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What Do Teachers Do? Teacher Quality vis–a-vis Teacher Quantity in a Model of Public Education and Growth

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Abstract

This paper analyses the contribution of teachers in a public education system and its implication for growth. We focus exclusively on two teacher-specific inputs - teacher quality and teacher quantity, and two student-specific inputs - ability and effort. We argue that all these factors enter separately in the education technology and therefore have differential impacts on the process of human capital formation. In a public education system where teachers' remunerations are paid by the goverment and financed by taxation, for any given amount of government revenue there exists a trade-off between teacher quality and teacher quantity. At the same time, the imposed tax rate also impinges on the effort choice of an agent. Thus human capital formation and growth in the model depends on a complex interaction between teacher quality, teacher quantity, student ability and student effort. In this context we discuss the optimal education policy as well the optimal taxation policy of the government and analyse their implications for growth.

Keywords: Public Education, Tecaher Quality, Teacher Quantity, Growth
JEL Classification: I28,O40

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1 Introduction

This paper examines the role of teachers in the process of human capital formation and growth. Modern growth theory emphasizes the role of human capital in economic development and growth. Accordingly education and schooling constitute important components of development strategies in almost all countries. But schooling *per se* does not necessarily lead to higher growth (Benhabib and Spiegel, 1994; Pritchett, 2001; Easterly, 2001). Obviously the quality of schooling matters. There exist significant differences in the quality of schooling in the developed vis-a-vis the developing world, which evidently have impacted upon their respective growth trajectories. As Hanushek and Woessmann (2007) observe: "Most people would, in casual conversation, acknowledge that a year of schooling in a school in a Brazilian Amazon village was not the same as a year of schooling in a school in Belgium. ... The data suggest that the casual conversation may actually tend to understate the magnitude of differences. ... Ignoring quality differences significantly distorts the picture about the relationship between education and economic outcome." Yet there are very few contributions in the theoretical literature on growth that take the quality factor explicitly into account. This paper is an attempt in this direction.

We focus explicitly on quality of schooling in a public education system and analyse its impact on human capital formation and growth. Quality of schooling has many different dimensions - some of which are teacher-specific and some of which are related to the school environment. Each of these factors plays a distinct role in the learning process of a student. In this paper we concentrate on two teacher-specific inputs: the teacher-student ratio and the quality of teaching. The teacher-student ratio (which we call 'teacher quantity') signifies how much personal attention a teacher can give to a student. The quality of teaching on the other hand refers to the proficiency of a teacher in imparting knowledge through class room instructions and/or the ability to communicate with the large body of students in a class room. While this second aspect of the formal schooling process is not directly measurable, we infer that a better qualified (or better trained) teacher would in general be able to convey the teaching material to her students more lucidly. Thus, this particular aspect of schooling quality is captured in our model by the average level human capital of teachers. We provide a theoretical framework whereby these two aspects of quality of schooling interact with the student-specific characteristics (ability, effort) to generate certain learning outcome (human capital) for each student, which in turn influences growth.

The growth-education interlinkage is explored here in terms of an endogenous growth model where the quality of schooling affects the skill level acquired by the future generation.

Quality of schooling improves if the number of teachers (per student) goes up and if better quality teachers are employed. But, in order to attract better quality teachers from their alternative profession, the government has to pay higher salaries. Thus for any given amount of total budgetary expenditure on schooling, there is a trade-off: the government has to decide whether to go for better teacher-quality or higher teacher-quantity. In this context we derive the optimal education policy of the government. Indeed, depending on whether teacher quality is more (less) important than teacher quantity in the education technology, it is optimal for the government to employ the most (least) qualified agents from the available pool of potential teachers.

But quite apart from this allocative efficiency of a given budgetary provision on schooling (and the associated teacher quality-quantity trade off), the total public expenditure on education itself is endogenous and depends on the choice of the tax rate. Thus the tax rate is an additional instrument that the government can use to influence schooling outcomes and the consequent growth process. However, we argue that for any historically given stock of human capital, the impact of taxation on the overall schooling quality is ambiguous. An increase in the tax rate, and the concomitant rise in the budgetary allocation on schooling, does not necessarily raise the overall schooling quality. Typically this happens when teacher quality is more important than teacher quantity. This happens because when teacher quality is the most important factor, with a limited budget, the government would have already employed the best ability teachers from the existing pool. Now an increase in the budgetary allocation would allow the goverment to employ more teachers but that would necessarily entail picking up agents with relatively lower ability, which may lead to a fall in overall schooling quality. In this context we show that there exists an optimal level of taxation that maximises the overall schooling quality.

In addition to overall schooling quality, human capital accumulation also depends on the effort level exerted by an agent. And this entails a different kind of trade off: an increase in the tax rate lowers the incentives of the students to exert effort (since a part of the product of this effort would be taxed away). This negative effect on effort interacts with the schooling quality to generate an optimal level of taxation that would maximise growth. This growth maximising tax rate differs from the tax rate that maximises overall schooling quality. Morover, to the extent that agents treat the schooling quality in this public education system as exogenous, the majority-preferred tax rate diverge from the growth maximising tax rate. In fact, the majority-preferred tax rate is likely to be lower than the growth-maximisng tax rate, implying that the agents under majority voting would underinvest in children's education.

There exists a large body of theoretical work pertaining to public education and growth (see, for example, Glomm and Ravikumar, 1992, 2001; Eckstein and Zilcha, 1994; Benabou, 1996; Zhang, 1996; Blankenau and Simpson, 2004; Boldrin, 2005; Viaene and Zilcha, 2009). However, most of these papers focus on the overall public expenditure on education and do not differentiate between quality and quantity. In contrast, we focus both on the optimal *level* of schooling expenditure (via the choice of tax rate) as well as the optimal *distribution* of this total expenditure across various factors that augment the overall quality of schooling. In this sense our model complements the existing body of theoretical work on education and growth.

Two papers which do take the teacher quality-quantity trade-off explicitly into account are those by Tamura (2001) and Gilpin and Kaganovich (2012). Indeed our specification of the human capital formation technology closely follows that of Tamura (2001). But we depart from Tamura in two crucial directions. First, Tamura (2001) considers a two-region set up where the regions differ in terms of the initial human capital endowment. In this context, he explores the possibility of human capital (and therefore, income) convergence across regions, driven by the mobility of teachers from one region to the other. Ours is an autarkic set up where such inter-regional mobility of teachers is not feasible. Nonetheless, we demonstrate possibilities of convergence under specific parametric conditions and analyse the economic significance of these parametric configurations. Secondly, we extend Tamura's model to an heterogenous agents framework where agents differ in terms of ability. In this context we discuss the optimal recruitment policy in terms of the skill profile of the teachers.

Gilpin and Kaganovich (2012) also analyse a similar quantity-quality trade-off in a heterogenous agents framework with a two-tiered public education system, comprising of basic education and college education. In Gilpin and Kaganovich, the gain from college education is positive if and only if the agent has attained a minimum threshold level of pre-college human capital. This effectively implies that only the relatively high ability people will opt for college education which, along with the assumption that only a college-educated agent can enter the teaching profession, automatically ensures a certain minimum quality of teachers. To our mind, the exact quality of teachers to be employed and the corresponding ability threshold itself is a policy variable and therefore imposing a minimum human capital threshold from outside undermines the quality-quantity trade-off implicit therein. Thus, in contrast, in our model the human capital threshold for teachers is determined endogenously. This allows us to get starker results in terms of the teacher-quality and teacher-quantity choices of the government. There is a second important distinction between the two papers. In Gilpin and Kaganovich, the income tax rate levied to finance education, and consequently the budgetary expenditure on education, is treated as exogenous. We, on the other hand, allow this tax rate to be optimally determined. Indeed, as we have argued above, for a given stock of human capital, an exogenous rise in the budgetary expenditure on education does not necessarily lead to higher overall quality of education. Since the government in a public education system is concerned about the overall quality of education, to allow for an optimal quality-quantity choice without allowing for an optimal determination of the total budgetary expenditure may generate inefficiencies that would undermine the optimality of the former choice. By allowing the tax rate also to be optimally determined, we remove this potential source of inefficiency.

This paper also addresses a growing concern among policymakers about the declining trend in the avearge aptitute of teachers which is perceived to be associated with lower quality of schooling. The falling aptitute of teachers over the years is well-documented in the emprical literature (see, for example, Bollou and Podgursky, 1995; Corcoran et al, 2004a, 2004b; Hoxby and Leigh, 2004). Gilpin and Kaganovich (2012) offer an explanation of this observed trend in terms of skill-biased technological progress. Our take on this is rather different. First, a falling average aptitude of teachers may arise mechanically with increasing budgetary allocation on education over time. As we have pointed out above, an increase in budgetary allocation on education may actually lead to lower overall quality of education when teacher quality is more important than teacher quantity. With an increased budgetary expenditure, as the goverment goes on to employ more teachers, it necessarily moves down the ability ladder and employs agents with lower ability. Thus a historical trend of rising budgetary allocation for education may be accompanied by falling average aptitude of teachers, even when there is no skill-biased technological progress. Secondly, and more importantly, a declining trend in the average aptitude of teachers does not necessarily imply lower growth. In view of the above mentioned trade-off between teacher-quality and teacher-quantity, we show that under certain scenarios, it might be *optimal* to employ teachers from the very bottom rung of the ability distribution - even from the perspective of long run economic growth. Hence, without specific knowledge about the precise nature of the education technology, such concerns about falling teachers' aptitude may be somewhat misplaced.

The rest of the paper is organized as follows. The next section describes the general framework of the model. Section 3 deliberates on the optimal education policy. Section 4 describes the human capital dynamics and growth . Section 5 derives the optimal taxation policy of the government. It also compares various optimal tax rates when objective functions differ. Section 6 concludes the paper.

2 The Model

2.1 General Framework

Time is discrete, represented by $t = 0, 1, 2, ...$ At any point of time the economy is populated by two successive overlapping generations of dynasties. Each generation consists of a continuum of population of measure one.

Each agent is born with some innate ability[1] which varies across agents within a cohort. An agent knows his own ability but it is not observable to others, although the distribution of ability is known to all. We assume that innate ability within a cohort is uniformly distributed over the unit interval $[0, 1]$. Thus agents belonging to the same generation can be indexed by their inherent ability factor, x , such that $x \in [0, 1]$. Also, innate ability is i.i.d. accross generations which implies that parental ability does not directly impact on childrens' ability.

The life cycle of a representative agent of any cohort is as follows. The agent lives for exactly two periods - defined for convenience as childhood and adulthood, and has exactly one offspring born to her during adulthood. During her childhood the agent consumes nothing and only exerts effort in acquiring education/skill. Upon adulthood, she works and earns a certain wage income (depending on the skill level acquired by her during childhood) - of which a part is contributed towards the education of the next generation (publicly provided) and the rest is consumed. The agent dies at the end of this period.

2.2 Preferences

Agents within a generation and across generations have identical preferences. An agent derives positive utility from own adulthood consumption, denoted by c, and from the contribution made towards the next generation's education, denoted by b. The latter can be thought of as a proxy for educational bequest - although in a public education system the actual amount spent on educating each child would depend on the *average* education ex-

[1] The term 'ability' does not necessarily mean intelligence or merit. It could represent a combination of factors like patience, tenacity, motivation, ambition or any other individual-specific factor which determines educational achievement over and above effort.

penditure.[2] The agent also derives negative utility from efforts exerted during childhood in acquiring education. Consider a young agent born at time t who exerts an effort level e_t during childhood in acquiring education, enjoys an adulthood consumption of c_{t+1}, and contributes b_{t+1} towards the public education system. The lifetime utility of this agent is represented by the following quasi-linear utility function:

$$U \equiv (c_{t+1})^{\epsilon} (b_{t+1})^{1-\epsilon} - e_t; \quad 0 < \epsilon < 1. \tag{1}$$

2.3 Production

A single final commodity is produced using human capital/skill (H). Technology for final good production is of a standard AK type:

$$Y_t = wH_t^Y \tag{2}$$

where H^Y denotes the part of the total stock of human capital that is employed in final goods production and w is a positive constant. The final good sector is characterized by competitive firms who earn zero profit and pay a constant wage rate per unit of skill, given by w (which is the constant marginal product of human capital/skill in this AK-type technological set up).

The AK-type production structure for the final goods sector implies that it is only the total human capital employed (H_t^Y) that is relevant for the final goods production; not its distribution across agents. To put it differently, it does not matter whether the supply of this total human capital comes from agents at the top end of the ability distribution or the bottom end; as long as they add up to H_t^Y, the total output remains the same.

2.4 Human Capital Formation

Human capital is acquired through compulsory schooling in public schools. We postulate that human capital acquired by a child through formal schooling depends on two broad sets of factors: (a) the overall quality of schooling, and (b) the absorbtive capacity of the student.

[2] This is equivalent to the 'warm glow' bequest assumption in Galor-Zeira (1993). Under a private education regime the agent herself would have optimally chosen the exact amount to be received by her progeny as educational bequest. However with public provision of education, a constant proportion of agents' income is taxed away to finance the education bill of the next generation. Thus the contribution made towards next generation's education is only a proxy for the educational bequest. The actual amount spent on her own progeny (i.e., per child education expenditure) does not necessarily equal the amount of contribution made by the parent.

The overall quality of schooling is determined by various inputs provided by the teachers as well as school infrastructure. We focus on two specific teacher-related inputs: (i) how much personal attention a student gets from the teacher, and (ii) the average quality of teaching. The former is captured by the teacher-student ratio (θ_t), while the latter is proxied by the average skill level of teachers (h_t^{TA}). These two teacher-related inputs determine the overall quality of schooling through the following education technology:

$$Q_t = (\theta_t)^{\alpha} \left(h_t^{TA}\right)^{1-\alpha}; 0 < \alpha < 1. \tag{3}$$

For any given quality of schooling, the skill level acquired by a student also depends on her absorptive capacity. Consider an young agent with inherent ability x who has an absorptive capacity of A_x. Then the skill level acquired by this young agent (to be employed in the next period) is given by:

$$h_{t+1}^{x} = A_x Q_t^{\gamma} \ ; 0 < \gamma < 1. \tag{4}$$

The absorptive capacity of a student of course depends on her innate ability. But innate ability can be complemented by hard work. Accordingly, the absorptive capacity of a student of ability x, who puts in an effort level e, is given by:

$$A_x = e^{\beta} x^{1-\beta}; 0 < \beta < 1. \tag{5}$$

There are several features of the above-mentioned education and skill formation technologies that require further elucidation. First, notice that the education technology that determines the quality of schooling (as specified by equation (3)) is identical to Tamura (2001). However our specification of the skill/human capital formation technology (as specified by equation (4)) is different from Tamura in that it completely ignores the role of 'home education' in influencing the level of human capital acquired by the next generation. In other words, we do not consider the impact of parental human capital on that of the children. This is **not** to deny the role of parents in the learning process of a child. While the quality of education is certainly influenced by non-school factors such as parental education, home environment etc., in this paper our focus is exclusively on school-specific factors asociated with the formal learning process. Therefore we deliberately shut off other mechanisms of human capital transmission across generations.[3]

[3] There exists some empirical evidence that supports this assumption. For example, Card and Krueger (1992) write: "Controlling for measures of school quality, however, we find no evidence that returns to education are related to the income or schooling levels of the parents' generation." However, there are other empirical studies which have re-iterated the importance of 'home education' (e.g., Woessmann, 2003).

Secondly, human capital formation exhibits diminishing returns with respect to individual effort level: greater effort increases the level human capital acquired, but at a decreasing rate ($\beta < 1$). Moreover, inherent ability and effort level are complementary[4]: a person who has higher innate ability *ceteris paribus* also has higher incentive to exert more effort. However whether a more able person actually exerts greater effort in equilibrium will depend upon the wage incentive. This issue will be taken up in detail later in the context of different education policies and concommitant teachers' salary schemes.

2.5 Individual Choices

Recall that agents derive positive utility from own consumption and the contribution made towards public education and derive negative utility from effort exerted in acquiring education (refer to the utility function specified by (1)). Under a tax-financed public education regime however, a part of the agent's adulthood income is taxed away at a predetermined rate to provide for the education of the next generation. Thus, the optimal consumption and bequest choices are trivial: the agent simply contributes the stipulated taxed amount towards educational bequest and consumes her entire after-tax income.[5] Consider a young agent of generation t who is born with an innate ability x and exerts an effort e_t during childhood that generates an adulthood income of $y_{t+1}^x \equiv wh_{t+1}^x$, where $h_{t+1}^x = \left(e_t^\beta x^{1-\beta}\right) Q_t^\gamma$ (by (4) and (5)). Let τ be the rate at which her income is taxed to pay for the education of the next generation. Substituting $(1-\tau)y_{t+1}^x$ for c_{t+1}, and τy_{t+1}^x for b_{t+1}, we get the indirect utility of the agent defined in terms of her effort level as:

$$\hat{U} \equiv \tau^{1-\epsilon}(1-\tau)^\epsilon w \left(e_t^\beta x^{1-\beta}\right) Q_t^\gamma - e_t. \tag{6}$$

A *forward-looking*, *rational* agent, who can perfectly ancipate her post-tax adulthood income,[6] would maximize $\hat{U}$ to choose her optimal effort level in childhood such that

$$\tau^{1-\epsilon}(1-\tau)^\epsilon w\beta Q_t^\gamma \left(e_t\right)^{\beta-1} x^{1-\beta} - 1 = 0.$$

It is easy to verify that an interior solution exists (since β is a positive fraction). Thus the optimal effort choice of an agent of generation t who is born with an innate ability x is given

[4] The second order cross partial derivative is positive.

[5] Instead of allowing for warm glow bequest (b), we could have defined the utility of an agent as a function of the human capital acquired by her child (e.g. Tamura, 2001), or the quality of education received by her child (e.g. Glomm and Ravikumar, 1992). The qualitative results would have been the same. Assuming a 'warm glow' educational bequest only simplifies the exposition.

[6] Notice that there is no uncertainty in this model. An agent precisely knows her ability upon birth.

by

$$e_t^x = \left(\beta\tau^{1-\epsilon}(1-\tau)^{\epsilon} w Q_t^{\gamma}\right)^{1/(1-\beta)} x. \tag{7}$$

The corresponding level of human capital acquired by this agent is

$$\begin{aligned} h_{t+1}^x &= \left[\left(\beta\tau^{1-\epsilon}(1-\tau)^{\epsilon} w Q_t^{\gamma}\right)^{1/(1-\beta)} x\right]^{\beta} x^{1-\beta} Q_t^{\gamma} \\ &= \left[\beta\tau^{1-\epsilon}(1-\tau)^{\epsilon} w\right]^{\beta/(1-\beta)} Q_t^{\gamma/(1-\beta)} x. \end{aligned} \tag{8}$$

Notice that in the current scheme of things when each agent's income is proportional to her qualification (level of human capital), the optimal effort level of the agent is proprtional to her ability.

3 Education Policy

Compulsory schooling is provided to every child free of cost and the salary bill of the teachers is financed by the government. We assume that the government offers each teacher her opportunity wage, i.e., the wage that she would be able to earn by employing her skills elsewhere (merit pay scheme). This implies that if the government wants to employ an agent with human capital h, then it has to pay her a salary of wh. (We shall assume that whenever the government pays the market-equivalent salary to any person, that person is willing to engage in teaching, even though wage-wise she is actually indifferent). The government pays the salary bill of the teachers by taxing the income of the entire working population (current adults) at a constant rate while maintaining a balanced budget.

The government imposes a proportional income tax at a time-invariant rate τ which, at this point, is fixed arbitrarily.[7] This generates a total tax revenue at time t, given by $T_t \equiv \tau \int_0^1 w h_t^x dx = \tau w H_t$, where H_t denotes the aggregate stock of human capital time at t and is defined as the sum total of all adult agents' human capital, i.e., $H_t \equiv \int_0^1 h_t^x dx$. On the other hand, the total salary bill of the teachers employed by the government is given by wH_t^T,where H_t^T denotes the aggregate human capital of the teachers. Thus, at any point of

[7] Eventually we shall allow for the optimal choice of τ.

time t, the balanced budget condition of the government is derived as follows[8]:

$$\begin{aligned} \tau w H_t &= w H_t^T \\ &\Rightarrow H_t^T = \tau H_t. \end{aligned} \tag{9}$$

The objective of the education policy is to allocate the government tax revenue efficiently so as to maximise the overall quality of schooling (Q_t), as in Tamura (2001) and Gilpin and Kaganovich (2012). But unlike these two papers, we do not impose any pre-determined minimum qualification (i.e., minimum human capital requirement) for teaching appointments. The purpose here is to explore a generic education technology where any agent can *potentially* be employed as a teacher, irrespective of her level of education. The exact quality of teachers to be employed itself is a policy variable, which is to be determined endogenously. Hence, imposing additional constraints on the qualification of teachers would impede the optimal policy choice of the government.

In maximising the overall schooling quality, the binding constraint faced by the government is of course its revenue constraint - as represented by the balanced budget condition (equation (9)). But its choices are also restricted by the distribution of human capital in the economy in the following way. For any given teacher-student ratio (θ), the best quality teachers in the economy consists of the θ measure of people who are at the very top end of the distribution. This generates an upper bound on the possible average quality of teachers. Likewise, the worst quality teachers in the economy consists of the θ measure of people who are at the very bottom end of the distribution. This generates an lower bound on the possible average quality of teachers. These upper and lower bounds act as additional feasibility constraints on the choice set of the government. The exact specification of these two constraints are discussed below.

Notice that the balanced budget condition itself captures the implied trade-off between teacher quality and teacher quantity. For any given tax rate τ and for any historically given stock of human capital H_t, the RHS of the above equation is fixed. The LHS on the

[8]By employing a set of the current adults as teachers, the government shifts a part of the working population from final goods production to teaching. Since salaries of these people have to be paid in terms of the final good, effectively the government has to tax away a part of the final output produced by the rest to pay for the salary bill of the teachers. Let $\hat{w}$ denote the effective wage rate per unit of skill for the teachers. Then $\tau w H_t^Y = \hat{w} H_t^T$. However, the effective wage rates in the teaching and non-teaching (final goods production) sectors being equal (which implies $\hat{w} = (1-\tau)\, w$), this is equivalent to taxing the income of the entire working population while providing everybody with the same wage rate w. The algebra below makes this point more precise.

$\tau w H_t^Y = \hat{w} H_t^T \Rightarrow \tau w H_t^Y = (1-\tau)\, w H_t^T \Rightarrow \tau w \left(H_t^Y + H_t^T\right) = w H_t^T \Rightarrow \tau w H_t = w H_t^T.$

other hand can be mechanically written as $H_t^T = \theta_t h_t^{TA}$.[9] Substituting this in the above balanced-budget condition, one can immediately see the quality-quantity tradeoff involved here:

$$\theta_t h_t^{TA} = \tau H_t. \tag{10}$$

The precise values of θ_t and h_t^{TA} are to be determined by the education policy of the government. However, the RHS of equation (10) being a constant (for a given τ), the equation is represented by a rectangular hyperbola. Thus if the government opts for a higher average quality of teachers then it must compromise in terms of the number of teachers.[10]

As mentioned above, the distribution of human capital at any point of time t also imposes two additional constraints on the choice of θ_t and h_t^{TA}. First, note that for any given value of θ_t, $H_t^T \leqq \int_{1-\theta}^1 h_t^x dx$, where the RHS represents the aggregate human capital of the top θ-proportion of the population. Noting that $H_t^T = \theta_t h_t^{TA}$, we can write this condition as follows[11]:

$$\begin{aligned} h_t^{TA} &\leq \frac{1}{\theta_t}\int_{1-\theta}^1 h_t^x dx \\ &= (2-\theta_t) H_t. \end{aligned} \tag{11}$$

Similarly, for any given value of θ_t, $H_t^T \geqq \int_0^\theta h_t^x dx$, where the RHS represents the aggregate human capital of the bottom θ-proportion of the population. Once again, we can write this condition as follows[12]:

$$\begin{aligned} h_t^{TA} &\geq \frac{1}{\theta_t}\int_0^\theta h_t^x dx \\ &= \theta_t H_t. \end{aligned} \tag{12}$$

[9] Recall that h_t^{TA} is the average skill level/human capital of the teachers.

[10] Das and Guha (2011) has explored the relationship between public education and growth in a similar set up. However in their framework, the only teacher specific input that enters the human capital formation technology is average quality of teachers. Thus even though there is an implicit trade off between teacher quality and teacher quantity, this trade off does not affect the overall schooling quality or human capital formation.

[11] This is because: $\frac{1}{\theta_t}\int_{1-\theta}^1 h_t^x dx = \frac{1}{\theta_t} H_t \frac{\int_{1-\theta}^1 h_t^x dx}{\int_0^1 h_t^x dx} = \frac{1}{\theta_t} H_t \frac{\left[\beta\tau^{1-\epsilon}(1-\tau)^\epsilon w\right]^{\beta/(1-\beta)} Q_{t-1}^{\gamma/(1-\beta)} \int_{1-\theta}^1 x dx}{\left[\beta\tau^{1-\epsilon}(1-\tau)^\epsilon w\right]^{\beta/(1-\beta)} Q_{t-1}^{\gamma/(1-\beta)} \int_0^1 x dx}$.
Solving,
$\frac{1}{\theta_t} H_t \frac{\int_{1-\theta}^1 x dx}{\int_0^1 x dx} = H_t \frac{1-(1-\theta_t)^2}{\theta_t} = (2-\theta_t) H_t$.

[12] This is because: $\frac{1}{\theta_t}\int_0^\theta h_t^x dx = \frac{1}{\theta_t} H_t \frac{\int_0^\theta h_t^x dx}{\int_0^1 h_t^x dx} = \frac{1}{\theta_t} H_t \frac{\left[\beta\tau^{1-\epsilon}(1-\tau)^\epsilon w\right]^{\beta/(1-\beta)} Q_{t-1}^{\gamma/(1-\beta)} \int_0^\theta x dx}{\left[\beta\tau^{1-\epsilon}(1-\tau)^\epsilon w\right]^{\beta/(1-\beta)} Q_{t-1}^{\gamma/(1-\beta)} \int_0^1 x dx}$.
Solving, $\frac{1}{\theta_t} H_t \frac{\int_0^\theta x dx}{\int_0^1 x dx} = H_t \frac{(\theta_t)^2}{\theta_t} = \theta_t H_t$.

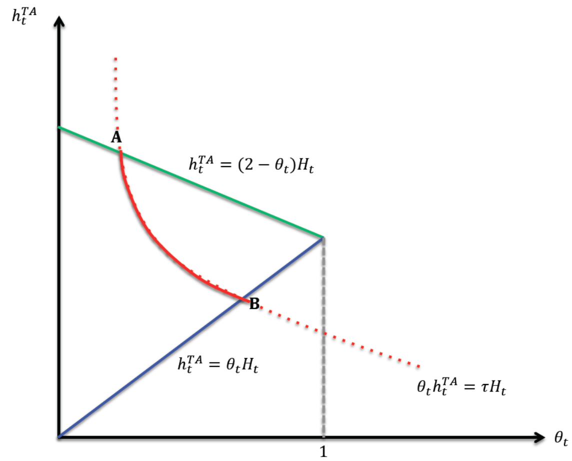

Figure 1

In view of the balanced budget condition (equation (10)) and the two distributional constraints (equations (11) and (12) respectively), the optimal education policy of the government entails finding a solution to the following constrained optimization exercise:

$$\underset{\{\theta_t, h_t^{TA}\}}{Max}\ Q_t = (\theta_t)^{\alpha} \left(h_t^{TA}\right)^{1-\alpha}$$

subject to

$$\begin{aligned} \text{(i) } h_t^{TA} &= \frac{\tau H_t}{\theta_t}; \\ \text{(ii) } h_t^{TA} &\leqq (2-\theta_t) H_t; \\ \text{(iii) } h_t^{TA} &\geqq \theta_t H_t. \end{aligned}$$

In solving the above problem, the government takes H_t, the distribution of H_t, and τ as given. Figure 1 characterizes the constraint space for this optimization problem.

The green line in Figure 1 represents the boundary points of constraint (ii), while the blue line represents the boundary points of constraint (iii). The closed triangular space identified by these two lines represents the feasible set for h_t^{TA} and θ_t, as determined by the distribution of human capital at any point of time t. The red dotted line in Figure 1 represents the budget constarint of the government. The segment of the red line that lies within the triangular space (denoted by the solid stretch AB) constitutes the budget set for the government.

3.1 Optimal Solutions for θ and h_t^{TA}

Since the balanced budget condition (constraint (i)) is always binding, we can use this condition to eliminate one of the choice variables from the objective function. Eliminating h_t^{TA}, we get the reduced-form objective function as: $\underset{\{\theta_t\}}{Max}\left[(\theta_t)^{2\alpha-1}(\tau H_t)^{1-\alpha}\right]$. From this reduced-form expression it is obvious that depending on the parameter value α, it is optimal for the government to go either for the maximum possible teacher quantity (whenever $\alpha > \frac{1}{2}$) or for the best possible teacher quality (whenever $\alpha < \frac{1}{2}$).[13] In terms of Figure 1, the optimal policy choice thus involves moving along the $\theta_t h_t^{TA} = \tau H_t$ curve (depicted by the solid red line drawn in bold) until one of the distributional constraints binds. More specifically, if $\alpha > \frac{1}{2}$, then it is optimal to move downward along the $\theta_t h_t^{TA} = \tau H_t$ curve until one hits the straight line representing $h_t^{TA} = \theta_t H_t$. On the other hand, if $\alpha < \frac{1}{2}$, then it is optimal to move upward along the $\theta_t h_t^{TA} = \tau H_t$ curve until one hits the straight line representing $h_t^{TA} = (2-\theta_t) H_t$. Formally, the optimal solutions for θ_t and corresponding h_t^{TA} for various parametric conditions are described below[14]:

Case A: $\alpha > 1/2$

$$\theta^* = \sqrt{\tau}\ ; \tag{13}$$

$$\left(h_t^{TA}\right)^* = H_t\sqrt{\tau}. \tag{14}$$

Case B: $\alpha < 1/2$

$$\theta^* = 1-\sqrt{1-\tau}\ ; \tag{15}$$

$$\left(h_t^{TA}\right)^* = H_t\frac{\tau}{1-\sqrt{1-\tau}}. \tag{16}$$

The corresponding optimal quality of schooling in the two cases are given repectively by:

$$Q_t^* = \begin{cases} \sqrt{\tau}\ \left(H_t\right)^{1-\alpha} & \text{for } \alpha > 1/2; \\ \\ \dfrac{(\tau)^{1-\alpha}}{\left(1-\sqrt{1-\tau}\right)^{1-2\alpha}}\left(H_t\right)^{1-\alpha} & \text{for } \alpha < 1/2. \end{cases} \tag{17}$$

[13] When α is exactly equal to $\frac{1}{2}$, the teacher quality-quantity choice becomes irrelevant and the overall quality of schooling depends only on the aggregate expenditure on schooling. We ignore this case.

[14] These solutions are obtained by solving a set of simultaneous equations represented by
$(1')$ $\theta_t h_t^{TA} = \tau H_t$; $(2')$ $h_t^{TA} = \theta_t H_t$ for the case $\alpha > 1/2$;
and by
$(1'')$ $\theta_t h_t^{TA} = \tau H_t$ and $(2'')$ $h_t^{TA} = (2-\theta_t)H_t$ for the case $\alpha < 1/2$.
Also note that the only admissible solutions for θ are those which lie within the interval $[0,1]$.

Thus the optimal schooling quality depends positively on the current available stock of human capital (H_t). Growth of human capital (i.e., next generation's human capital) in turn depends on the schooling quality. Thus the optimal education policy of the government will have a direct bearing on the rate of growth in the economy, which we shall analyse in the next section.

Notice that the optimal schooling quality Q_t^* also depends on the tax rate τ. Hence the choice of the tax rate itself could be an important policy tool in influencing the educational outcome of an economy. However the exact functional relationship between Q^* and τ differs depending on whether α is greater or less than 1/2. In particular, for any given stock of aggregate human capital, if $\alpha > 1/2$ then optimal schooling quality is monotonically increasing in τ. But the result is ambiguous if $\alpha < 1/2$. In fact one can easily verify that when $\alpha < 1/2$, optimal schooling quality Q_t^* initially increases as τ rises, reaches a maxima at $\tau = 4\alpha(1-\alpha)$ and decreases thereafter.[15]

Proposition 1 below summarises the key characteristics of the optimal education policy under various parametric conditions. Corollary 1 specifies the corresponding relationship between optimal schooling quality and the tax rate, which at this point is exogenously given.

Proposition 1 *Consider any given time-invariant tax rate τ. If the education technology is such that teacher-quantity is more important that teacher-quality (i.e., $\alpha > 1/2$) then the optimal education policy for the government consists of maintaining a constant teacher-student ratio at the level $\sqrt{\tau}$ and employing the least qualified (and lowest ability) agents as teachers in every period. On the other hand if the education technology is such that teacher-quality is more important that teacher-quantity (i.e., $\alpha < 1/2$) then the optimal education policy for the government consists of maintaining a constant teacher-student ratio at the level $1-\sqrt{1-\tau}$ and employ the most qualified (and highest ability) agents as teachers in every period.*

Corollary 1: *Suppose the government is following the optimal education policy as specified in Proposition 1. If the education technology is such that teacher-quantity is more important that teacher-quality (i.e., $\alpha > 1/2$), then the overall quality of schooling is monotonically increasing in the tax rate (τ). On the other hand if the education technology is such that teacher-quality is more important that teacher-quantity (i.e., $\alpha < 1/2$), then the overall*

[15] Taking the partial derivative of Q_t^* with respect to τ, after simplification, one can show that $\frac{\partial (Q_t)^*}{\partial \tau} \gtreqqless 0$ according as $\tau^2 - 4\alpha(1-\alpha)\tau \lesseqqgtr 0$,where $4\alpha(1-\alpha)$ is a positive fraction (since $\alpha < 1/2$). Thus for any τ such that $0 < \tau < 4\alpha(1-\alpha)$, optimal schoolong quality is increasing in τ, while for any τ such that $4\alpha(1-\alpha) < \tau \leqq 1$, optimal schooling quality is decreasing in τ.

quality of schooling is inverted U-shaped in the tax rate (τ) and there exists an optimal tax rate which maximises the overall schooling quality.

This non-monotonic relationship between optimal schooling quality and the tax rate is a rather novel result in the context of the existing literature on public education and growth. Typically, in the absence of any finer distinction between teacher quality and teacher quantity, a higher rate of taxation and the concomitant rise in public expenditure on education unequivocally improves the quality of schooling. However, as Corollay 1 shows, in the presence of a tecaher quality-quantity trade off, having more resources does not necessarily lead to an improvement in schooling quality. In particular, if $\alpha < 1/2$, the relationship between the tax rate and optimal schooling quality becomes inverted U-shaped. The reason for this non-monotonic relationship between Q^* and τ is quite intuitive. When $\alpha < 1/2$, then teachers' average human capital is a more significant input than the number of teachers in the education technology. So it is always optimal for the government to employ the best quality teachers, subject to its budget constraint and subject to the distribution of human capital. In terms of Figure 1, this optimal choice is represented by the point of intersection of the $\theta_t h_t^{TA} = \tau H_t$ curve with the $h_t^{TA} = (2 - \theta_t) H_t$ line. Now if available resources go up while the stock of human capital remains unchanged, there is a rightward shift in the $\theta_t h_t^{TA} = \tau H_t$ curve while positions of the two distributional constraints remain the same. Thus the new optimal (h, θ) configuration now shifts further down along the $h_t^{TA} = (2 - \theta_t) H_t$ line, which implies that at this new optima, *average teacher quality is lower while number of teachers is higher*. This is so because the government had already employed the poeple who were at the top end of the distribution. Now as it attempts to employ more teachers (due to a relaxation of the budget constarint), it has to move to a lower quality pool, which necessarily lowers the average quality of teachers. At the same time number of teachers goes up. To a certain extent more teachers compensate for the fall in the teacher average quality. But α being less than $1/2$, teacher quantity is a poor substitute for teacher quality and eventually this shows up in terms of a decline in overall schooling quality.

4 Human Capital Dynamics & Growth

Before we discuss the precise dynamics of human capital formation and the associated growth path of the economy, recall that the term H_t^T / H_t denotes the proportion of the aggregate human capital stock that is employed in the teaching professon. Equation (9) tells us that this proportion is directly measured by the tax rate τ. Now output in this economy depends linearly on the part of aggregate stock of human capital that is employed in final good production (H_t^Y). Since τ proportion of the total human capital stock H_t is employed in the teaching professon, it follows that $H_t^Y = H_t - H_t^T = (1-\tau)\, H_t$, i.e., H_t^Y is also proportional to H_t. This implies that the rate of growth of output in this economy can be measured by the rate of growth of the aggregate stock of human capital. In other words,

$$\frac{Y_{t+1}}{Y_t} - 1 \equiv g_t = \frac{H_{t+1}}{H_t} - 1. \tag{18}$$

Now aggregate human capital stock in the next period, $H_{t+1} \equiv \int_0^1 h_{t+1}^x dx$, is determined by the governement's education policy (h_t^{TA} and θ_t) as well as by the effort spent today by each agent in acquiring education (e_t) during childhood. We have already solved for the optimal effort choice by a rational forward-looking agent with ability x and derived the corresponding level of human capital acquired by her (refer to equations (7) and (8) repectively). Thus substituing for h_{t+1}^x and aggregating over all agents who differ in terms of innate abilities, next period's human capital is given by:

$$\begin{aligned} H_{t+1} &= \left[\beta\tau^{1-\epsilon}(1-\tau)^{\epsilon} w\right]^{\beta/(1-\beta)} Q_t^{\gamma/(1-\beta)} \int_0^1 x dx \\ &= \frac{1}{2}\left[\beta\tau^{1-\epsilon}(1-\tau)^{\epsilon} w\right]^{\beta/(1-\beta)} Q_t^{\gamma/(1-\beta)}. \end{aligned} \tag{19}$$

Using equation (17) to substitute for Q_t , we get the following equation determining the dynamics of the aggregate human capital in this economy:

$$H_{t+1} = \begin{cases} \frac{1}{2}\left[\beta\tau^{1-\epsilon}(1-\tau)^{\epsilon} w\right]^{\beta/(1-\beta)} \left[\sqrt{\tau}\ \right]^{\gamma/(1-\beta)} (H_t)^{(1-\alpha)\gamma/(1-\beta)} & \text{for } \alpha > 1/2\ ; \\ \frac{1}{2}\left[\beta\tau^{1-\epsilon}(1-\tau)^{\epsilon} w\right]^{\beta/(1-\beta)} \left[\dfrac{(\tau)^{1-\alpha}}{\left(1-\sqrt{1-\tau}\right)^{1-2\alpha}}\ \right]^{\gamma/(1-\beta)} (H_t)^{(1-\alpha)\gamma/(1-\beta)} & \text{for } \alpha < 1/2. \end{cases} \tag{20}$$

Notice that for any given time-invariant tax rate τ, the economy will converge to a long run steady state if $(1-\alpha)\gamma < (1-\beta)$, i.e., if $(1-\alpha) < \dfrac{(1-\beta)}{\gamma}$. If this condition is satisfied, then initial differences in human capital endowment (as well as initial differences in teacher quality across regions) do not matter in the long run; all economies eventually converge to the same steady state (provided the tax rate and other parameters across the countries are the same). Thus we get a strong Solow-type convergence result.[16] This possibility arises when

[16]Notice that unlike Tamura (2001), the convergence happens here even in a autrakic set up whereby hiring of better quality teachers from other region is ruled out.

(a) teacher quality is not a very significant input in the education production technology (low $(1-\alpha)$); and/or (b) innate ability is an important determinant of students' absorptive capacity (high $(1-\beta)$); and/or (c) human capital formation technology is not very elastic with respect to overall schooling quality (low γ). These parametric configurations are quite intuitive. Recall that when τ is the same for all economies, a low human capital endowment shows up only in terms of the a low average human capital of teachers. If teacher quality is not an important factor in the education technology (case (a)) or human capital formation is not very responsive to schooling quality (case (c)), then having a low average quality of teachers does not make much difference in terms of the dynamic path of the economy. Likewise, if innate ability is very important in human capital formation (case (b)), then again the dynamic paths of all economies would be similar, since (by assumption) ability distribution across agents in all the economies are identical.

Although convergence can happen under specific parametric configuations, we shall however focus on the possibility of balanced growth. Note that the economy will exhibit a balanced growth path if and only if

$$(1-\alpha)\gamma = (1-\beta)\,. \qquad (\textbf{Assumption 1})$$

Henceforth we shall assume that (**Assumption 1**) is satisfied. The corresponding balanced growth rate for Case (A) and Case (B) are given below:

$$g = \begin{cases} (\beta w)^{\beta/(1-\beta)}\left[\tau^{1-\epsilon}(1-\tau)^{\epsilon}\right]^{\beta/(1-\beta)}\left[\sqrt{\tau}\,\right]^{1/(1-\alpha)} - 1 \quad \text{for } \alpha > 1/2\ ; \\ (\beta w)^{\beta/(1-\beta)}\left[\tau^{1-\epsilon}(1-\tau)^{\epsilon}\right]^{\beta/(1-\beta)}\left[\dfrac{(\tau)^{1-\alpha}}{\left(1-\sqrt{1-\tau}\right)^{1-2\alpha}}\right]^{1/(1-\alpha)} - 1 \quad \text{for } \alpha < 1/2. \end{cases} \tag{21}$$

Since the human capital stock in this economy is growing at a constant rate (given Assumption 1), it is easy to verify (from equations (14), (16), (13), (15) and (17)) that along the balanced growth path (for both Case A and B), optimal teacher-student ratio remains constant but teacher quality increases over time and so does the overall quality of schooling. Proposition 2 summarises these results for the balanced growth path.

Proposition 2 *Suppose* **Assumption 1** *holds. Then for any given time-invariant tax rate, the economy grows at a constant rate. Along the balanced growth path, the relative quality of teachers (as characterized by their innate abilities) remains constant while the absolute teacher-quality as well as overall schooling quality increase over time.*

5 Taxation Policy: Optimal Tax Rate

So far we have assumed that the tax rate τ is fixed arbitrarily. However, as is evident from (17), the optimal schooling quality depends crucially on the tax rate chosen by the government. Moreover the choice of tax rate directly affects the growth rate of the economy (see equation (21)). Thus the tax rate itself is another policy instrument which the government can use to influence the educational outcome and the growth of the economy. One cannot talk of the optimal education policy independent of the optimal taxation policy. In this section we discuss the choice of optimal tax rate by the government. We then compare the optimal choice of tax rate by the government with the majority-preferred tax rate and analyse the difference (if any).

It is worth mentioning here that even though τ is to be chosen optimally, we shall restrict our analysis to time-inavriant values of τ only. The reason for maintaining a time-invariant tax rate is two-fold: (a) we want to focus on a balanced growth path which cannot be attained unless the tax rate is constant; and (b) in determining the majority-preferred tax rate, we need to keep the incentive structure the same for different generations which implies retaining the same policy parameters across generations.

In analysing the optimal choice of tax rate, we consider two alternative regimes. In the first regime the government itself chooses the tax rate with the objective of maximizing of growth. In the second regime the tax rate is decided by majority voting: in each period the government asks the current adults to vote for their most-preferred tax rate and implements the one which is preferred by majority.[17] As we already know from the literature on public spending and growth (e.g., Barro (1990); Alesina-Rodrik (1994)), the growth maximising policy need not always co-incide with the majority preferred policy. Here we re-examine this issue in the context of public education.

5.1 Growth Maximizing Tax Rate

Suppose the government chooses the tax rate so as to maximise growth rate of output. From equation (21), it is clear that the relationship between the balanced growth rate (g) and the tax rate (τ) is non-monotonic. Indeed, there are two τ-dependent terms here, represented

[17] For the second regime to generate meaningful results (which are comparable to the first regime) two conditions need to be satisfied: (i) there exists a unique majority voting equilibrium tax rate; and (ii) this tax rate is time-invariant. As we shall see, both these conditions are fulfilled in the current set up.

respectively by

$$F(\tau) \equiv \tau^{1-\epsilon}(1-\tau)^{\epsilon};$$

$$G(\tau) \equiv \begin{cases} \sqrt{\tau} \text{ for } \alpha > 1/2; \\ \dfrac{(\tau)^{1-\alpha}}{\left(1-\sqrt{1-\tau}\right)^{1-2\alpha}} \text{ for } \alpha < 1/2. \end{cases}$$

In fact, the latter term is related to the optimal schooling quality Q_t^*, which itself is a function of τ (refer to equation (17)). The two terms $F(\tau)$ and $G(\tau)$ capture two very different effects (and the associated trade offs) of an increase in the tax rate.

The first term, $F(\tau)$, captures the trade-off related to optimal effort choice. We have seen that the human capital acquired by an agent depends on the effort exerted by him in school. But optimal effort choice depends non-monotonically on τ. On the one hand, a higher tax rate means that a lower proportion of the agents' income will be available to them for consumption tomorrow. This reduces their incentive to exert effort. On the other hand, a higher tax rate also means that they will effectively leave more educational bequests for their progeny. To the extent agents care for their progeny (due to the presence of 'warm glow'), this gives them some satisfaction and therefore increases their incentive to exert effort. The net effect depends on the relative strengths of these two countering mechanisms.

The second term, $G(\tau)$, captures the trade off related to schooling quality. We have already noted that the optimal schooling quality is a function of the tax rate (see corollary 1). When $\alpha > \frac{1}{2}$, the optimal schooling quality is a monotonically increasing function of τ. This positive influence of the tax rate in terms of improving the schooling quality somewhat mitigates the effort-related trade-off but does not eliminate it completely. When $\alpha < \frac{1}{2}$, the optimal schooling quality itself is non-monotonic in τ due to the presence of teacher quality-quantity trade-off (as we have explained in the previous section). In choosing the optimal tax rate in this case, one now has to take into account both these trade offs.

Note that trade off related to the effort level is quite commonly observed in the public expenditure and growth literature[18]. The trade off related to schooling quality is a rather novel feature of our framework and it arises purely due to the finer distinction between teacher quality and teacher quantity that we have emphasized in this paper. It is worth reiterating here that when the effort-related trade off is the *only* trade off in the economy (i.e, τ enters the human capital formation equation only through the $F(\tau)$ term), then

[18] For instance, such a trade off exists in Barro (1990), Alesina & Rodrik (1994) and other papers which use taxation to finance productivity enhancing public inputs.

the optimal tax rate would be given by $(1-\epsilon)$. Introducing the teacher quality- quantity consideration brings in a new effect which may counteract or reinforce the earlier effect - depending on parameter conditions. In particular, nature of the interation between the two affects would differ depending on whether $\alpha > \frac{1}{2}$ or $< \frac{1}{2}$. We now analyse these two cases separately.

5.1.1 Case A: $\alpha > 1/2$

Maximising the relevant balanced growth rate with respect to τ, from the first order condition we get:

$$(\beta w)^{\beta/(1-\beta)} (\tau\ )^{[\gamma+2\beta(1-\epsilon)]/2(1-\beta)} (1-\tau)^{\epsilon\beta/(1-\beta)} \left[\frac{\gamma+2\beta(1-\epsilon)}{2(1-\beta)}\left(\frac{1}{\tau}\right) - \frac{\epsilon\beta}{(1-\beta)}\left(\frac{1}{1-\tau}\right)\right] = 0$$

The resulting optimal tax rate is given by:

$$\tau^* = 1 - \left(\frac{2\beta}{\gamma+2\beta}\right)\epsilon > (1-\epsilon). \tag{22}$$

5.1.2 Case B: $\alpha < 1/2$

The derivation in this case is more involved for the simple reason that now there are two trade-offs associated with an increase in τ : one related to the effort choice; other related to the schooling quality - which are captured by the terms $F(\tau) \equiv \tau^{1-\epsilon}(1-\tau)^{\epsilon}$ and $G(\tau) \equiv \frac{(\tau)^{1-\alpha}}{\left(1-\sqrt{1-\tau}\right)^{1-2\alpha}}$ respectively. Since both $F(\tau)$ and $G(\tau)$ enter with postive powers in the balanced growth rate expression (see equation (21)), what happens to the growth rate depends crucially on what happens to these two functions as τ increases. It is easy to verify that $F(\tau)$ is maximised at $\tau = 1-\epsilon$, while we have already seen that $G(\tau)$ (which is nothing but the optimal schooling quality) is maximized at $\tau = 4\alpha(1-\alpha)$ (see footnote 13). Thus it is obvious that the balanced growth rate will be increasing in τ for any $\tau \leqq Min\left[1-\epsilon, 4\alpha(1-\alpha)\right]$; will be decreasing in τ for any $\tau \geqq Max\left[1-\epsilon, 4\alpha(1-\alpha)\right]$; and will attain a maxima somewhere in between. In other words, there exits an optimal tax rate τ^* such that $1-\epsilon < \tau^* < 4\alpha(1-\alpha)$ if $(1-\epsilon) < 4\alpha(1-\alpha)$; and $4\alpha(1-\alpha) < \tau^* < 1-\epsilon$ if $4\alpha(1-\alpha) < (1-\epsilon)$.

The results from these two cases are summarized in the proposition below.

Proposition 3 *Consider an economy which is on a balanced growth path. There exists a unique tax rate that maximizes the rate of growth output in this economy. When* $\alpha > \frac{1}{2}$*, the* *optimal (growth maximizing) tax rate is given by* $\tau^* = 1 - \left(\frac{2\beta}{\gamma+2\beta}\right)\epsilon > (1-\epsilon)$. *When* $\alpha < \frac{1}{2}$, *the optimal (growth maximizing) tax rate* τ^* *lies between* $(1-\epsilon)$ *and* $4\alpha(1-\alpha)$. *In the latter case,* $\tau^* \gtreqless (1-\epsilon)$ *according as* $(1-\epsilon) \lesseqgtr 4\alpha(1-\alpha)$.

5.2 Majority-preferred Tax Rate

So far we have assumed that the government chooses the optimal tax rate so as to maximise the growth rate of the economy. Suppose now we allow the tax rate to be chosen by majority voting. Accordingly, in each period the government chooses a tax rate τ_t on the basis of the preferences of the *current adults.* Notice that in our formulation, this tax rate τ_t enters into the utility function an adult agent in two ways. The current tax rate enters directly through the disposable income $(1-\tau_t)y_t$ which goes into her consumption and through the bequest $\tau_t y_t$ which goes into the education of her child. But the (anticipated) tax rate τ_t must have also influneced her optimal effort choice in the previous period: e_{t-1}. Thus when they are asked to choose that tax rate today, the 'rational' agents would choose the current tax rate so as to maximise their indirect utility:

$$\hat{U}^x = (1-\tau_t)^{\epsilon}(\tau_t)^{1-\epsilon} y^x(e^*(\tau_t)) - e^*(\tau_t).$$

By Envelope theorem, this indirect utility is maximized at

$$\tau^{**} = 1 - \epsilon. \tag{23}$$

There are several important implications of this result. First, note that the tax rate chosen by an agent is independent of x. In other words, all agents, irrespective of their innate ability will choose the same tax rate τ^{**}, which will obviously be the majority-preferred tax rate. Secondly, the majority-preferred tax rate in indeed time-invariant: in every period the current adults choose the same tax rate $1-\epsilon$. Thirdly, a comparison of the majority-preferred tax rate with the growth maximising tax rate clearly reveals that as long as schooling quality is increasing in τ, $\tau^{**} < \tau^*$, i.e., the growth maximizing tax rate is always higher than the majority-preferred tax rate.[19] This result is due to the fact the in choosing their most-preferred tax rate, the agents only look at their consumption-bequest trade-off (which has an impact on their optimal effort choice as well). But if the tax rate is time-invariant, then it also has an indirect positive effect on the income of the current adults through the quality of schooling factor. But the current adults do not care about this latter effect since

[19] This happens if either $\alpha > 1/2$, or $\alpha < 1/2$ and $(1-\epsilon) > 4\alpha(1-\alpha)$.

it operates though a tax that was paid by their parents. Thus the 'myopic' adults will spend less towards children's education and the economy will attain a growth rate which is lower than the maximum. Proposition 4 below summarises these results.

Proposition 4 *Consider an economy where the tax rate for eduaction finance in every period is chosen by majority voting (by the current adults). Then each adult agent would choose a time-invariant tax rate $\tau^{**} = 1 - \epsilon$, implying that the economy will be on a balanced growth path. Moreover the tax rate chosen by majority voting will be less than the growth maximising tax rate as long as either $\alpha > \frac{1}{2}$ or $(1-\epsilon) < 4\alpha(1-\alpha)$.*

Remarks: Notice that the majority-preferred tax rate $(1-\epsilon)$ would have been exactly equal to the optimal tax rate if effort-related trade off was the *only* relevant trade off in this economy. Thus bringing in an additional dimension in terms of teacher quality and teacher quantity results in a divergnce between the optimal (growth maximising) tax rate and the majority-preferred tax rate. Indeed, since individual agents would ignore this quality-quantity trade off in their choice of the preferred tax rate, they would typically underinvest in children's human capital formation. This also suggests that if the government is also not careful in taking into account the relevant quality- quantity trade off in the hiring decisions of teachers, it might end up following an education policy which hampers growth.

6 Conclusion

In this paper we have analyzed the impact of overall quality of education on growth in a framework where better quality of schooling results from employing better quality teachers as well as from maintaining higher teacher-student ratios. We show that overall quality of schooling is an important factor contributing to growth. However in an economy where education is publicly provided and is financed by taxation, the requirement of maintaining a balanced budget on the part of the government imposes a trade off in terms of quality and quantity of teachers. For any given amount of tax revenue, in order to attract better quality teachers to teaching profession the government has to pay higher average wage, which in term implies that fewer teacher can be employed. Since the quality and quantity of teachers enter differently in the education technology, the government has to optimally allocate of its limited resources between teacher quality and teacher quantity in order to ensure efficiency.

Apart from the trade off between teacher quality and teacher quantity that works through the balanced budget condition (which is to be resolved by an efficient education policy), there is another trade off implicit here. This second trade off arises in the context of the optimal taxation policy of the government. Higher taxation eases the balanced budget constraint of the government, but it does not necessarily result in better quality of schooling - especially when teacher quality is the binding constraint. Thus the governement has to choose its tax rate optimally if it wants to maintain schooling quality at its maximum level.

At the same time, there is a trade off that works at the individual level. Any tax rate reduces the effective wage rate of an agent, thereby reducing her incentive to exert effort in schooling.While the agent is also somewhat compensated because higher taxation implies higher effective bequest to his progeny, to the extent that the disutility from lower income outweighs the utilty from higher effective bequest, the agent will cut down on his effort level, which in turn undermines the impetus to growth. Thus in choosing the optimal tax rate that maximises growth, the government has to take into account both the schooling related trade-offs as well as the effort related trade offs. To the extent that quality of schooling in this public eduaction system is treated as exogenous by the individual agents, the growth maximizing tax rate need not be welfare-maximizing (and majority preferred). In fact, the welfare maximising tax rate is likely to be lower that the growth maximizing one.

It is important to note that in our model we have assumed that individual ability is known to an agent when she decides how much effort to invest in acquiring human capital given his ability. This assumption however is not crucial for our results. One can easily show that the same conclusion will prevail even when agents do not know their exact ability but optimize on the basis of the expected value of the ability distribution, which is common knowledge. In this latter case, the optimal effort chosen will be the same across all agents; however the actual level of human capital will differ across agents depending on their realization of actual ability. Since the nature of the effort-related trade off is the same irrespective of the assumed level of ability, the basic results of the model will remain unchanged.

A more interesting scenario would arise if the human capital formation technology depends directly on the teachers' average ability rather than average human capital. If human capital is observable but innate ability is not, then employing teachers solely on the basis of the level of human capital acquired may give rise to adverse selection. Needless to say, under the merit pay regime, which is what we assume in this paper, ability and human capital acquirement are perfectly aligned; so possibility of such adverse selection would not arise. However under alternative forms of teachers salary scheme (e.g., a fixed pay regime where the wage rate is decided by collective bargaining), this may lead to inefficiencies which would not only lower overall schooling quality but would also hamper growth. Analysis of such alternative payment regime, its implication for human capital formation and growth however lies beyond the scope of the present paper. Indeed this is an agenda for future research.

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