standard preferences. See Figure 3.

We would not expect a Treatment Effect in this experiment if the reference point is the status quo, either. For example, suppose the status quo is $(0, 0)$ (absence of the good, and zero payment for it). Then the utility from bidding b, for an agent with value v for the good, is given by $U(v, b, \theta) = \int_0^b (v - p)\frac{1}{K}dp - \theta \int_0^b \frac{p}{K}dp$. The second term corresponds to loss sensations from paying p when the reference point is 0. It is easy to see that when the optimal bid is an interior optimum (i.e. less than K), it equals $v/(1 + \theta)$. While this is shaded below the value v, it does not depend on the value of K; and so the treatment effect should be zero over a relevant range of values.

Now consider the case of an agent with reference dependent preferences and a rational expectations-based reference point. Proposition (3) below establishes that for a large range of values, agents will bid *lower* in Treatment 2 than in Treatment 1. The intuition is that given a bid b, increasing the length of the interval $[0, K]$ from which the competing random draw is generated decreases the agent's probability of winning. At the margin, there is a utility decrease owing to increased loss sensations, that leads the agent to optimally bid lower.

For simplicity, we state the Proposition for loss aversion parameters that give rise to continuous optimal bid functions.

Proposition 3 *Treatment Effect:*

Let $\theta \leq \theta_c \equiv \sqrt{2} - 1$. Suppose an agent has reference dependent preferences. Let her optimal bid functions in the 2 treatments (bidding against draws from $U[0, K_1]$ and $U[0, K_2]$, $K_2 > K_1$) be denoted $b_1(v)$ and $b_2(v)$ respectively. Let $\bar{v}_1$ be the value at which her optimal bid function in Treatment 1 satisfies $lr(\bar{v}_1, \theta, K_1) = K_1$. There exists $v_\tau > \bar{v}_1$ s.t. for all $v \in (0, v_\tau)$, $b_2(v) < b_1(v)$.

Proof. Appendix.

Figure 2 illustrates Proposition (3). The two bid functions are increasing and convex on the stretches in which the bids are equal to the lower root of the quadratic first order condition. Since $b_2(v)$ lies below $b_1(v)$ initially, and is continuous, it cuts it at v_τ to the right of $\bar{v}_1$. For v greater than this, it is optimal to bid K_1 in Treatment 1, whereas the optimal bid in Treatment 2 is higher. Note that the Treatment effect says that for *any* v for which it is optimal to bid less than K_1 in Treatment 1, it is optimal to bid even lower in Treatment 2. Figure 2 is drawn for a value of $\theta = 0.2$.

3 Experiment, Data and a First Test

The data used in this paper consist of participants' bids in BDM auctions; they are extracted from an experiment that was replicated at 4 educational institutions in Delhi, India in 2010-11. The subjects/participants were students of various undergraduate and Master's courses. The commodity for which BDM auctions were conducted was chocolate; the particular chocolate bar used here was an 80 gram bar of premium dark chocolate. The brand was unknown to the students as this chocolate is a fairly recent boutique brand that is not marketed from stores; it is available at a single delicatessen.

The experiment was replicated with with one institution being covered in a day. The short time span and considerable distance between the institutions was designed to prevent any information about the experiment flowing from an institution that was recently visited and another that was not yet covered[8].

[8] The large size of the city and the heavy institution density also act as barriers to quick flow of information for experiments of this size

At each institution, students who signed up for the experiment were randomly assigned to one of the two treatments. The two treatments were done one immediately after the other, so no information about the experiment was shared between participants. The sequencing of the two treatments was randomly determined. Given the population chosen for the experiment, the subjects were homogeneous in terms of age(19-23) and their student background; gender was a principal source of possible heterogeneity. So the list of students that signed up was divided into the male and female subgroups, and randomized allocation to the treatments was done separately by gender. Overall, the number of male students was significantly larger. Also, because not everyone that signed up actually showed up for the experiment, the actual proportion of each gender across the treatments is not equal. However, the absentee rate was small.

The subjects were instructed on the auctions in a classroom kind of setting, with an attempt, however, to provide adequate private space to each participant. For their participation in the experiment, each subject was given Rupees 200 at the beginning; this money was one of the items in the folder of materials given to each of them. It was emphasized that for each participant, this was an exercise in individual decision-making. Following a display and description of chocolates (without revealing the brand), about a sixth of each bar of 80 gram chocolate was given to each subject for tasting. This was followed by a description of the auction. The BDM auction was explained in detail, contrasted with the more familiar first-price auction, and illustrated with an example. The consequences of incrementing a bid by some amount, on the probability of winning and on the increase in expected payment were brought out. Questions were taken and answered along the way and at the end of this training part. Since the BDM is increasingly used for preference elicitation, guides for standard subject training practice exist at the textbook level (e.g. Lusk's and Shogren's 2007 book on experimental auctions). The one departure from the usual practice, in our experiment, was: it is standard practice to give an example to explain to the subject that it is optimal for her to bid her value. Since this is not the case for loss averse bidders, we obviously excluded this practice from our instructions.

Subjects were asked to write down bids on the sheet of paper provided to each of them for this purpose, and put it in the provided envelope; they also filled in a short questionnaire on basic demographics and responses to the chocolate. The envelope and sheet of paper of each subject had a unique serial number. Random draws from the competing distribution for the treatment had been recorded on sheets of paper and put in envelopes by enumerators; these envelopes used the same set of serial numbers as the ones for the envelopes given to subjects to seal their bids. The envelopes were then matched using common serial number, and the result of each auction was implemented; winners got an 80 gram bar of the chocolate and paid the competing random draw as the price.

The reference dependence theory of Section 2 shows that if an individual is loss averse and has an intrinsic value v in an appropriate interval, then her bid is lower competing against a uniform distribution on $[0, K_2], K_2 > K_1$, than against that on $[0, K_1]$. The experiment design is however between-subject. We felt that having the same individual bid in separate auctions with two competing distributions ran the risk of confusion, unanticipated framing, and priming. If individuals are randomly assigned to the two treatments, with a large enough sample size, then we would expect intrinsic values of the object to be similar *on average* across the two treatments. So, if preferences were standard, or if the reference point is the status quo, we would expect mean bids to be similar across the treatments; whereas in the case of expectations-based reference dependence, we would expect the mean bid to be lower in Treatment 2.

In fact, we do see this treatment effect, that is implied by the reference dependence model, for each institution. However, owing to the large variances in bids, the small sample sizes at individual institutions (30-40 per treatment) imply that we are powered to detect significance at 3 out of the 4 institutions (even though the effect is of appreciable magnitude at all of them). So we begin our analysis by pooling the data from the 4 institutions. An institution-wise analysis is reported later, in Section 5 (and Table 5).

For this pooled sample, summary statistics for the two treatments are presented in Table 1. There were 301 subjects in all, quite evenly divided across the two treatments. The intervals for the uniform distributions in Treatments 1 and 2 had right supports at Rupees 150 and 200 respectively. While the mean bid is higher in Treatment 1 than in Treatment 2 (by about Rupees 5.52, with a standard error of about 3.58), a simple comparison of these means is not correct as we now argue.

It was shown in Section 2 that for values beyond a right support K, choosing to bid K is optimal (even with standard preferences: of course, in the standard preferences case a bid equal to one's value is optimal as well). Thus the data show that no one bid above 150 and 200 respectively, in the treatments. (The proportions of bids equal to 150 and 200 for the respective treatments are 6.5 percent and 1.4 percent). Under the null hypothesis that people have standard preferences, a subject with value v between 150 and 200 could then optimally bid 150 if she is in Treatment 1, and v if she is in Treatment 2. So the range of values for which we expect the *same* bid in the two treatments (under the null hypothesis) should be restricted to $[0, 150]$.

Since we observe bids (not values), we achieve this now by excluding from both treatments all bids that are greater than or equal to 150.

Table 2 summarizes the data for this truncated sample. The means for the two treatments are 64.15 (treatment 1) and 56.49 (treatment 2); the t-statistic equals 3.12. So the difference in means is highly significant. Since the treatment effect ranks the bids (with bids from treatment 2 being ranked lower), we also did the nonparametric Wilcoxon test. The p-value, at 0.0358, is again highly significant.

Table 3 reports a regression which establishes the same fact. We regress bids on a treatment dummy (=1 for Treatment 2) and gender (=1 for females); the coefficient of the treatment dummy is estimated to be -7.4 and its standard error is 4.3. For a one-tailed test of the absence of treatment effect versus its presence (in the direction that bids are lower in treatment 2 as predicted by the theory), the implied p-value for the treatment dummy coefficient is 0.044. So this regression clearly suggests that the treatment effect is negative and significant (at the 95% confidence level).

4 Estimation

Since the data set has only one bid per individual, we cannot estimate loss aversion parameters for each individual. In fact, it is standard in other empirical auction applications (e.g. estimating the degree of risk aversion of bidders) to assume and estimate a common parameter for the entire set of agents (e.g. Guerre, Perrigne and Vuong, 2009). The main source of heterogeneity in our data set that is possible to account for is gender. So, we estimate gender-specific loss aversion parameters.

4.1 Identification

Our estimation is enabled by the following identification result: loosely put, given a pair of observed population bid distributions, arising out of a population of agents submitting bids in 2 BDM treatments (using uniform distributions on $[0, K_1]$ and $[0, K_2], K_2 > K_1$), we can obtain a unique loss aversion coefficient θ and value distribution G for the population, that can generate the pair of observed bid distributions. We state this result formally now, for the case of continuous bid functions; so, we restrict the class of reference dependent models to come out of loss aversion parameters $\theta \in (0, \theta_c]$. The identification employs a certain α-quantile of the bid distributions. Limiting the space of the loss aversion parameter as above necessitates that the $\alpha-$quantile bid for Treatment 2 be bounded below as in Proposition 4 below. It can be shown that for the more general case of $0 < \theta < 1$, the implied bound is close to 0; so this bid restriction is quite weak for the more general case.

To state Proposition 4, we define α to be the quantile at which the bid in Treatment 1 equals $\frac{2}{3}K_1$. We use the notation $b_{1\alpha}, b_{2\alpha}$ to represent the bids for this α-quantile, for the two Treatments; so we have by construction $b_{1\alpha} = \frac{2}{3}K_1$.

Proposition 4 *Suppose G_1, G_2 are the observed bid distributions for the two Treatments, and have continuous densities upto their atoms at the bids K_1 and K_2 respectively. If $b_{2\alpha} \geq lr(\frac{2}{3}K_1, \theta_c, K_2)$, then there exists a unique loss aversion parameter $\tilde{\theta}$, and a value distribution G unique on $[0, \frac{K_2}{1+\tilde{\theta}})$, such that if the population of agents has the distribution G of values and loss aversion parameter $\tilde{\theta}$, their bids in the two Treatments will generate the bid distributions G_1 and G_2.*

Proof. Appendix.

The idea behind the proof is partly illustrated in Figure 4. It shows that the bid $b_{1\alpha} = (2/3)K_1$ is optimal for the value $(2/3)K_1$, regardless of the loss aversion parameter (this follows from Corollary 1). The proof goes on to show that then the corresponding α-quantile bid for Treatment 2, $b_{2\alpha}$, is the optimal bid for value $(2/3)K_1$, for a *unique* loss aversion parameter $\tilde{\theta}$. Having recovered $\tilde{\theta}$, we simply take the bid distribution over the longer interval (Treatment 2), and since values are a monotone transform of bids via the optimal bid function $b_2(v)$ (upto K_2), we can compute the value distribution G for v in the usual way.

4.2 Estimation

We use a parametric Maximum Likelihood Estimation strategy to estimate the value distribution and degree of loss aversion of the subjects from their bids in the experiment/BDM auction. We shall assume that the value v_i (or consumption utility from a unit of the commodity) for individual i is independent of other individuals' values and is distributed as a normal with mean μ and variance σ^2. After working with several specifications, the precise model we estimated assumes that males and females draw their values from normal distributions with different means, and that they have different loss aversion parameters. For the sake of simplifying the exposition, we however use a single distribution and a single loss aversion parameter. We represent this distribution and density by Φ and ϕ respectively; these are functions of μ and σ, which we suppress for simplicity.

Since the optimal bid function $b(v, \theta, K)$ for a treatment with BDM competing distribution being uniform on $[0, K]$ is strictly increasing for values in $[0, K/(1 + \theta))$ and equal to K thereafter, the contribution of a bid to the likelihood function is as follows.

For $b < K$, the likelihood contribution is given by $\phi(b^{-1}(b, \theta, K))$. For a bid equal to K, all we know is that the value exceeds $K/(1 + \theta)$. Thus the likelihood contribution equals $1 - \Phi(K/(1+\theta))$. The aggregate log-likelihood function adds up the logarithms of these terms.

5 Results

5.1 Loss Aversion Estimates and Treatment Effect

The estimated normal value distribution has a mean of 66.05 and standard deviation of 31.97 for males (see Table 4). Values for females are estimated to have a significantly higher mean of 73.27. The degree of loss aversion for male subjects is estimated to be 0.196 (corresponding to a marginal WTA/WTP ratio of 1.43). For females it is almost 40% higher at 0.272 (WTA/WTP equals 1.62). (see Table 4).

Armed with estimates of the value distributions and loss aversion parameters, we are also able to get more informed estimates of weighted means and mean differences for the two treatments. In the test reported in section 3, all bids that are greater than or equal to 150 were dropped. But in fact, in the loss aversion model, which we have statistically shown to be closer to reality, Treatment 2 bids are lower than Treatment 1 bids upto the value v_τ in Figure 2. Separately for males and females, our estimates of the loss aversion parameter give us an estimate of v_τ. Of the bids equal to 150 in Treatment 1, we now *retain* some; we drop the proportion of bids equal to 150 that equals the probability that $v > v_\tau$, using our estimates of the distribution of values.

We then recompute the difference between the average bids in the two treatments. We do this exercise separately for males and females and then recompute the weighted means. The weighted mean bid for Treatment 1 is estimated at 68.21, while for Treatment 2 it is 57.17, the t-value is 3.69, thus reconfirming a highly significant treatment effect, and one of fairly large magnitude (16 percent).

We present the estimated bid functions for males and females for each of the two treatments in Figure 5. The bid functions for Treatment 2 lie below those for Treatment 1, till $v = b_2^{-1}(150)$. Beyond this, $b_2()$ lies above $b_1()$, as it does not pay to bid higher than 150 in Treatment 1.

Corollary 1 established that the subjects whose values belong to $(0, \frac{2}{3}K)$ are expected to be bidding conservatively relative to their values, while those whose values are greater than $\frac{2}{3}k$ (for a stretch) bid aggressively. Note that such departure from bidding their true values is therefore more pronounced for females (who have a higher loss aversion θ) than for males.

Table 5 presents estimates of the treatment effect by institution, corresponding therefore to the 4 replications of the experiment. The treatment effect is in the correct direction for all 4 institutions. Its magnitude varies approximately from 10 to 30 percent. The t-statistic is highly significant for 3 of the 4 institutions, and the p-values for the Wilcoxon test indicates signficance for these institutions (between 2 percent and 10 percent); for the 4th institution, the sample size was too small to detect significance.

5.2 An Application to Preference Elicitation

The BDM auction is a popular way to elicit preferences or willingness to pay. In such applications, it is assumed that the observed bids are the true values of subjects (i.e. the BDM is incentive compatible). But if agents are loss averse, this is not an accurate assumption; we quantify the consequences of this inaccuracy for our data set as follows. For this application, we use the data and estimates for males.

First, using the loss aversion model, we estimated a latent distribution of values for males. Suppose loss aversion is the true model. The mean value is Rupees 66.05. The cumulative distribution can also be regarded as a demand curve for chocolate, assuming unit demand for each individual; we simply have to put values on the vertical axis, and the *proportion* of population that has values greater than or equal to each given value, on the horizontal axis. (See Figure 6).

If a researcher assumes that preferences are standard, however, she would run the BDM experiment with a single distribution. Suppose this is uniform $[0, 200]$. Assuming that the bids are equal to values, we estimate a parametric normal value distribution by the method of Maximum Likelihood, with the Treatment 2 data for males. The mean value is now estimated to be about 54. This is considerably lower than when we assume loss aversion, because in that case values are greater than the optimal bids for much of the value distribution. We plot the corresponding demand curve in Figure 6; it is considerably lower than the estimate under loss aversion. We then repeat this entire exercise for Treatment 1 data for males; this corresponds to assuming that the researcher uses this narrower distribution. The estimated mean value is now about 64; this is actually quite close to the one we estimated in the loss aversion model. The corresponding demand curve (Figure 6) is also much closer to the estimated demand under loss aversion. The reason is the direction of the Treatment Effect: with loss aversion, the bids are more aggressive with the narrower range (upto 150; Treatment 1), than in Treatment 2; and hence are closer to the underlying values.

One important implication of this is that if we do a conventional preference elicitation exercise, using the incentive compatibility assumption for the BDM auction, and if preferences are actually loss averse, then our estimate of the value distribution will be closer to the truth if our choice of BDM distribution is a smaller interval. Of course, the choice of interval must be judicious; if it is too narrow, we run the risk of a lot of bid censoring.

6 Conclusion

In conclusion, our novel auction experiment design enables testing for the presence of reference dependent preferences using observed bids from BDM auctions. The bid data from an experiment with this design rejects the hypothesis that bidders have 'standard' preferences, and affirm the alternative of reference dependence. Facilitated by our identification strategy (that rests on this experiment design), we also estimate the extent of loss aversion in this auction context. We apply our estimates to quantify the underestimation of values if the BDM auction is used for preference elicitation under the assumption of 'standard' preferences.

The loss aversion estimates that we get are important in their own way. They enable a comparison with estimates from other papers in other contexts. They also suggest that females are more loss averse than males. Furthermore, they can be used to throw fresh light on the use of the BDM auction as a preference elicitation mechanism. Our data show that people are loss averse, and on average, their bids are lower than their values for the object. Incorrectly assuming that the BDM auction is incentive compatible would therefore lead to an underestimation of values, willingness to pay, and demand. We quantify this showing in particular that a judicious choice of the BDM distribution can still minimize this underestimation.

7 Tables and Figures

Table 1

Summary statistics of the data

Statistic	Treatment 1 (K=150) Estimate	Treatment 2 (K=200) Estimate
Mean	68.03	62.68
Standard error	3.41	3.71
Number of bids	155	146
Male-female ratio	1.18	1.86
Mean (Males)	65.58	59.93
Standard error (Males)	4.31	4.20
Mean (Females)	70.93	67.82
Standard error (Females)	5.43	7.19
Number of bids equal to K	10	2
Proportion of bids equal to k	0.065	0.014

Table 2

Summary statistics of the *truncated** data

Statistic	Treatment 1 (K=150) Estimate	Treatment 2 (K=200) Estimate
Mean	64.15	56.49
Standard error	3.10	2.95
Number of bids	141	136
Male-female ratio	1.27	2.02
Mean (Males)	64.04	54.76
Standard error (Males)	4.02	3.47
Mean (Females)	64.29	59.98
Standard error (Females)	4.88	5.49

* *Retaining bids that are strictly between zero and* 150

Table 3

Regressing Bids (truncated sample) on Treatment and Gender Dummies

Variable	Estimate	Std Error	p value
Intercept	63.024	3.578	$< 2e - 16$
Treatment	-7.385	4.312	0.0879
Gender	2.559	4.427	0.5637

Table 4

Maximum Likelihood Estimation - Results

Variable	Estimate	Std error	t value	p value
Mean (males)	66.0478755	2.0975472	31.4881	$< 2e-16^{***}$
Std dev	31.9686438	1.0486600	30.4852	$< 2e-16^{***}$
Theta (males)	0.1956383	0.0013571	144.1574	$< 2e-16^{***}$
Difference in theta	0.0765039	0.0015927	48.0343	$< 2e-16^{***}$
Difference in means	7.2203240	3.1459692	2.2951	0.02173^{*}

Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 .

Mean (females) = Mean (males) + Difference in means

Theta (females) = Theta (males) + Difference in theta

Table 5

Institution-wise Treatment Effect Estimates

Institution	Treatment 1 Weighted Mean	Treatment 2 Weighted Mean	t-statistic	wilcox test p-value
A	85.95	73.02	9.18	0.0586
B	65	51.49	3.57	0.09287
C	58.27	52.46	0.49	0.3483
D	60.48	39.08	3.21	0.0156

Figure 1 : Optimal Bid Function with Loss Aversion

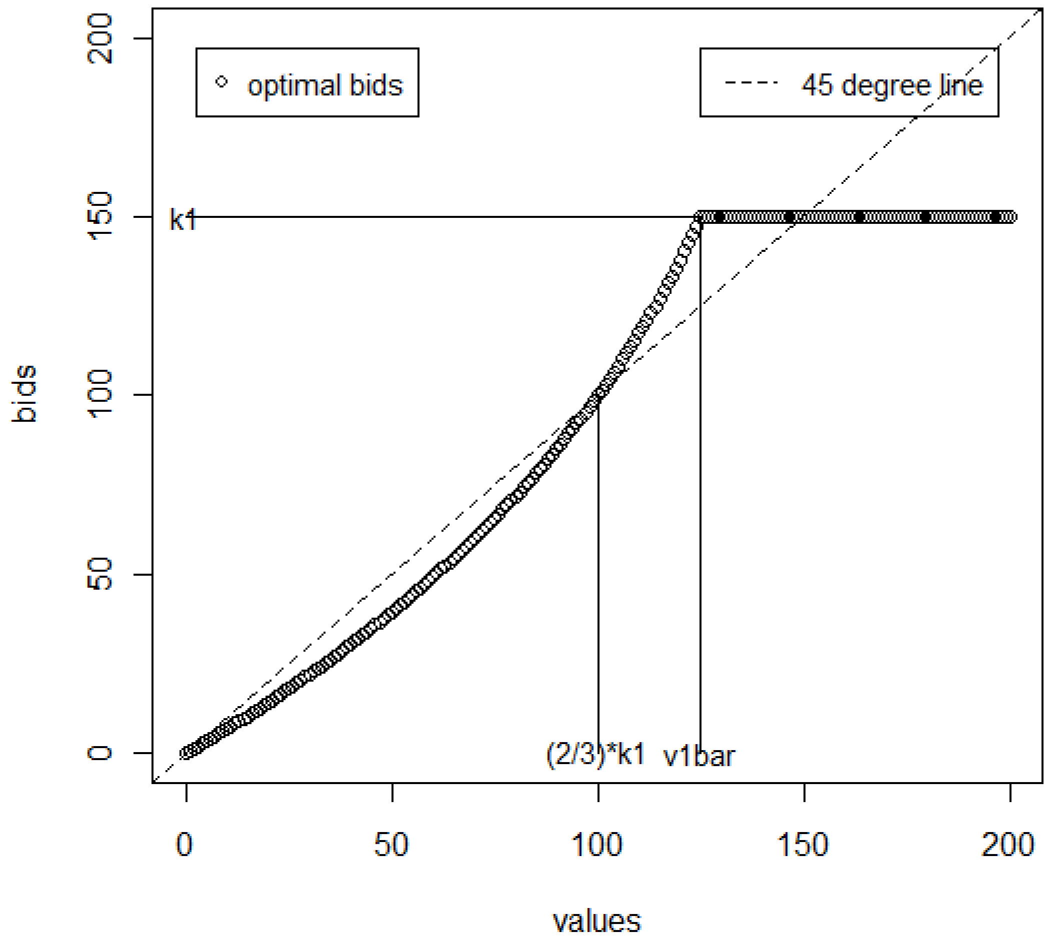


Figure 2 : Treatment Effect

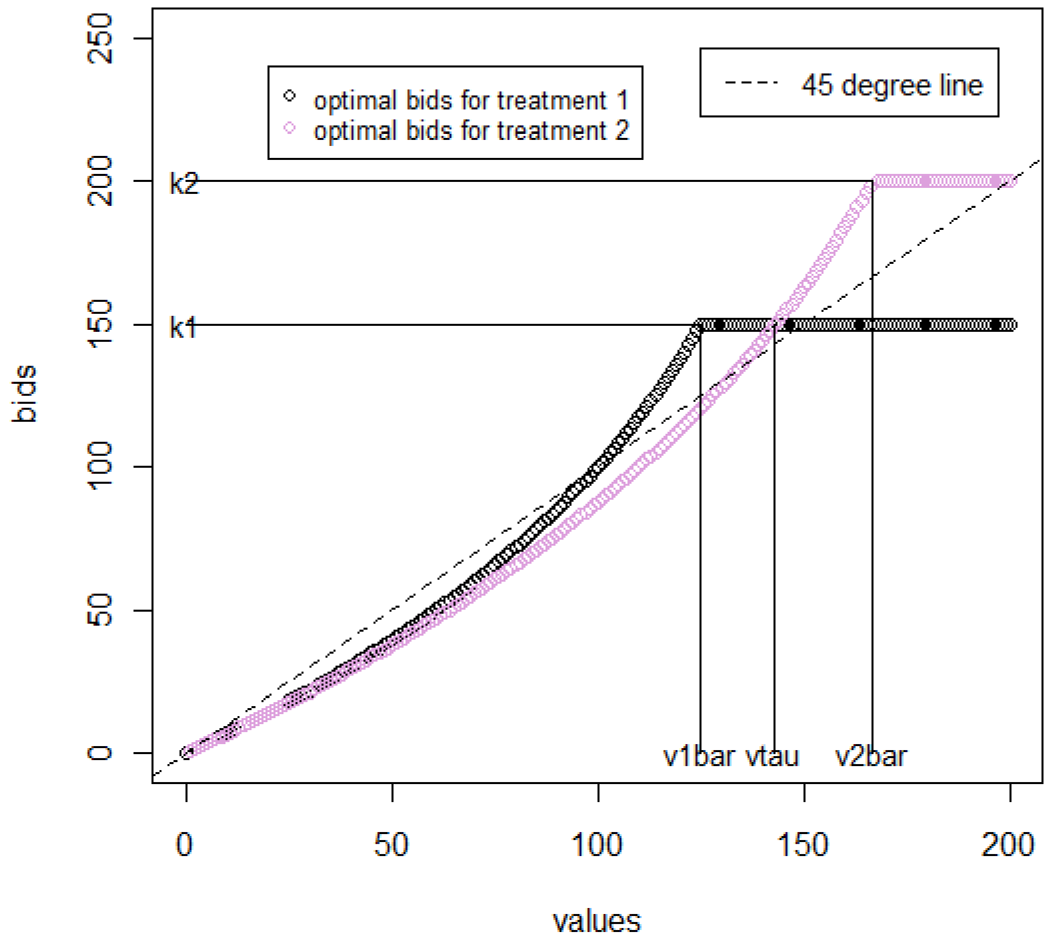


Figure 3 : Optimal bid function with standard preferences

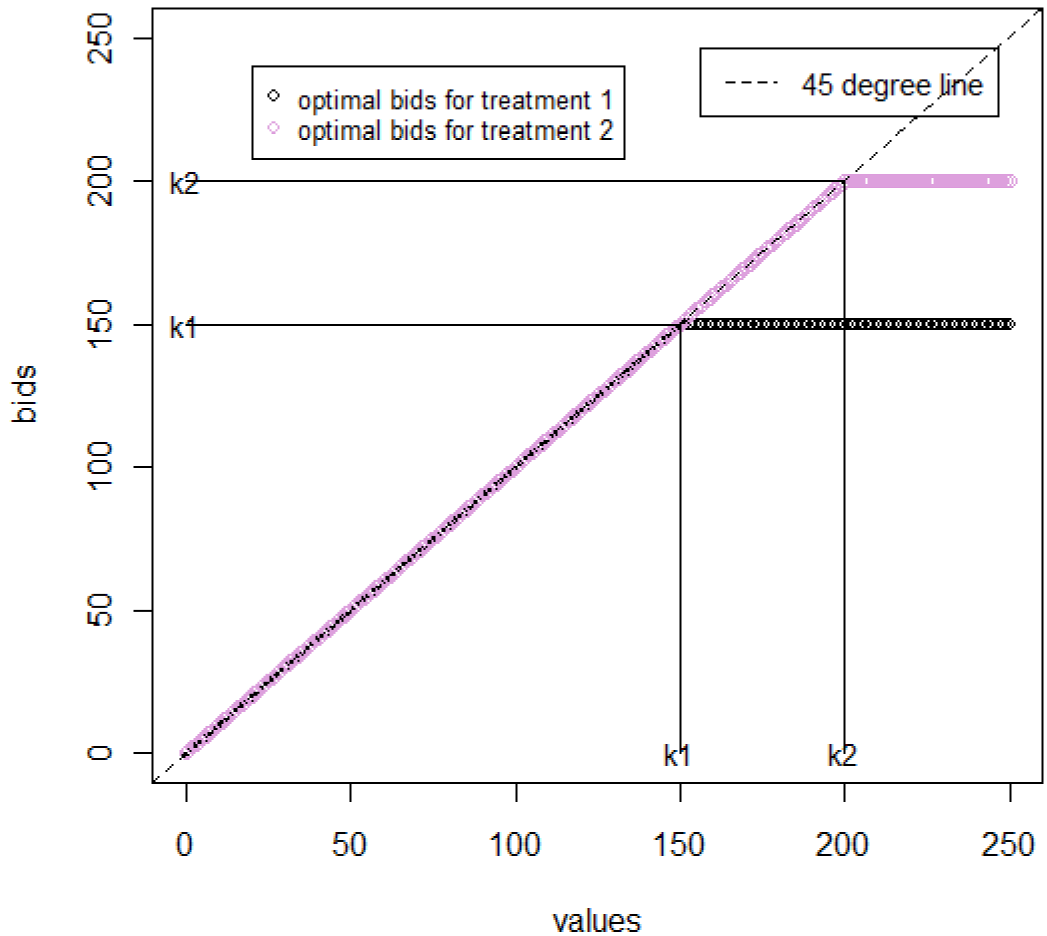


Figure 4 : Impact of change in theta on bids

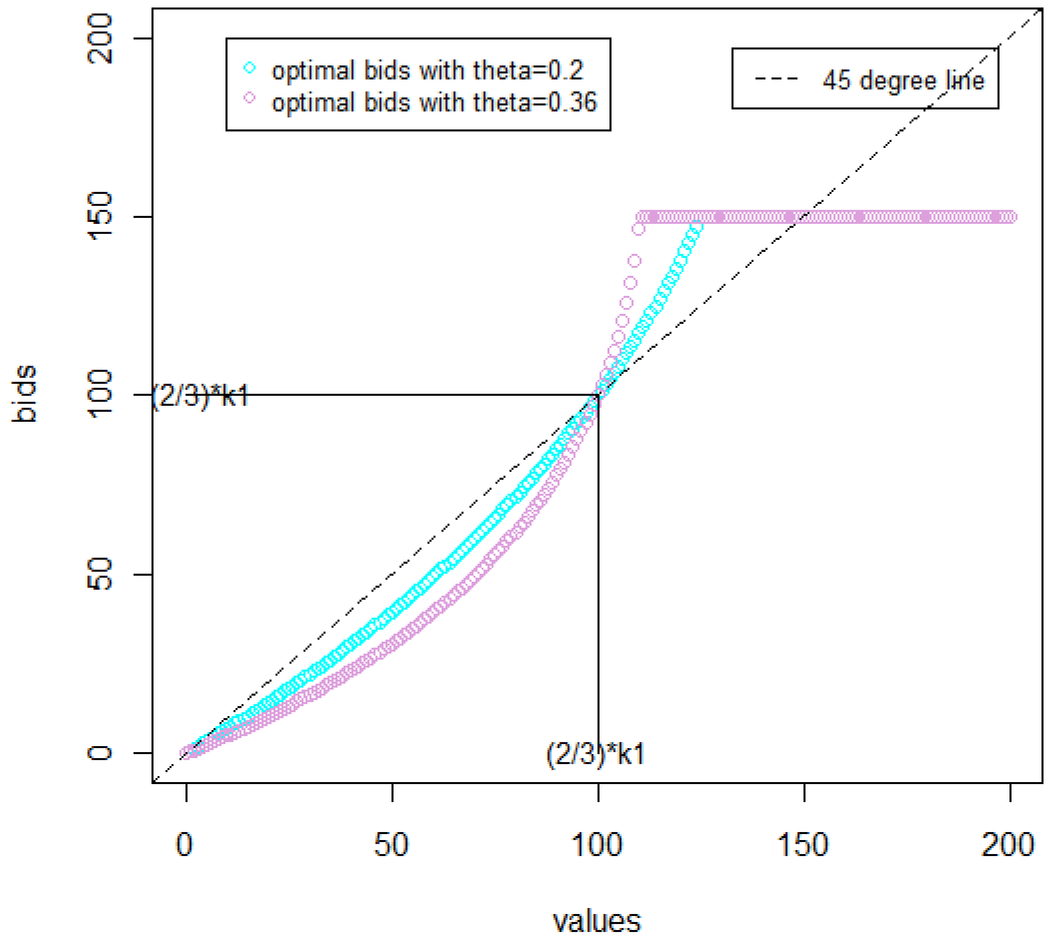


Figure 5 : Estimated bid functions (loss aversion specification)

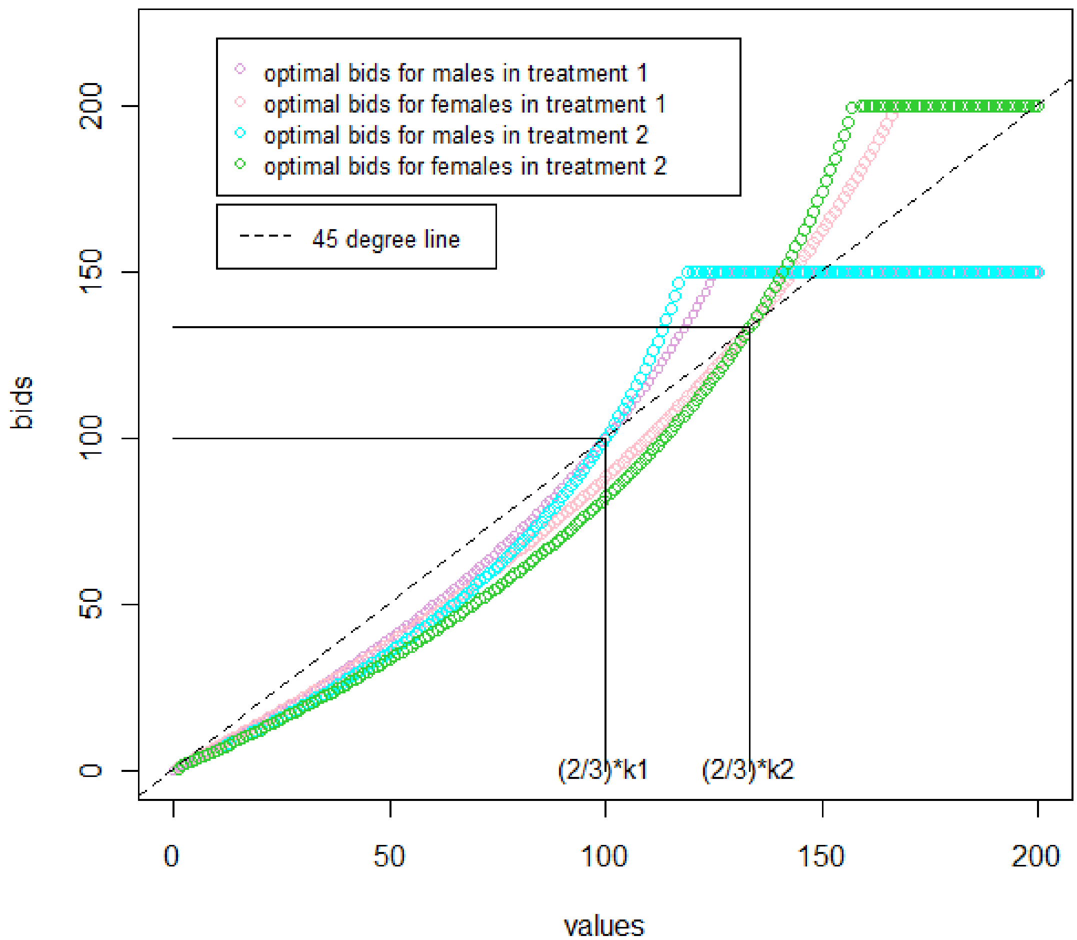
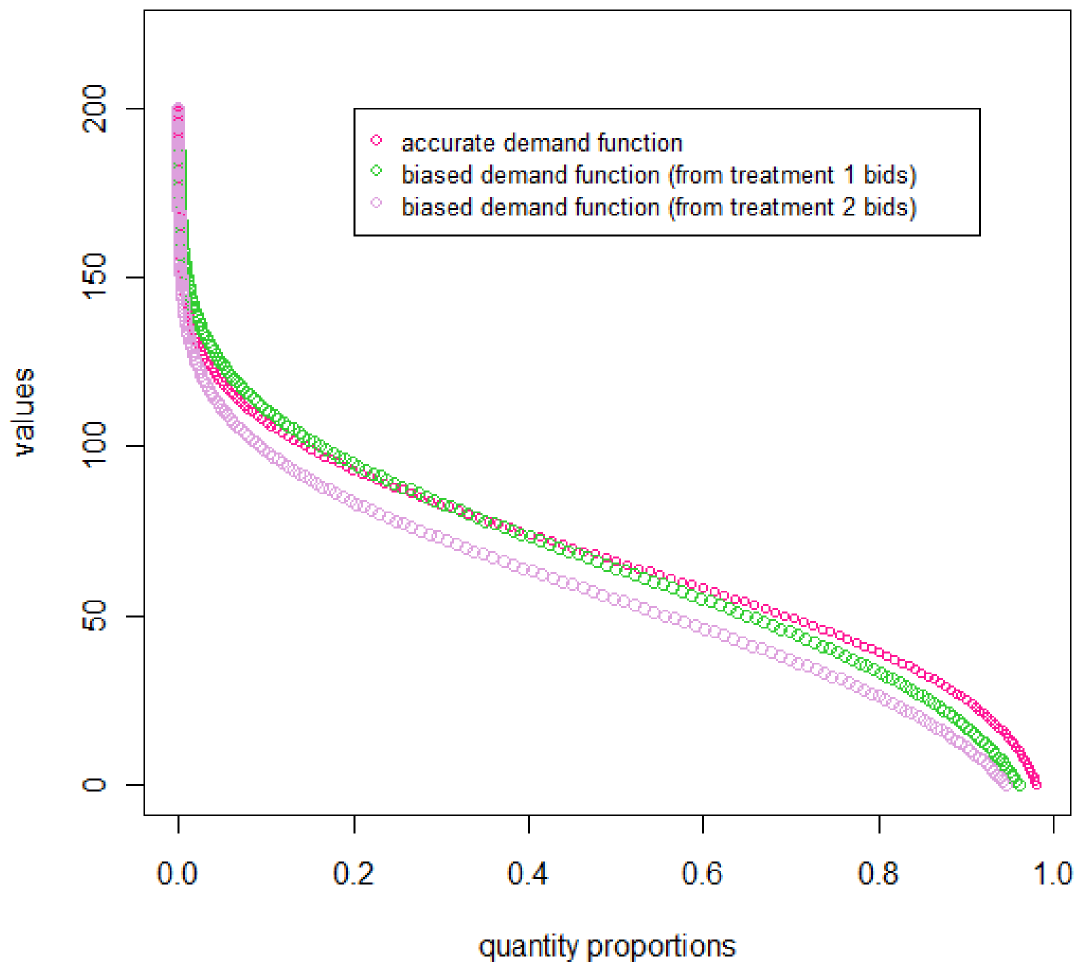


Figure 6 : Estimated demand functions for male subjects



8 Appendix

Proof of Proposition (1). Note from Eq.(1)that the payoff from a bid $b > K$ equals the payoff from bidding exactly K. Thus we can take the range of optimal bids to be a subset of $[0, K]$. So the optimal bid for intrinsic value v is derived by choosing $b \in [0, K]$ to maximize the utility function (Eq.(2)). Differentiating the utility function w.r.t bid b, we get a first order condition for an interior local maximum:

$$v(1-\theta)/K + \frac{2}{K}[\theta(\frac{v}{K} - \frac{1}{2}) - \frac{1}{2}]b + \frac{\theta b^2}{K^2} = 0 \tag{4}$$

Let the lower root of this quadratic equation be denoted $lr(v, \theta, K)$; this is the expression on the RHS of Equation (3). When there is no confusion, we will use the shorthand $lr(v)$ for the lower root. We first note some properties of $lr(v)$.

Step 1. There is a threshold $\hat{v} > 0$ such that for all $0 \leq v \leq \hat{v}$, $lr(v)$ is real.

Proof. The discriminant of $lr(v)$ can be written as

$$g(v/K) = 4\theta^2(\frac{v}{K})^2 - 8\theta\frac{v}{K} + (1+\theta)^2$$

.

This is a convex quadratic in $\frac{v}{K}$, with left root (say $\hat{v}(\theta)/K$) given by

$$\hat{v}(\theta)/K = \theta^{-1}\left[1 - (1/2)\sqrt{4 - (1+\theta)^2}\right]$$

.

If $0 < \theta < 1$, then this is positive; and for all $0 \leq v < \hat{v}(\theta)$, $g(v/K) > 0$; so, $lr(v)$ is real.

Step 2. $lr(0) = 0$. $lr(v)$ is increasing on $v \in [0, \hat{v}(\theta))$.

Proof. $lr(0) = 0$ follows by substituting 0 for v in the expression for $lr(v)$.

Differentiating $lr(v)$, we have

$$lr^{'}(v) = -1 + 2(1 - \frac{\theta v}{K})\delta^{-1/2}$$

where $\delta = [\theta(\frac{2v}{k} - 1) - 1]^2 - 4\theta(1-\theta)\frac{v}{K}$ is the discriminant of $lr(v)$, and hence is positive for $v \in [0, \hat{v}(\theta))$. Rearranging and squaring gives us that $lr^{'}(v) > 0$ if and only if $4 > (1+\theta)^2$, which holds since $0 < \theta < 1$.

Step 3. For $v \in [0, \hat{v}(\theta))$, the optimal bid is either $lr(v)$ or K.

Proof. Since the first order condition is a convex quadratic, the lower root $lr(v)$ satisfies the necessary first and second order conditions for a local max. So, the utility function $u(v, b, \theta, K)$ is locally concave at $b = lr(v)$. Since utility is a cubic in b (with positive coefficient on the cubic term), after the local max at $lr(v)$ and the subsequent local min at the larger root, utility increases in b. So if $0 \leq lr(v) \leq K$, the utility max will occur either at $lr(v)$ or at the boundary K, unless utility at both these bids is negative. But this last is not possible if $0 < \theta < 1$. For in this case, for $v > 0$, a small positive bid always gives positive utility (the first term of Eq. (2) is then positive, and dominates the other terms which are an order or two magnitude lower for small b). The utility at the local max is thus at least this positive amount.

If $lr(v) > K$ for some v, we can take K to be the optimal bid by the argument in the first line of this proof.

Step 4. There exists $v^{'} > 0$ such that for all $v \in [0, v^{'})$ the optimal bid $b(v) = lr(v)$.

Proof. For an agent with value v, bidding $b = K$ gives a utility equal to $u(v, b = K, \theta, K) = v - \frac{K}{2}(1 + \frac{\theta}{3})$. This is negative for an agent with $v = 0$; whereas bidding $lr(v) = lr(0) = 0$ gives a payoff of 0, which is greater; i.e. $u(0, lr(0), \theta, K) > u(0, K, \theta, K)$. From Step 3, this implies that it is optimal for type $v = 0$ to bid $lr(0)$.

From Step 1, $lr(v)$ is real for $v \in [0, \hat{v}(\theta)]$. Since $lr(v)$ is continuous in this interval, and $u(v, b, \theta, K)$ is continuous (see Eq. (2)), $u(v, lr(v), \theta, K)$ is continuous in v. So by continuity, $u(v, lr(v), \theta, K) > u(v, K, \theta, K)$ for some interval of $v \in [0, v^{'})$.

Step 5. The optimal bid for an agent with value $\hat{v}(\theta)$ equals K.

Proof. When $v = \hat{v}(\theta)$, the cubic utility function (Eq.(2)) is increasing in b with a point of inflection at $b = lr(\hat{v}(\theta))$. It follows that it is maximized at a bid equal to K.

Step 6. The optimal bid function $b(v)$ is non decreasing in v.

Proof. From Equation (2), we get the cross-partial

$$\frac{\partial^2 u}{\partial v \partial b} = \frac{1-\theta}{K} + \frac{2\theta b}{K^2}$$

If $0 < \theta < 1$ (and bid $b \geq 0$), this is strictly positive. So, u is supermodular. Hence, $b(v)$ is non decreasing. (See Topkis (1978)).

From Step 5, $u(\hat{v}(\theta), lr(\hat{v}(\theta)), \theta, K) \leq u(\hat{v}(\theta), K, \theta, K)$. Let $\bar{v}$ be the lowest $v \in [0, \hat{v}(\theta)]$ such that $u(v, lr(v), \theta, K) = u(v, K, \theta, K)$. By Step 4 and the monotonicity of the bid function (Step 6), it follows that the bid function $b(v) = lr(v)$ for $v \in [0, \bar{v})$ and $b(v) = K$ for $v \geq \bar{v}$. ■

Proof of Proposition (2). As in the proof of Proposition 1 (Step 1), let $\hat{v}(\theta)$ be the threshold v at which the lower and higher real roots of the quadratic FOC equal each other. That is,

$$\hat{v}(\theta)/K = \theta^{-1}\left[1 - (1/2)\sqrt{4 - (1+\theta)^2}\right]$$

.

So the lower root (strictly speaking the single real repeated root)

$$lr(\hat{v}(\theta), \theta, K) = \frac{1 - \theta(2\frac{\hat{v}}{K} - 1)}{2\frac{\theta}{K}}$$

Notice that $\frac{lr(\hat{v}(\theta),\theta,K)}{\theta}K > (=,<) 1$ according as $\theta < (=,>) \sqrt{2} - 1 \equiv \theta_c$. Three cases are then possible.

Case 1. $\theta > \theta_c$. So $lr(\hat{v}(\theta), \theta, K) < K$. Parametrized at $\hat{v}(\theta)$, the utility function is increasing in the bid, and the single repeated root is at the point of inflection $lr(\hat{v}(\theta), \theta, K)$, which is lower than K. So, K is the optimal bid: $u(\hat{v}, lr(\hat{v}), \theta, K) < u(\hat{v}, K, \theta, K)$.

On the other hand, when $v = 0$, we have seen in the earlier proof that $u(0, lr(0), \theta, K) > u(0, K, \theta, K)$. By continuity (the Intermediate Value Theorem), there exists a smallest $\bar{v}$ satisfying $0 < \bar{v} < \hat{v}$ s.t. $u(\bar{v}, lr(\bar{v}), \theta, K) = u(\bar{v}, K, \theta, K)$. Also, since $lr(v)$ is increasing, $lr(\bar{v}) < lr(\hat{v}) < K$. From the proof of Step 6 of Proposition 1, it follows that the optimal bid function $b(v) = lr(v)$ for $v \in [0, \bar{v})$, and $b(v) = K$ for higher v. Thus there is a jump from $lr(\bar{v})$ to K at the value $\bar{v}$.

Case 2. $\theta < \theta_c$. So $lr(\hat{v}(\theta), \theta, K) > K$. On the other hand, we know that $lr(0, \theta, K) = 0 < K$. By continuity and monotonicity of $lr(v)$, therefore, there is a unique $\bar{v}$ s.t. $lr(\bar{v}, \theta, K) = K$. So trivially,

$$u(\bar{v}, lr(\bar{v}), \theta, K) = u(\bar{v}, K, \theta, K) \qquad A1$$

We claim that then, for every $v < \bar{v}$, $u(v, lr(v), \theta, K) > u(v, K, \theta, K)$.

For suppose not. Then for some $v' < \bar{v}$, $u(v', lr(v'), \theta, K) \leq u(v', K, \theta, K)$. Differentiate both sides of this equation w.r.t. v.

Derivative of the LHS: Since there is a local optimum at $lr(v')$, the envelope theorem implies that the derivative is equal to u_1, the partial derivative w.r.t. the first argument (evaluated at the bid $lr(v')$. Partially differentiating Eq.(2) gives

$$u_1 = (1 - \theta)\frac{lr(v')}{K} + \theta\left(\frac{lr(v')}{K}\right)^2 < 1$$

because $v' < \bar{v}$ implies $lr(v') < lr(\bar{v}) = K$.

On the other hand, the RHS $u(v', K, \theta, K) = v' - \frac{K}{2}(1 + \frac{\theta}{3})$, so the partial derivative w.r.t. v equals 1.

So, $v > v'$ implies $u(v, lr(v), \theta, K) < u(v, K, \theta, K)$. This contradicts Equation $A1$.

Therefore, for $v \in [0, \bar{v}]$, $u(v, lr(v), \theta, K) \geq u(K, \theta, K)$. Thus the optimal bid function $b(v)$ equals the lower root $lr(v)$ until this root equals K; and then stays at K. So the bid function does not have any jump.

Case 3. $\theta = \theta_c$. So $lr(\hat{v}(\theta), \theta, K) = K$, and therefore $u(\hat{(v)}, lr(\hat{v}), \theta, K) = u(\hat{v}, K, \theta, K)$. We can then show that for every $v < \hat{v}$, $u(v, lr(v), \theta, K) > u(v, K, \theta, K)$.

The proof is identical to that for the similar claim in Case 2. And rules out any jump in the optimal bid function. ∎

Proof of Proposition (3). Suppose an agent with intrinsic value v bids against a random draw from $U[0, K]$. In the proof, we keep the value v and the loss aversion parameter θ fixed. We treat K as a parameter, and study the response of the optimal bid b to a change in K. For the duration of this proof, rename the utility function $u(v, b, \theta, K)$ as $u(b, K)$ for short, keeping v and θ fixed and suppressing them. Use the notations u_i and u_{ij} for the first and second partial derivatives of $u(b, K)$.

Suppose $v \in (0, \bar{v})$, so that the optimal bid equals the lower root of the quadratic FOC (Eq. 4 right now). Call this optimal bid $b(K)$, as it is parametrized by K. We show that $b'(K) < 0$.

Using the implicit function theorem on the first order condition $u_1(b, K) = 0$, we get $b'(K) = -u_{12}/u_{11}$. By local concavity at the optimal b, $u_{11} < 0$. So the signs of $b'(K)$ and u_{12} (evaluated at the optimal b) are the same. From the utility function (Eq. (2)), we get the cross partial derivative

$$u_{12} = \frac{-[v(1-\theta) - b(1+\theta)]}{K^2} - \frac{4\theta bv + 2\theta b^2}{K^3}$$

$u_{12} < 0$, therefore, if and only if

$$\frac{2\theta}{K}b^2 + \left[\frac{4\theta v}{K} - (1+\theta)\right]b + v(1-\theta) > 0 \qquad A2$$

Now, at the optimal bid b, notice that the FOC (Eq.(4)) is

$$\frac{\theta}{K}b^2 + \left[\frac{2\theta v}{K} - (1+\theta)\right]b + v(1-\theta) = 0$$

Comparing the LHS of this with the LHS of Eq.($A2$) above, we see that the inequality in $A2$ holds at the optimal bid b. So, $u_{12} < 0$, and therefore $b'(K) < 0$.

So, since $K_2 > K_1$, we have that for every $v \in (0, \bar{v}_1)$, the optimal bid is lower when $K = K_2$ (Treatment 2), than under Treatment 1. Due to the form of the continuous bid function, under Treatment 2, the optimal bid is lower than that under Treatment 1 for all $v > 0$ until the optimal bid function under Treatment 2 cuts the optimal bid function under Treatment 1 from below (at say v_τ). ■

Proof of Proposition (4). To prove the result, we first state and prove the following lemma, that we then use. $\bar{v}$ in the statement of the lemma is the minimum value at which the optimal bid equals K: it equals $K/(1+\theta)$.

Lemma 1 *Let the competing distribution be uniform on* $[0, K]$*. Fix a value* $v \in (0, \bar{v})$*. Let* $b(\theta)$ *denote the optimal bid when the value is* v*, when the loss aversion parameter equals* θ*. Then* $b'(\theta) < (=, >)0$ *according as* $v < (=, >)\frac{2}{3}K$*.*

Proof of Lemma. Holding v and K fixed, write the utility function as

$$u(b,\theta) = \frac{v}{K}(1-\theta)b + \frac{b^2}{K}\left(\theta(\frac{v}{K} - \frac{1}{2}) - \frac{1}{2}\right) + \frac{\theta b^3}{3K^2}$$

$b(\theta)$ is the solution to $\max\{u(b,\theta)|b \in [0, K]\}$. By the Implicit Function Theorem, $b'(\theta) = -\frac{u_{b\theta}}{u_{bb}}$, where on the right hand is a ratio of second partial derivatives.

Since $u_{bb} < 0$ at the optimum (established earlier), the sign of $b'(\theta)$ is the same as that of $u_{b\theta}$. We now sign $u_{b\theta}$; this depends on the value of v/K.

Note that

$$u_{b\theta} = (b/K)^2 + (2(v/K) - 1)(b/K) - (v/K)$$

Suppose $0 < (v/K) < (1/2)$. So $(b/K) < (v/K)$ (by Corollary 1, as $(v/K) < (2/3)$); so $(b/K)^2 < (v/K)$ as well. It follows that $u_{b\theta} < 0$.

Next, we sign $u_{b\theta}$ for the range $(4(v/K) - 1) > 0$, i.e. $(v/K) > (1/4)$.

$u_{b\theta}$ is a convex quadratic in (b/K) with real roots; the lower root is negative and so irrelevant; the positive root is

$$\frac{b}{K} = \frac{1 - (2v/K) + \sqrt{4(v/K)^2 + 1}}{2}$$

Call this positive root $R(v/K)$ for short. Because $u_{b\theta}$ is a convex quadratic, it follows that for positive values of (b/K), $u_{b\theta} < (=, >)0$ according as $(b/K) < (=, >)R(v/K)$.

Note now that $(v/K) = (2/3)$ solves the equation $(v/K) = R(v/K)$; this is established by a simple rearrangement of the equation $(v/K) = R(v/K)$. At the same time, Corollary 1 implies that for $(v/K) = (2/3)$, the optimal bid b satisfies $b = v$. So, for $(v/K) = (2/3)$, $(b/K) = R(b/K)$; so $u_{b\theta} = 0$. Next, if $(1/4) < (v/K) < R(v/K)$, a rearrangement of the second inequality establishes that $(v/K) < (2/3)$; so by Corollary 1, the optimal bid satisfies $(b/K) < (v/K)$, which is less than $R(v/K)$. So, $u_{b\theta} < 0$. The reverse inequality, for $(v/K) > (2/3)$, follows similarly. ■

Proof of the Proposition. The argument is recursive: from the observed population bid distributions for the two treatments, we can find a unique $\theta = \tilde{\theta}$ that will generate the bids from values. We then use $\tilde{\theta}$ to find the unique value distribution (unique on the interval $[0, K_2)$) that generates the two bid distributions.

Let α be the quantile of the bid distribution for treatment 1 at which the bid equals $(2/3)K_1$. Call this bid $b_{1\alpha}$. From Corollary 1, this bid is equal to the value, for Treatment 1 (i.e. for $K = K_1$). So $(2/3)K_1$ is the α-quantile of the distribution of values; call this value v_α. Since $K_2 > K_1$, $v_\alpha < (2/3)K_2$. Lemma 1 shows that for values less than $(2/3)K_2$ in Treatment 2, $b'(\theta) < 0$. So the lowest possible bid at the value v_α, for Treatment 2, over all admissible θ, occurs at the highest such $\theta = \theta_c$; the bid equalling $lr(v_\alpha, \theta_c, K_2)$. $b(\theta)$ is continuous by the Maximum Theorem; so for any observed α-quantile bid $b_{2\alpha} \geq lr(v_\alpha, \theta_c, K_2)$ for Treatment 2, there exists $\tilde{\theta}$ such that $b_{2\alpha} = lr(v_\alpha, \tilde{\theta}, K_2)$.

Moreover, because $v_\alpha < (2/3)K_2$, Lemma 1 implies that $b(\theta)$ is strictly decreasing in θ; so the $\tilde{\theta}$ at which it is optimal to bid $b_{1\alpha}$ at the value v_α is unique.

So, from our specifically chosen α-quantile, we have established that there is a unique $\theta = \tilde{\theta}$ for which the observed α-quantile bids for both treatments $b_{1\alpha}, b_{2\alpha}$, are optimal.

Having obtained $\tilde{\theta}$, consider Treatment 2, which has the larger interval $[0, K_2]$ for the competing distribution. The optimal bid function $b(v, \tilde{\theta}, K_2)$ is strictly increasing for values in $[0, \frac{K_2}{1+\tilde{\theta}})$; so the bid distribution for Treatment 2 can be inverted in the standard way to get the value distribution in this interval. ■

References

- Abeler, J., A. Falk, L. Gotte, and D. Huffman (2009): Reference Points and Effort Provision, American Economic Review, forthcoming.

- Barberis, Nicholas, Ming Huang, and Tano Santos. 2001. Prospect Theory and Asset Prices, Quarterly Journal of Economics, 116(1): 1-53.

- Benartzi, S. and R. Thaler (1995): Myopic loss aversion and the equity premium puzzle, Quarterly Journal of Economics, 110, 7392.

- Camerer, C., L. Babcock, G. Loewenstein, and R. Thaler (1997): Labor Supply of New York City Cabdrivers: One Day At A Time, Quarterly Journal of Economics, 112, 407441.

- Cox, James C., Roberson, Bruce and Vernon L. Smith. 1982. Theory and Behavior of Single Object Auctions. In Research in Experimental Economics Vol. 2,ed. Vernon L. Smith, 1-43. Greenwich, CT: JAI Press.

- Cox, James C., Smith, Vernon L., and James Walker. 1988. Theory and Individual Behavior of First-Price Auctions. Journal of Risk and Uncertainty, 1: 61-99.

- Crawford, V. P. and J. Meng, 2011. "New York City Cabdrivers' Labor Supply Revisited: Reference-Dependent Preferences with Rational-Expectations Targets for Hours and Income". American Economic Review, 101(5), 1912-32.

- DellaVigna, Stefano (2009) Psychology and Economics: Evidence from the Field, Journal of Economic Literature, 47 (2): 315-372.

- Ericson, Keith M. M and Fuster, Andreas (2010). Expectations as Endowments: Evidence on Reference-Dependent Preferences from Exchange and Valuation Experiments. Mimeo: Princeton University.

- Fehr, E. and L. Gotte (2007): Do Workers Work More if Wages Are High? Evidence from a Randomized Field Experiment, American Economic Review, 97, 298317.

- Genesove, D. and C. Mayer (2001): Loss aversion and seller behavior: Evidence from the housing market, Quarterly Journal of Economics, 116, 12331260.

- Guerre, E., Perrigne, I., Vuong, Q., 2009. Nonparametric Identification of Risk Aversion in First-Price Auctions Under Exclusion Restrictions. Econometrica, 77(4), 1193-1227.

- Harrison, Glenn W. 1989. Theory and Misbehavior of First-Price Auctions. American Economic Review, 79(4), 74962.

- Heidhues, P. and B. Koszegi (2008): Competition and price variation when consumers are loss averse, American Economic Review, 98, 12451268.

- Herweg, F., D. Muller, and P. Weinschenk (2009): Binary Payment Schemes: Moral Hazard and Loss Aversion, American Economic Review, forthcoming.

- Kahneman, D., Tversky, A., 1979. Prospect theory: An analysis of decision under risk. Econometrica 47, 263-291.

- Knetsch, J. L. and W.-K. Wong (2009): The endowment effect and the reference state: Evidence and manipulations, Journal of Economic Behavior and Organization, 71, 407413.

- Koszegi, B., Rabin, M., 2006. A model of reference-dependent preferences. Quarterly Journal of Economics 121, 1133-1165.

- Lange, A. and A. Ratan (2010): Multi-dimensional reference-dependent preferences in sealed-bid auctionshow (most) laboratory experiments differ from the field, Games and Economic Behavior, 68, 634645.

- Lusk, J. L. and Shogren, J. F., 2007. Experimental Auctions. Cambridge Books, Cambridge University Press.

- Macera, R. (2010): Intertemporal Incentives Under Loss Aversion, Working Paper, UC Berkeley.
- Ratan, A., 2010. Reference-Dependent Preferences in First Price Auctions. Mimeo: Monash University.
- Smith, A. (2008): Lagged Beliefs and Reference-Dependent Utility, Working Paper 08-03, Department of Economics, University of Arizona.
- Spears, D., 2011. Low Expectations: Reference-Dependent Preferences and Labor Supply in Cape Town, South Africa. Mimeo: Princeton University.
- Sydnor, Justin. 2006. Sweating the Small Stuff: Risk Aversion in Home Insurance. Working paper.