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ABSTRACT

In a two-country infinite-horizon model, with two traded goods and two factors of production and no international borrowing and lending, there is no convergence of incomes if there is factor-price equalization. With factor-price equalization, the Euler equations of the two economies become identical. I show that in such a set-up if agents have a non-zero probability of death, then we do get convergence. In the steady state the two economies have identical capital-labor ratios and revert to autarky.

JEL Classification: F10, F11, F43.

Key Words: Convergence, Dynamic Heckscher-Ohlin Model, Factor-Price Equalization, Blanchard-Yaari Model, Capital Accumulation.

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1. INTRODUCTION

In the last two decades, there has been a revival of interest in growth theory. Researchers have grappled with the question whether economies, irrespective of their starting points, converge to the same steady state? The consensus in the literature argues against absolute convergence but points to "conditional" or "club" convergence.[i]

The initial responses to this question were in a closed economy framework. It was pointed out, however, that empirically it does not make sense to look at the convergence issue in a closed economy framework—international trade plays an important role in generating convergence, via factor-price equalization (or at least by generating forces that bring factor prices of trading economies closer together). The question can then be reframed as: whether theoretical models predict convergence in, say, a Heckscher-Ohlin-Samuelson (H-O-S) structure in a growth setting?

The initial answer to this question was, surprisingly, negative. In a two factor-two-good-two country H-O-S model with infinitely-lived consumers, identical technologies and incomplete specialization, convergence does not occur (first shown by Chen (1992)). With factor-price-equalization, we have the Euler equations for the consumers (say, the representative consumers in both countries have CRRA felicity functions[ii]) in the two economies:

$$\frac{C_{t+1}}{C_t} = \{\beta(1+r_{t+1})\}^{1/\sigma} = \frac{C^*_{t+1}}{C^*_t} \qquad (1)$$

C is consumption, r the interest rate β the discount factor, and σ the inverse of the intertemporal elasticity of consumption. The domestic marginal rate of substitution between consumption today and consumption tomorrow is equal to that of the foreign consumer (an asterisk denotes a foreign variable), since tastes are identical and they face the same interest rate. Since they face the same interest rate at each period, their growth rates of consumption are always equal. Ergo, an economy starting with a lower <u>level</u> of consumption never catches up with its richer trading partner. The reason is straightforward—in a factor-price equalization setting, trade replaces the "missing" asset markets (in these models international borrowing and lending are absent by assumption). Hence, loosely speaking, a poorer economy always remains a "fraction" of the richer one.

This finding has spawned a sizable literature, see e.g. Atkeson and Kehoe (2000), Bajona and Kehoe (2010), Baxter (1992), Bianconi (1996), Brecher et al. (2002)), Caliendo (2009), Chatterjee and Shukayev (2012), Cuñat, and Maffezzoli (2004a and 2004b), Francois and Shiells (2008), Kaneko (2006), Mountford (1998), Sen and Shimomura (2013) and Ventura (1997).

In this paper, remaining within the H-O-S tradition, I ask how robust is this non-convergence result? The point of departure is (retaining the H–O-S structure) that agents have uncertain lifetimes (as in the Blanchard-Yaari (B-Y) model--see Blanchard (1986)).[iii][iv] Does the non-convergence result survive this tweak in the demographic structure? It is worthwhile doing this experiment because, while the B-Y model is called the continuous-time overlapping generations model, it shares many attributes of the Ramsey-Cass Koopmans (R-C-K) model—e.g. in equilibrium, dynamic inefficiency is not possible, the dynamics looks very similar to the R-C-K model etc. The R-C-K model is, of course, the limiting case of the B-Y model, as the probability of death goes to zero.

In my model then with identical homothetic preferences and technologies, and with incomplete specialization, factor-price equalization occurs. With factor price equalization (that is the countries always produce in a common cone of diversification), individual Euler equations are identical. Although individual Euler equations are identical between agents within an economy and internationally, aggregate consumption growth is different between a rich and a poor economy. This is due to the new-born not being linked altruistically to the others in the economy and, therefore, have no financial assets at birth.[v]

The mechanism outlined in the previous paragraph can be explained further as follows: Individual Euler equations are identical across economies (as in equation (1) above), but since the newborn are born without financial assets they pull down the aggregate consumption growth rate i.e. there is a wedge between average consumption growth and the consumption of the marginal (new-born) households. A richer economy's aggregate consumption growth rate is pulled down more (assuming identical birth-rates) because the difference in financial wealth is greater between the newborn and existing cohorts. This enables a poorer economy to "catch up". If the economies are identical in every other respect except initial wealth, then its consumption growth ceases (i.e. in the steady state) when they reach the same level of wealth. Thus in the steady state, under incomplete specialization, the two economies converge to the same per capita income and consumption levels. They also converge to the same capital-labor ratios, thus destroying any reason for international trade.

2. THE MODEL

The household sector of an economy and its demographics follow the Blanchard-Yaari model. A proportion η of the population dies at each instant and is replaced by a new cohort of exactly the same size. This implies that we are abstracting from population growth. Each agent faces a time-independent probability of death η and seeks insurance so that on the death of the agent an insurance company assumes all liabilities (or assets) of this agent.[vi] The new-born are not altruistically linked to any existing agent, so they are born without any financial wealth.

Preferences and technology are identical across the two economies. In particular, preferences are homothetic to make aggregation across cohorts possible—in fact we work with instantaneous utility that is logarithmic. Also the rate of time preference is the same in the two economies.

On the production side, there are two intermediate goods that are produced and traded internationally. These are then combined costlessly to produce a non-traded final good that can be either consumed or invested. There is no borrowing or lending internationally.

The only difference between the two economies stems from different initial capital stocks: we refer to the economy with the lower capital stock (say, the foreign country) as the initially poor economy.

We confine our attention to factor endowments in the world economy being such that both economies are always diversified in production. This causes factor prices to be equalized internationally.

2.1 The Households

The agents in the two countries (called home and foreign, with foreign variables denoted by an asterisk) have identical homothetic tastes. In particular, the rate of time preference is identical across countries and they face a common time invariant probability of death at each instant.[vii] The representative home consumer of generation v (i.e. born on date v) maximizes the following utility functional (a similar specification holds for the foreign country)[viii]:

$$\int_0^\infty [\log C(v,t) \exp.\{-(\rho+\eta)t\}]dt \qquad (2)$$

where C(ν, t) is the consumption on date t of an individual born on ν, ρ is the rate of time preference and η is the instantaneous time-invariant probability of death. Labor is supplied inelastically.

The budget identity facing this individual of vintage v on date τ is:

$$\dot{A}(\nu,\tau) = [r^A(\tau)+\eta]\, A(\nu,\tau) + w(\tau) - C(\nu,\tau) \qquad (3)$$

where A is the assets of the household and r^A is the market return on these assets and η is the transfer from (to) the insurance company if A(ν,τ)is positive (negative).[ix] Note that in this "model of perpetual youth", all agents on date τ earn w(τ)—hence it does not have a generational index.

In addition the household must satisfy a No-Ponzi-Game (NPG) condition:[x]

$$\lim{}_{\tau\to\infty}\ e^{-\int_t^{\tau}[r^A(s)+\eta]ds} A(\nu,\tau)=0, \tag{4}$$

The decision rule for consumption is given by:

$$C(\nu,t)=(\rho+\eta)\,[A(\nu,t)+H(t)] \tag{5}$$

where

$$H(t)\equiv\int_t^{\infty} w(\tau)\, e^{-\int_t^{\tau}[r^A(s)+\eta]ds}\, d\tau \tag{6}$$

The consumption Euler equation for an individual of vintage v is given by:

$$\frac{\dot{C}(\nu,t)}{C(\nu,t)}=\{r^A(t)+\eta-(\rho+\eta)\}=r^A(t)-\rho \tag{7}$$

Aggregating over all households alive on date t (see the Appendix 1 for details), we have from equation (5):

$$C(t)=(\rho+\eta)\,[A(t)+H(t)] \tag{8}$$

I note that aggregation across cohorts with different wealth levels requires homothetic preferences. Buiter (1988) derives the conditions for aggregation with isoelastic utility functions where the intertemporal consumption elasticity is different from unity. This can be incorporated in my analysis, but the gain from doing so is probably more than offset by the notational complexity.[xi]

Similarly, the aggregate budget identity is obtained from equation (3) as follows:

$$\dot{A}(t)=r^A(t)A(t)+w(t)-C(t) \tag{9}$$

The insurance firm offers actuarially-fair risk premium to individuals in return for inheriting their wealth (or liabilities) on their death. That is why the probability of death does not appear in the aggregate budget constraint—it just constitutes a transfer from the dead to the living.

The aggregate Euler equation is obtained by aggregating equation (7) over all cohorts.

$$\frac{\dot{C}(t)}{C(t)} = [r^A(t)-\rho]-\eta(\rho+\eta)\left(\frac{A(t)}{C(t)}\right)$$

$$= \frac{\dot{C}(\nu,t)}{C(\nu,t)}-\eta\left(\frac{C(t)-C(t,t)}{C(t)}\right) \tag{10}$$

As is well understood by now, the first term on the right and side of (10) is the usual Euler equation term from the R-C-K models. The second term is the so-called “generational turnover effect”. It is there because agents are born only with human capital and without financial assets. They accumulate financial assets by saving. The newborn thus pull down the “average” Euler equation.

A similar specification holds for the foreign country, where the consumers’ preferences, the rate of time preference and the expected probability of death is the same.

2.2 The Firms

In each of the two economies, two intermediate goods (X and Y) are produced using two factors of production (K and L). Both factors are instantaneously mobile across sectors but not across borders. [xii]

$$X = F(K^x, L^x) \tag{11a}$$

$$X^* = F(K^{x*}, L^{x*}) \tag{11b}$$

F is linearly homogeneous, with positive but diminishing marginal productivity of the two inputs. It is twice continuously differentiable. Both inputs are essential for production. The production function also satisfies the Inada conditions. We have (a subscript denotes a partial derivative):

$$F(0, L^x) = F(K^x, 0) = 0$$

$$i \to 0,\ F_i(.) \to \infty;\ i \to \infty,\ F_i(.) \to 0,\quad i = K^x, L^x$$

The Y good is also produced via a linear homogeneous technology G(.) that satisfies positive but diminishing marginal products for factors. Essentiality of inputs, twice continuous differentiability and Inada conditions similar to F(.) hold.

$$Y = G(K^y, L^y) \tag{12a}$$

$$Y* = G(K^{y*}, L^{y*}) \tag{12b}$$

2.3 Factor Market equilibrium

For full employment for L and K in the two countries we have,

$$K = K^x + K^y \tag{13a}$$

$$L = L^x + L^y \tag{13b}$$

$$K* = K^{x*} + K^{y*} \tag{14a}$$

$$L* = L^{x*} + L^{y*} \tag{14b}$$

We assume that good Y is relatively capital-intensive than good X for all factor-price ratios—i.e. there is no factor-intensity reversals.

To simplify some of the algebra below, we use GDP functions for the two countries defined as (see Dixit (1980) and Woodland (1982) for discussion of the properties):

$$R(1, p, K, L) \equiv \max_{K^x, K^y, L^x, L^y} \{X(K^x, L^x) + pY(K^y, L^y) \big| K^x + K^y = K, \big| L^x + L^y = L\} \tag{15}$$

$$R^*(1, p, K^*, L^*) \equiv \max_{K^{x*}, K^{y*}, L^{x*}, L^{y*}} \{X(K^{x*}, L^{x*}) + pY(K^{y*}, L^{y*}) \big| K^{x*} + K^{y*} = K^*, \big| L^{x*} + L^{y*} = L^*\} \tag{16}$$

Thus, given the capital stock and labor supply in each economy and the relative price of the intermediate inputs, the two factors move across the sectors till their marginal revenue products are equalized.

$$R_K = X_K(.) = pY_K(.) \tag{17a}$$

$$R_L = X_L(.) = pY_L(.) \tag{17b}$$

$$R_K^* = X_K^*(.) = pY_K^*(.) \tag{18a}$$

$$R_L^* = X_L^*(.) = pY_L^*(.) \tag{18b}$$

We have assumed that the two economies are diversified in production. Then given p, linear homogeneity of F(.) and G(.), from (17a), (17b), (18a), and (18b) we have factor price equalization.

2.4 Trade: Incompletely Specialized Economies

Intermediate inputs are traded internationally and used to produce a homogeneous good, Q. This good can be used either for consumption or capital accumulation. Trade in the intermediate inputs requires (a variable with a tilde denotes the quantity demanded of the intermediate input):

$$X + pY = \tilde{X} + p\tilde{Y} \tag{19}$$

$$X^* + pY^* = \tilde{X}^* + p\tilde{Y}^* \tag{20}$$

Both economies are diversified in production and face the same prices. The X good is the numeraire and p is the (free trade) relative price of the Y good. Note that we have assumed that trade is balanced in every period, i.e. there is no borrowing or lending.

The final good (Q) is an assembly of the two inputs procured in international markets by trading the intermediate goods output of the economy in question. The relative price of Q is z (in terms of the numeraire). The price index z is linearly homogeneous in the prices of the inputs (1 and p)—it is the unit cost of producing Q. Since both economies face the same input prices (and have the same technology), z is identical across the two countries. We have:

$$Q = H(\tilde{X}, \tilde{Y}) \tag{21}$$

$$Q^* = H(\tilde{X}^*, \tilde{Y}^*) \tag{22}$$

The function H(.) is also increasing, homogeneous of degree one in its arguments, is twice continuously differentiable and satisfies the Inada conditions.

The final good can be consumed or invested:

$$Q(t) = C(t) + I(t) \tag{23}$$

$$Q^*(t) = C^*(t) + I^*(t) \tag{24}$$

2.5 Market-clearing

First, for both countries, we substitute A(t)=K(t), i.e. capital is the only store of value—remember we ruling out international borrowing and lending-- and $r^A(t)=\{R_K(t)/z((t)\}-\delta)$ (similarly for the foreign economy).

Market-clearing in the world economy (i.e. given the state variables K(t) and K*(t)) requires:

(i) The factor markets in both economies clear (equations (13a). (13b), (14a) and (14b).

(ii) That the value of the final good be equal to the GDP of the respective economies given by equations (25) and (26).[xiii]

$$z(1,p)Q = R(1,p,K,L) \tag{25}$$

$$z(1,p)Q^* = R^*(1,p,K^*,L^*) \tag{26}$$

(iii) In addition to the above two budget constraints, we must ensure that the markets for the two traded goods clear. By Walras' Law if one of the markets clear, then so does the other one. The market-clearing condition for the Y-good is:

$$R_p + R_p^* = (Q+Q^*)z_p \tag{27}$$

2.6 Capital Accumulation and Dynamics

We have the aggregate (average) Euler equations for the two countries:

$$\dot{C}(t) = \quad [\{R_K(t)/z(t)\}-(\rho+\delta)].C(t)-\eta(\rho+\eta).K(t) \tag{28}$$

$$\dot{C}^*(t) = \quad [\{R^*_{K^*}(t)/z(t)\}-(\rho+\delta)].C^*(t)-\eta(\rho+\eta).K^*(t) \tag{29}$$

Note, as mentioned in the Introduction, as the probability of death η (more correctly the birth rate) goes to zero, with factor price equalization, equations (28) and (29) give us equal rates of consumption growth in the two countries.

The capital accumulation equations are given by:

$$\dot{K}(t) = Q(t) - \delta K(t) - C(t) \tag{30}$$

$$\dot{K}^*(t) = Q^*(t) - \delta K^*(t) - C^*(t) \tag{31}$$

As long as economies are diversified in production, factor-price equalization holds. We can then write:

$$R_K(t)/z(t) = R^*_{K^*}(t)/z(t) \equiv r(t),$$

and

$$R_L(t)/z(t) = R^*_{L^*}(t)/z(t) \equiv w(t).$$

3. EQUILIBRIUM

Definition 1: An equilibrium path of the world economy is a sequence of consumption and investment in the two countries {C(t), C*(t), I(t), I*(t)}, $t \in (0, \infty)$, such that:

(1) Given the path {p(t), w(t), r(t)}, $t \in (0, \infty)$, the consumers maximize their utility in both countries;

(2) Given the path {p(t), w(t), r(t)},)}, $t \in (0, \infty)$, firms maximize profits;

(3) The consumption paths and capital stocks {C(t), C*(t), K(t), K*(t)}, $t \in (0, \infty)$, satisfy the feasibility conditions (21) to (27), and the initial conditions (K(0), K*(0)).

4. THE STEADY STATE

Definition 2: A steady state of the dynamic system above is a constant level of (an overbar denotes a steady state magnitude) $(\overline{C},\overline{C}^*,\overline{I},\overline{I}^*,\overline{K},\overline{K}^*)$, outputs $(\overline{X},\overline{Y},\overline{Q},\overline{X}^*,\overline{Y}^*,\overline{Q}^*)$ and prices $(\overline{p},\overline{w},\overline{r})$ consistent with equilibrium of the world economy and the initial conditions (K(0) K*(0) with K(0) >K*(0)).

The steady state of the model is obtained by setting equations (28), (29), (30) and (31) to zero—the stability of the system is discussed in the next section.

We have from equations (28) and (29):

$$(\overline{r}-(\rho+\delta))=\eta(\eta+\rho)\overline{K}/\overline{C}, \tag{32}$$

and

$$(\overline{r}-(\rho+\delta))=\eta(\eta+\rho)\overline{K}^*/\overline{C}^* \tag{33}$$

Note in equations (32) and (33), the capital stock is less than the one given by the "modified golden rule" (in the R-C-K model).[xiv]

From equations (30) and (31) we have:

$$\{R(\overline{p},\overline{K},..)/z(1,\overline{p})\}-\delta\overline{K}=\overline{C} \tag{34}$$

$$\{R^*(\overline{p},\overline{K}^*,..)/z(1.\overline{p})\}-\delta\overline{K}^*=\overline{C}^* \tag{35}$$

Proposition 1: The steady state of the two-country model under factor price equalization is unique.

The proof is straightforward. Given $\overline{p}$ and an $\overline{r}$, equations (32) and (34) solve for unique levels of $\overline{K}$ and $\overline{C}$ (similarly for the foreign economy). R(.) is concave in $\overline{K}$ (from the property of the GDP function). Given that, $\overline{C}$ is concave in $\overline{K}$ (from equation (34)).■

Proposition 2: In the steady state, given incomplete specialization in the two countries, the capital-labor ratios will equalize globally. This implies that countries revert to autarky.

Equations (32) and (33) imply that the left hand sides of the two equations are identical. Hence we have

$$\overline{C} / \overline{K} = \overline{C}^* / \overline{K}^*$$

Inserting this in (34) and (35) we get

$$\overline{Q} / \overline{K} = \overline{Q}^* / \overline{K}^* \tag{36}$$

Equation (36) implies:

$$\overline{w}(L / \overline{K}) + \overline{r} = \overline{w}(L^* / \overline{K}^*) + \overline{r}$$

or

$$L / \overline{K} = L^* / \overline{K}^* \equiv \overline{k} \tag{37}$$

Thus, in the presence of factor-price equalization, the two countries will have the same capital labor ratios in the steady state. ■

The larger labor-endowment economy accumulates more capital until the capital-labor ratios are equalized. Thus, as long as the economies converge to an equilibrium in the cone of diversification, they go to the same capital-labor ratio and the reason for international trade evaporates.

5. DYNAMICS

The behavior of the world economy over time can be represented by the four differential equations given by (28), (29), (30) and (31).

Linearizing these four differential equations around the initial steady state and writing in a matrix form, we have:[xv]

$$\begin{bmatrix} \dot{K} \\ \dot{K}^* \\ \dot{C} \\ \dot{C}^* \end{bmatrix} = \begin{bmatrix} (R_K / z) - \delta & 0 & -1 & 0 \\ 0 & (R_K / z) - \delta & 0 & -1 \\ -\eta(\eta + \rho) + CM & CM & (R_K / z) - \rho - \delta & 0 \\ C^* M & -\eta(\eta + \rho) + C^* M & 0 & (R_K / z) - \rho - \delta \end{bmatrix} \begin{bmatrix} C - \overline{C} \\ C^* - \overline{C^*} \\ K - \overline{K} \\ K^* - \overline{K^*} \end{bmatrix} \tag{38}$$

Or compactly:

$\dot{V} = A(V - \overline{V})$

where V≡[K, K*, C,C*]^T,

$M \equiv z(R_{Kp} - R_K / z)^2 / \Delta < 0$. Recalling $(R_K / z) \equiv \bar{r}$, we have

$$\begin{aligned} Det.A = {} & (\bar{r} - \delta)^2 (\bar{r} - \delta - \rho)^2 + [\eta(\eta + \rho)(\bar{r} - \delta - \rho)(\overline{w}\bar{k} - (\bar{r} - \delta)] \\ & - M(C + C^*)[\eta(\eta + \rho) - (\bar{r} - \delta\}(\bar{r} - \delta - \rho)] \end{aligned} \tag{39a}$$

$$Tr.A = 4(\bar{r} - \delta) - 2\rho > 0 \tag{39b}$$

$$\Sigma_{2\times 2} = (\bar{r} - \delta)^2 + (\bar{r} - \delta - \rho)^2 + 2\{(\bar{r} - \delta)(\bar{r} - \delta - \rho)\} > 0 \tag{39c}$$

$$\Sigma_{3\times 3} = 2[(\bar{r} - \rho) + (\bar{r} - \delta - \rho)]\{(\bar{r} - \rho)(\bar{r} - \delta - \rho) - \eta(\eta + \rho)\} - [(\bar{r} - \rho) + (\bar{r} - \delta - \rho)]M(C + C^*) < 0 \tag{39d}$$

Proposition 3: Matrix A has two "stable" roots and two "unstable" roots if the wage share is greater than the net of depreciation capital share. Thus, if this sufficient condition is satisfied, the steady state is a saddle-point.[xvi]

Proof: If the sufficient condition above is satisfied, the determinant of the coefficient matrix A is positive. This implies one of the following possibilities: (i) that there are four unstable roots i.e. with positive real parts (if complex conjugates); (ii) four stable roots; and (iii) two unstable and two stable roots. Note that the positive value of the determinant rules out the possibility of a zero root (or hysteresis).[xvii] The trace (39b) is always positive, thereby ruling out all negative roots (possibility (ii)). The sum of the product of three roots at a time ($\sum_{3\times3}$) (39d), is always negative, so all the roots cannot be positive. We are then left with possibility (iii) i.e., exactly two negative roots. The steady state is therefore a saddle-point and the stable arm is a plane. Since there are two predetermined variables (K and K*), and two forward-looking (or "jump") variables (C and C*), we can associate an initial condition with each of the stable roots. The two transversality conditions (i.e. the No-Ponzi Game conditions) rule out explosive behavior due to unstable roots.■

Discussion: (1) In determining the sign of ($\sum_{3\times3}$) we need to show that

$$\eta(\eta+\rho) > (\bar{r}-\delta)(\bar{r}-\delta-\rho).$$

Now:

$$\eta(\eta+\rho) = (\bar{r}-\delta-\rho)\bar{C}/\bar{K} = [(\bar{r}-\delta-\rho)\{w(L/\bar{K}) + (\bar{r}-\delta)\}] \geq (\bar{r}-\delta-\rho)(\bar{r}-\delta).$$

(2) A sufficient condition for Det A >0 is $\bar{C}/\bar{K} = w(L/\bar{K}) + (\bar{r}-\delta) \geq 2(\bar{r}-\delta)$. Or

$w(L/\bar{K}) \geq (\bar{r}-\delta)$[xviii]

(3) Note that if η=0, Det. A = 0, because we have $(R_K/z) = \delta+\rho$--i.e. we revert back to the infinitely-lived Ramsey model (as discussed in the Introduction).

(4) Note that in all of the results above (i.e., Propositions 1, 2 and 3), we did not need to assume anything about the factor-intensities of the two intermediate inputs X and Y.

Thus at any moment there are two economies—say a richer North and a poorer South. They both produce in the cone of diversification implying factor-price-equalization. Per capita income is higher in the North because it has more capital per capita. In line with comparative advantage, the South exports the labor-intensive good X, and the North exports the capital-intensive good Y. The South accumulates capital faster and (asymptotically) it attains exactly the same capital-labor ratio as the North. Then the two economies revert to autarky.

To understand the mechanism generating convergence, subtract equation (29) from equation (28).

$$\frac{\dot{C}(t)}{C(t)} - \frac{\dot{C}^*(t)}{C^*(t)} = \quad -\eta(\rho+\eta).\{\frac{K(t)}{C(t)} - \frac{K^*(t)}{C^*(t)}\} \tag{40}$$

And from equation (8), we have for the North (a similar equation holds for the South):

$$\{C(t)/K(t)\} = (\rho+\eta)[1+\{H(t)/K(t)\}] \tag{41}$$

H (t) is a forward-looking variable and thus H/K is higher for the poorer economy (it is capital-poor but will become richer with time). Agents consume a part of the future increase in wages that would result due to capital accumulation. This implies that the right-hand side of (40) is negative (because K(t)>K*(t)). And thus the growth rate of consumption is higher for the capital-poor country. This continues until the steady state is reached.

6. CONCLUSIONS

In this paper we had a two-by-two-by-two H-O-S model in a dynamic setting but with individuals with uncertain lifetimes. We found that if both countries are incompletely specialized along the adjustment path, then there is indeed convergence of per-capita incomes and per-capita capital stocks. Given that trade is caused by differences in factor endowments, a convergence to the same capital stock per capita implies that in the steady state there is no trade (i.e. the economies revert to autarky). As noted above, none of these results require any restrictions on relative capital-intensities.

The conclusion of the paper is that factor-price equalization is incompatible with convergence if agents are infinitely-lived. But if we introduce new-born agents who are not linked altruistically to the existing population, then convergence is indeed obtained--the zero-root problem in a continuous time infinite horizon H-O-S model is, therefore, not “robust” to a small perturbation.

Given the results of this paper the next step would be to do a numerical simulation. Some of the restrictive assumptions could be dropped (e.g. log utility, a common cone of diversification) and we could check how well the model performs with realistic parameter values put in.

APPENDIX 1

Individual household behavior

Expected lifetime utility of agent of cohort ν in period *t*:

$$E\Lambda(\nu,t) \equiv \int_t^\infty [1-F(\tau-t)]\log C(\nu,\tau)e^{\rho(t-\tau)}d\tau$$

$$= \int_t^\infty \log C(\nu,\tau)e^{(\rho+\eta)(t-\tau)}d\tau$$

The budget identity:

$$\dot{A}(\nu,\tau) = [r^A(\tau)+\eta]\,A(\nu,\tau)+W(\tau)-C(\nu,\tau)$$

There is also a No Ponzi Game (NPG) condition:

$$\lim_{\tau-\infty} e^{-\int_t^\tau \{r^A(s)+\eta\}ds} A(\nu,\tau) = 0,$$

Decision rule for consumption:

$$C(\nu,t) = (\rho+\eta)\,[A(\nu,t)+H(t)]$$

$$H(t) \equiv \int_t^{\infty} W(\tau)\, e^{-\int_t^{\tau}\{r^A(s)+\eta\}ds} d\tau$$

Note that the human wealth is discounted at the annuity rate of interest, $r^A(\tau)+\eta$.

Aggregate household behavior

We know that the size of cohort ν at time *t* is $\eta e^{\eta(\nu-t)}$. This means that we can define aggregate variables by aggregating over all existing agents at time *t*. For example, aggregate consumption is:

$$C(t) \equiv \eta \int_{-\infty}^{t} e^{\eta(\nu-t)} C(\nu,t) d\nu \qquad \text{(A1.1)}$$

In view of (A1.1) aggregate consumption satisfies:

$$\begin{aligned} C(t) &\equiv \eta \int_{-\infty}^{t} e^{\eta(\nu-t)} (\rho+\eta)\,[A(\nu,t)+H(t)]\,d\nu \\ &= (\rho+\eta)\,[\underbrace{\beta\eta \int_{-\infty}^{t} e^{\eta(\nu-t)} A(\nu,t)\,d\nu}_{A(t)} + \underbrace{\eta \int_{-\infty}^{t} e^{\eta(\nu-t)} H(t)\,d\nu}_{H(t)}\ ] \\ &= (\rho+\eta)\,[A(t)+H(t)] \end{aligned}$$

Similarly, the aggregate budget identity can be delivered:

$$\dot{A}(t) = r^A(t)A(t) + w(t) - T(t) - C(t)$$

The market rate of interest (but not the annuity rate) features in the aggregate budget identity: the term $\eta A(t)$ is a transfer--via the life insurance companies-- from agents who die to agents who are alive.

The consumption Euler equation for individual agents is:

$$\frac{\dot{C}(\nu,t)}{C(\nu,t)} = r^A(t) - \rho$$

The “aggregate Euler equation” satisfies:

$$\frac{\dot{C}(t)}{C(t)} = \eta \int_{-\infty}^{t} e^{\eta(\nu-t)}[r^A(t)-\rho]dt = (r^A(t)-\rho)-\eta(\rho+\eta)\left(\frac{A(t)}{C(t)}\right) \qquad \text{(A1.2)}$$

$$= \frac{\dot{C}(\nu,t)}{C(\nu,t)} - \eta\left(\frac{C(t)-C(t,t)}{C(t)}\right)$$

Note that aggregate consumption growth differs from individual consumption growth due to the turnover of generations. Newborns are poorer than the average household and therefore drag down aggregate consumption growth.

APPENDIX 2

Totally differentiating equations (19), (20) and (27) we get equations (A2.1), (A2.2) and (A2.3) below. Written in matrix form we get (A2.4). Equation (A2.5) is the positive because a_{32} (in the coefficient matrix A in equation (A2.4) below) is the partial derivative of excess demands with respect to price.

$$zdQ + (Qz_p - R_p)dp = R_K dK \qquad \text{(A2.1)}$$

$$zdQ* + (Q* z_p^* - R_p^*)dp = R_{K*}^* dK* \qquad \text{(A2.2)}$$

$$z_p(dQ + dQ*) + [(Q+Q*)z_{pp} - (R_{pp} + R_{pp}^*)]dp = R_{pK}dK + R_{pK*}^* dK* \qquad \text{(A2.3)}$$

$$\begin{bmatrix} z & (Qz_p - R_p) & 0 \\ 0 & (R_p - Qz_p) & z \\ Lz_p & a_{32} & L*z_p \end{bmatrix} \begin{bmatrix} dQ \\ dp \\ dQ* \end{bmatrix} = \begin{bmatrix} R_K dK \\ R_K dK* \\ LR_{pK}dK + L*R_{pK*}^* dK* \end{bmatrix} \qquad \text{(A2.4)}$$

where

$$a_{32} = -(R_{pp} + R_{pp}^*) < 0$$

$$\Delta = -b_{32}z^2 > 0 \qquad \text{(A2.5)}$$

$$dQ/dK = \{(Qz_p - R_p)(zR_{pK} - R_K z_p) - zR_K a_{32}\}/\Delta = z^{-1}\{-M^y(dp/dK) + R_K\} \qquad \text{(A2.6a)}$$

$$dQ/dK^* = (Qz_p - R_p)(zR_{pK*}^* - R_{K*}^* z_p)/\Delta = -(M^y z^{-1})(dp/dK*) \qquad \text{(A2.6b)}$$

$$dp/dK^* = -z(zR^*_{pK^*} - R^*_{K^*}z_p)/\Delta \tag{A2.6c}$$

$$dp/dK = -z(zR_{pK} - R_K z_p)/\Delta \tag{A2.6d}$$

$$dQ^*/dK = (R_p - Qz_p)(zR_{pK} - R_K z_p)/\Delta = z^{-1}M^y(dp/dK) \tag{A2.6e}$$

$$dQ^*/dK^* = \{-(Qz_p - R_p)(zR^*_{pK^*} - R^*_{K^*}z_p) - zR^*_{K^*}a_{32}\}/\Delta = z^{-1}\{M^y(dp/dK^*) + R^*_{K^*}\} \tag{A2.6f}$$

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[i] See, for instance, Barro and Sala-i-Martin (2004).

[ii] $u(C_t) = (C_t^{1-\sigma})/(1-\sigma), \;\; \sigma > 0.$

[iii] Weil (1989) is identical to this set up except he focuses on population growth. I use the simpler model of Blanchard for the problem at hand.

[iv] See Kaneko (2006) for a small country model with uncertain lives and a two-sector production structure. He showed that the small open economy is diversified in production (see also Baxter (1992) and Bianconi (1996) on this).

[v] Everyone has the same labor productivity. Thus human capital is not different between the new-born and the rest of the population.

[vi] This ensures that assets and liabilities of dead individuals pass to the living ones.

[vii] Hence the description of this model as 'a model of perpetual youth".

[viii] It would be straightforward, if tedious, to extend the analysis to the case of the instantaneous utility function being of the CRRA variety but with an intertemporal consumption elasticity different from unity (see Buiter (1988)). The marginal propensity to consume would be given by:

$$\int_0^{\infty} \{\exp.(\sigma^{-1}-1)(R^A(t)+\eta t)-\rho\sigma^{-1})\}dt \text{ where } R^A(t) \equiv \int_0^t r^A(\tau)d\tau.$$

[ix] The rate η is the actuarially-fair return (that enables the insurance companies to end up with zero profits).

[x] This is a transversality condition that is written as an equality.

[xi] I thank a referee for asking for a clarification on aggregation with factor-price-equalization.

[xii] Where there is no chance of confusion, we do not explicitly write the time index.

[xiii] This is essentially reproducing the static duality analysis of Dixit and Norman (1980) and Woodland (1982).

[xiv] In this respect it is like a closed economy B-Y model. I am grateful to a referee who wanted this clarified.

[xv] We are going to use the results given in the Appendix 2.

[xvi] Whenever we say a root is negative (positive) or stable (unstable), we refer to the real part.

[xvii] I.e., the case that occurs with zero population growth (remember here it is births that are crucial in determining the wedge between average and marginal growth rates of consumption, see Buiter (1988)).

[xviii] This requires the wage share in GDP to be greater than the share of capital *net of depreciation*.