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Inequality and Expectations in a Model of Technology Adoption and Growth

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Abstract

This paper highlights the role of initial wealth inequality in determining the technology adoption decision of firms, which in turn impacts upon the overall productivity in an economy. Wealth inequality interacts with producers' expectations to generate mutiple equilibria: poor economies where initial wealth inequality is too high are perpetually stuck at a bad equilibrium with poor technology; economies with moderate degree of inequality can oscillate between the bad and the good equilibria depending on producers' expectations; and rich economies with sufficiently low degree of wealth inequality always enjoy a self-sustaining good equilibrium, characterized by the adoption of advanced technology.

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1 Introduction

The basic question that this paper seeks to address pertains to the observed large and persistent differences in the levels of technology and consequent productivity differences across countries. We provide a simple theoretical framework to analyse the technology adoption decisions of profit-seeking private firms and examine how their incentives to adopt the modern technology could vary across countries depending on the overall macroeconomic scenario, even when the latest, state of the art technical know-hows are readily available.

That differences in total factor productivity, or TFP, are key for understanding income differences across countries is well-recongnised in the literature. Seminar works by Parente and Prescott (1994), Klenow & Rodríguez-Clare (1997) and Hall & Jones (1999) have shown that there are major differences in cross-country levels of TFP, which account for half or more of level differences in GDP per worker. Jerzmanowski (2007) argues that the bulk of cross-country TFP differences are due to the inefficient use or delayed adoption of new technologies by the developing countries. There exists a parallel theoretical literature that has tried to understand why firms in poorer economies do not implement the modern, state of the art technologies. Lack of adoption of the latest technology has been attributed to various institutional factors like barriers to trade, lack of competetion (protecting vested interest), and lack of property rights (e.g., Parente & Prescott (1994)). Another set of the literature emphasizes on the differences in relative factor endowmnets in the advanced vis-a-vis the backward economies (in particular, relatively low endowments of skilled vis-a-vis unskilled labour in the latter) which either hamper the process of adaptation and diffusion of newer technologies (e.g., Nelson & Phelps (1966); Benhabib & Spiegel (2005)), or lead to adoption of inappropriate technologies (e.g, Atkinson & Stiglitz (1969); Basu & Weil (1998); Acemoglu & Zilibotti (2001)). In this paper we argue that initial wealth distribution could be a key determinant in influencing firms' decision to adopt the modern technology - quite independent of the other institutional factors highlighted in the literature. Wealth inequality interacts with producers' expectations to generate mutiple equilibria: poor economies where initial wealth inequality is too high are perpetually stuck at a bad equilibrium with poor technology; economies with moderate degree of inequality can oscillate between the bad and the good equilibria depending on producers' expectations; and rich economies with sufficiently low degree of wealth inequality always stay in a self-sustaining good equilibrium with advanced technology.

We develop a dynamic general equilibrium model where skill formation and technology adoption feed into each other. The model starts with the basic presumption that firms' decision to adopt the latest technology depends on the overall profitability of such a decision. In the presence of a fixed cost, the profitability of firms depends on the relative productivity of the technology (in comparison to the fixed cost) as well as on the level of aggregate demand. We postulate that the the productivity parameter associated with the modern technology is an increasing function of the ratio of skilled to unskilled labourforce in the economy (spillover effect). In such a scenario, wealth inequality enters into the picture through two distinct channels: (a) it influences skill formation and composition of the labour force - thereby influencing the productivity parameter of the modern technology; (b) it has an impact on the aggregate demand - thereby affecting the overall profitability. In a less developed economy with a high degree of inequality and consequently with a high proportion of unskilled people, the above two channels work in tandem to ensure that the better technology is never adopted - the productivity (relative to the cost) is too low and aggregate demand is too low as well. A middle income economy (with moderate inequality) demonstrates the classic mutiple equilibria scenario where productivity (relative to the cost) is moderately high but adequate demand is generated if and only if sufficient number of firms coordinate; not otherwise. However once the modern technology is succesfully adopted, income thus generated boosts skill formation in the next period. This enhances productivity in the next period, thereby creating a postive loop that may allow a middle income economy to escape the bad equilibrium in the long run. This is also the mechanism through which the rich countries can perpetually enjoy a good equilibrium with continuous adoption of the better technology.

The skill formation mechanism depicted here is an adaptation of Galor & Zeira (1993) where acquiring skill requires a lumpy investment. In the absence of credit markets, households are compelled to use own resources to acquire skill. This immediately implies that households earning below a threshold will not be able to opt for skill formation. Thus the initial distribution of income will generate an initial skilled-unskilled labour ratio which in turn will have important implications for the production side of the story.

On the production side, the structure is close to Murphy, Shleifer & Vishny (1989) where at any point of time two types of technologies co-exist - a constant returns to scale cottage technology with low return and an increasing returns to scale modern technology with high return but an associated fixed set up cost. However, we make two crucial departures from Murphy et al. (1989).

First, we allow for a heterogenous labour force - comprising of skilled and unskilled labour such that only the former is employed in the modern technology while the latter is employed in cottage production. Ownership of firms is also heterogenously distributed in our model. These two forms of heterogenity allow us to exploit a rich and diverse occupational structure where different occupations are affected differently by the process of modernization and will therefore have differential impact on the aggregate demand. We then explore the implications of endogenous determination of optimal decisions to invest in skill by individuals on the one hand and the optimal decision to invest in modern technology by firms in the economy on the other hand. Our results show that expectations and history (as represented by the initial distribution of wealth) together determine the path of technology adoption in an economy.

Secondly, and more importantly, we incorporate a dynamic feedback mechanism from skill formation to technology adoption and vice-versa. This two-way feed back mechnism, which is a novel feature of our model, is crucial in bringing back a role of history in the standard multiple equilibria story of Murphy et al. (1989). Whether the modern technology is adopted or not depends on its profitability - which in turn depends on the existing skilled-unskilled ratio. On the other hand, skill acquisition is costly and forward-looking agents opt for skill formation if and only if the modern firms operate. Thus if the initial inequality is too high so that very few people can afford to acquire skill to begin with, then modern firms may not operate at all, which dampens the incentives to acquire skills even for those who can afford to do so. Hence cottage equlibrium perpetuates. The opposite happens when the initial inequality if sufficiently low. In other words, allowing for this two-way linkage between skill formation and technology adoption enables us to analyse the long run consequences of initial wealth distribution and to clearly demarcate the process of transition from multiple equilibria (where expectation rules the roost) to steady growth (where expectation ceases to matter).

The rest of the paper is organised as follows. Section 2 presents the basic set-up of the model and details the consumer side and the producer side of the economy. In section 3, we discuss the static equilibrium in the economy. In section 4, we give a brief outline of dynamic extension of our model. Section 5 concludes.

2 The Basic Structure of the Model

Consider a closed economy, comprising of a continuum of dynastic households of measure $\bar{L} > 1$. Each dynasty is indexed by a number $i \in [0, \bar{L}]$.

Households differ in terms of their inherited wealth. Apart from the material wealth (tangible resources) households also inherit ownership of production units (firms). For analytical convenience we assume simple point distributions for both wealth ownership and firm ownership. Thus on the basis of wealth ownership we divide all the households in two catergories: Rich and Poor. Similarly on the basis of firm ownership all the households are divided into two categories: Capitalists (those who own a firm) and Workers (those who do not own a firm). Thus, at any point of time, the economy comprises of four mutually exclusive sub-categories: Rich Capitalists (RE), Rich Workers (RW), Poor Capitalists (PE) and Poor Workers (PW). Households belonging to each of these four sub-groups are identical in every respect.

Wealth ownership evolves over time on the basis of the bequest dynamics across generations. However, we assume that firm ownership is exogenously given and does not change over time. In other words, there is no market where people can buy or sell a production unit. Production units belonging to a dynasty are simply handed over from one generation to the next.

2.1 Household Side Story

Each individual lives for exactly one period and in the next period an exact replica is born who carries on the dynastic link.

2.1.1 Skill Formation Technology

An agent is born with one unit of unskilled labour. In addition to this labour endowment, he also inherits some wealth from his parent in the form of bequests. At the beginning of his life-time, before production takes place, the agent can convert his one unit of unskilled labour into one unit of skilled labour by investing a fixed amount h. We assume this conversion of unskilled labour to skill labour happens instantaneously. We futher assume that there exists no market for borrowing. This implies that the entire investment in skill formation must be financed from the bequest received. Thus an agent can opt for skill formation if and only if his inherited wealth level at the beginning of the period (x) is at least as high as h. Any residual inherited wealth (over an above the amount spent on skill formation) can be carried forward to the end of the period for consumption and bequest purposes.

We use the resource cost for education, h, as the threshold for classifying the initial population into wealth categories: 'rich' and 'poor'. Thus, the initial 'rich' are those individuals for whom $x_0 > h$ and the initial 'poor' are those with $x_0 < h$.

Having decided about skill formation, the agent can then work either as an unskilled labour or a skilled labour to earn a corresponding wage income. Further, if the agent belongs to a family of capitalists then he also inherits the ownership rights to the family firms which, if operational during this period, generate some profit income. Thus for agents belonging to the 'Worker' category, total income consists only of labour income, while for agents belonging to the 'Capitalist' category, total income consists of labour income as well as possibly some profit income.

2.1.2 Preferences

All agents have identical preferences. They derive utility from own consumption (C) as well as from the amount of bequest (b) left to their progeny (warm glow). At any point of time in the economy there exists a continuum of different varieties of final goods, represented by the inverval $[0, 1]$ such that a variety is indexed by a number $q \in [0, 1]$. Each of these final goods can be converted one-to-one into a generic bequest good used for transferring resources across generations.[1] Let c_q denote the quantity consumed of each variety q. The preferences of an agent is represented by the following love-for-variety (Dixit-Stiglitz) utility function:

$$U = U(C, b) = \beta \int_0^1 (\log c_q) dq + (1 - \beta) \log b \tag{1}$$

Thus the utility maximisation exercise for the agent is given by:

$$\max U(C, b) = \beta \int_0^1 (\log c_q) dq + (1 - \beta) \log b$$

subject to

$$\int_0^1 c_q p_q dq + b = \hat{y} \equiv y + \hat{x}$$

where y is the income of the individual and $\hat{x}$ reprsents his residual inherited wealth.[2]

In a symmetric equilibrium where $c_q = \bar{c}$ and $p_q = \bar{p}$ for all q, solution to the above utility maximization exercise generate the following optimal consumption level for each variety:

$$\bar{p}\bar{c} = \beta(y + \hat{x}). \tag{2}$$

[1] Alternatively, we can treat any one of these varieties as the numeraire good. In asymmetric equilibrium where all prices are equal, all varities can be costlessly converted into one another. Therefore bequest can be held in any form.

[2] The income of each individual would vary depending upon the wealth and ownership category that he belongs to. We calculate these incomes later, in section 2.5.

Corresponding optimal bequest is given by;

$$b = (1 - \beta)(y + \hat{x}). \tag{3}$$

2.1.3 Demand for each variety

Notice that households' demand for any variety comes from two sources: (i) from their current income (y); and (ii) from their inherited wealth (x). Out of these two, only the first one constitutes the demand for current production, the latter being already accounted for in the previous period's production.

Also recall that any variety can be coverted one-to-one into the bequest good. Thus an agent's total expenditure for any variety $q \in [0, 1]$ out of his current income (y) is given by y itself - part of which is used consumption purposes and the other part is used for bequest purposes. Hence total expenditure on any variety that is currently produced is given by $Y = \int y_i di$, where y_i is the income of the i-th agents and Y represents the aggregate income in that period. In other words, the demand function for current production of any variety q is given by:

$$D_q = \frac{1}{\bar{p}} Y \quad \text{for all } q \in [0, 1]. \tag{4}$$

Note that the demand function for each variety exhibits unit price elasticity. We shall come back to this point later.

2.2 Distribution of Wealth and Ownership of Production Units

Let γ_t denote the proportion of the population at time t that are rich. Accordingly $1 - \gamma_t$ is the proportion of the people that are currently poor.

Likewise, let μ_t denote the proportion of the population at period t who are capitalists (i.e, have inherited ownership rights to the production units). Accordingly $1 - \mu_t$ is the proportion of current population that are workers. Recall that in our model ownership rights to firms are inherited; these are transferred form one generation to the next within the same dynasties. Thus, μ_t remains constant over time at some value $\bar{\mu}$, which is the historically given and hereditary firm ownership pattern in the economy.

However, our emphasis here is more on the distrbution of wealth than the distribution of ownership rights. Hence we shall focus on two polar cases: one where the ownerhip rights to firms are equally distributed across the entire population such that $\bar{\mu} = 1$; the other where the distribution of firm ownership coincides with the initial wealth distribution, i.e., $\bar{\mu} = \gamma_0$.

In what follows we shall proceed with the first case. It is easy to show that our results are accentuated if the distribution of firm ownership is as concentrated as the distribution of wealth.

2.3 Production Side Story

At any point of time the economy produces a given variety of final goods of measure 1, which are indexed by $q \in [0, 1]$. The measure of different varieties does not change over time. In other words, technical progress in our model does not stem from invention of newer varieties.[3] Technological advancement is our model is associated with the usage of a more productive technology (vis-a-vis a less producive one) for producing the same set of goods, as explained below.

Each variety q can be potentially produced using two different technologies: a highly productive modern technology and a less productive cottage technology. Production technologies are symmetric for all varieties.

2.3.1 Modern Technology

Production under modern technology is organized in a proper production unit (a firm) and involves skilled labour as the only input. Operating this technology requires a fixed set up cost of $\hat{L}$ units of skilled labour. After incurring the fixed cost, every additional unit of skilled labour employed produces α units of output under the modern technology. The associated production function is represented by:

$$Y^s = \alpha \left(L^s - \hat{L} \right)$$

where, L^s is the total skilled labour employed by the modern firm; $\hat{L}$ is the fixed set up cost in terms of skilled labour and α represents labour productivity in the modern technology.

A crucial assumption in our model (which sets it apart from Murphy et al (1989)) is that the labour productivity index in the modern sector, α, is not exogenously given; it is positively related to the proportion of skilled labour in the economy. This assumption is motivated by the new growth theory literature, which justifies the existence of such spillover effects in modern production. Let λ_t be the the proportion of skilled labour in the economy at time t. Then :

$$\alpha_t = \alpha(\lambda_t),$$

[3]This is where we differ from the existing models of technology adoption and growth (e.g., Nelson and Phelps (1966) or Benhabib and Spiegel (2005)).

such that $\alpha(0) = \underline{\alpha} > 1$ and $\alpha' > 0$.

Note that due to presence of the fixed cost, the modern technology exhibits increasing returns to scale at any point of time. Moreover due to the spillover effect there is a positive externality from skill formation which operates over time.

The IRS nature of the technology implies that it cannot be operated under competitive market structure; we assume that each of these technologies (for each $q \in [0, 1]$) is operated by a monopolist firm.

2.3.2 Cottage Technology

Cottage technology on the other hand can be thought of as a home production technology that does not require a formal production unit. This technology uses unskilled labour and there is no fixed cost involved in production. One unit of unskilled labour employed produces 1 unit of output under the cottage technology. The associated production function is represented by:

$$Y^n = L^n,$$

where L^n represents unskilled labour. This technology exhibits constant returns to scale.

One should note here that a skilled worker can always masquerade as unskilled worker; but not the other way round. Thus even though the cottage production technology is specified in terms of unskilled labour, in practice both skilled and unskilled labour can be employed in this production technology. However, the productivity of both types of worker under the cottage technology will be the same, equal to unity.

2.4 Wages and Profits

It is obvious from the cottage production technology specified above that productivity of labour in cottage production is unity. We shall use the unskilled wage rate as the numeraire. Thus implicit wage rate in the cottage production is given by:

$$w_N = 1.$$

In the modern production, the production technique involves skilled labour. Since skill acquisition is costly, the skilled wage rate sould be higher than that unskilled labour. We assume that wage rate in the modern production is a constant, given by:

$$w_M = 1 + v,$$

where $v > 0$ is the skill premium.

We assume that $\bar{\alpha} > 1 + v$ such that modern production is always technically viable, even when the labour productivity is at its minimum.

The profit for a representative monopolist firm in the modern sector of the economy can now be written as:

$$\pi_q = p_q Y^s - w_M L^s.$$

Due to the presence of the constant returns to scale technology which can produce the same variety at unit cost, prices charged by the monopolist firms engaged in modern technology cannot be greater than 1. On the other hand, facing a unit elastic demand curve, he would not charge anything less. We thus deduce that prices are constant across all varieties and is equal to unity:

$$p_q = 1 \quad \forall q.$$

Let Y be the aggregate income in the economy. Recall that poeple spend β proportion of their income on consumption and leave $(1 - \beta)$ proportion of their income as bequest. Since any variety can be coverted one-to-one into the bequest good, the aggegate demand for each variety (comprising of consumption as well as bequest demand) is given by $D_q = \frac{1}{p_q} Y$. Using $L_s - \hat{L} = \dfrac{Y^s}{\alpha(\lambda)}$, $Y^s = \frac{1}{p_q} Y$, $p_q = 1$ and $w_M = 1 + v$, we can re-write the profit function for each monopolist firm as:

$$\pi_q = Y - (1 + v) \left[\frac{Y}{\alpha(\lambda)} + \hat{L} \right] \equiv \pi \tag{5}$$

Or,

$$\pi = Y a(\lambda) - (1 + v)\hat{L}$$

where $a(\lambda) \equiv \frac{\alpha(\lambda) - (1+v)}{\alpha(\lambda)}$.

2.5 Investment in Skill Formation

We have already solved for the consumption and bequest decisions of an agent with income y and residual wealth $\hat{x}$ (refer to equation 2 and equation 3 in section 2.1.) Plugging back these solutions in the utility function of an agent, and using $\bar{p} = 1$ we get the following indirect utility function:

$$\beta \log \beta + (1 - \beta) \log(1 - \beta) + \log \left(y + \hat{x} \right).$$

Notice that the indirect utility is an increasing function of his total *end-of the-period* wealth $\hat{y} \equiv (y + \hat{x})$ (i.e., income and resildual inherited wealth).

Thus any agent with inherited wealth $x > h$ will therefore decide to invest in skill formation if and only if the resulting income is higher *end-of the-period* wealth is higher when skilled than when unskilled. Recall that agents differ in terms of their inherited wealth although the ownership rights to firms (and therefore potential profit incomes) are equally distributed. Thus an agent with inherited wealth $x > h$ will therefore decide to invest in skill formation if and only if

$$w^S + (x - h) \geqq w^N + x;$$

$$i.e., \; v > h. \qquad \text{(Assumption 1)}$$

Thus, the wage premium in skilled sector should be sufficiently high so as to cover the opportunity cost of investing in skill formation (given by h). This is the incentive condition for the rich agents to opt for skill formation. We shall assume that the above parametric condition always holds.

Let us now calculate the income for alternative categories of households. Since ownership rights to firms are equally distributed, all agents earn some 'potential' profit income, given by $\frac{\hat{\Pi}}{\bar{L}}$, where $\hat{\Pi}$ represents aggregate profit in the economy.[4]

Rich Agents:

This set of agents receives a bequest $x_t^R > h$. Each of them also inherits ownership rights to modern firms that generates a profit income $\frac{\Pi}{\bar{L}}$. By Assumption 1, these agents necessarily invest in skill formation. Hence the corresponding income of a rich agent is given by:

$$y_t^R = (1 + v) + \frac{\hat{\Pi}}{\bar{L}}$$

Poor Agents:

This set of agents receives a bequest $x_t^P < h$. Hence they cannot opt for skill formation anyway. However each of them inherits ownership rights to modern firms that generate a profit income $\frac{\hat{\Pi}}{\bar{L}}$. Thus the corresponding income of a poor agent is given by:

$$y_t^P = 1 + \frac{\hat{\Pi}}{\bar{L}}$$

[4]Notice that thi profit income would accrue to the agents if and only if the modern firms are operating in the economy. Thus stirctly speaking, these are the 'potential' income levels of the agents. Whether the potential is realized or not would depend on the overall macroeconomic scenario, which we discuss later.

Notice that by Assumption 1, those who inherit enough to afford skill formation will definitely acquire skill (provided the modern sector is operating) while those who inherit less than the cost of skill formation, will not be able to acquire skill. A significant (and immediately useful) implication of this assumption is that the distribution of wealth alone determines the distribution of skilled labourforce. Thus, γ_t which denotes the proportion of population that is rich, now also represents the proportion of workforce that is skilled (λ_t), i.e.,

$$\gamma_t \equiv \lambda_t.$$

Thus we could use the terms 'rich' and 'skilled' interchangeably.

It is very important to recognise however that this conclusion presupposes that the modern sector is already operational. If none of the modern firms are operating, then the rich people, who had opted for skill formation, would not be able to find employment in the modern sector. In that case, under perfect foresight, the forward looking agents would **not** opt for skill formation in the first place and the equivalence bewteen γ_t and λ_t would break down. This is in fact a very important and novel feature of our model which ties up the skill formation decision of agents to market expectations. We shall come back to this point later.

3 Characterization of Static (Temporary) Equilibrium

3.1 Aggregate Output

Now we can add the wage and the profit components to obtain aggregate domestic output (GDP) in the economy when $n \in [0,1]$ varieties are being produced using the modern technology:

$$Y = W + \hat{\Pi} = W_N + W_M + n\pi$$

where

$$W = \text{Total Wage Bill}$$

$$W_M = \text{Wage Bill in the modern sector}$$

$$W_N = \text{Wage Bill in the traditional sector}$$

$$\hat{\Pi} = n\pi = \text{Aggregate Profit in the modern sector}$$

and,

$$W_N = w_N(\bar{L} - nL^s) = (\bar{L} - nL^s)$$

$$W_M = w_M n L^s = (1+v) n L^s = (1+v) n \left[\left(\frac{Y}{\alpha(\lambda)} + \hat{L} \right) \right]$$

$$\hat{\Pi} = n\pi = n \left[Y - (1+v) \left(\frac{Y}{\alpha(\lambda)} + \hat{L} \right) \right]$$

Then,

$$\begin{aligned}
Y &= (\bar{L} - nL^s) + (1+v) n \left[\left(\frac{Y}{\alpha(\lambda)} + \hat{L} \right) \right] + n \left[Y - (1+v) \left(\frac{Y}{\alpha(\lambda)} + \hat{L} \right) \right] \\
&\Rightarrow \; Y = (\bar{L} - nL^s) + nY \qquad (6) \\
&\Rightarrow \; (1-n)Y = \bar{L} - nL^s = \bar{L} - n \left[\left(\frac{Y}{\alpha(\lambda)} + \hat{L} \right) \right] \qquad (7) \\
&\Rightarrow \; [\alpha(\lambda)(1-n) + n] \, Y = \alpha(\lambda) \left[\bar{L} - n\hat{L} \right] \qquad (8)
\end{aligned}$$

or,

$$Y_t = \frac{\bar{L} - n_t \hat{L}}{1 - n_t \frac{\alpha(\lambda_t) - 1}{\alpha(\lambda_t)}} \qquad (9)$$

We now derive a sufficient condition for aggregate demand to respond positively to the degree of modernization in the economy (as represented by the numbers of sectors, n, which adopt the modern technology).

First note that relationship between aggregate income and number of modern firms in the economy is captured by the following derivative:

$$\frac{dY}{dn} = \frac{\bar{L} m(\lambda) - \hat{L}}{(1 - m(\lambda) n)^2}$$

where, $m(\lambda) \equiv \frac{\alpha(\lambda)-1}{\alpha(\lambda)}$

It is now easy to see that the condition required for aggregate output to respond positively to modern technology adoption is given by:

$$\begin{aligned}
\frac{dY}{dn} &> 0 \\
&\Rightarrow \; \bar{L} m(\lambda) > \hat{L} \qquad (10)
\end{aligned}$$

This condition implies that the threshold skilled labour (fixed cost) for production in modern firm should not exceed a fraction of the total labourforce in the economy. This fraction, $m(\lambda)$, is positively related to the productivity parameter $\alpha(\lambda)$. However, since the productivity parameter itself is an increasing function of the composition of labourforce, one can deduce that the above condition would always hold as long as it holds for the lowest possible value of the productivity parameter ($\underline{\alpha}$). Accordingly, we make the follwoing assumption:

$$\bar{L}(\frac{\underline{\alpha}-1}{\underline{\alpha}}) > \hat{L}. \qquad \text{(Assumption 2)}$$

This is a sufficient conditon for aggregate output to increase as the number of firms opting for modernization increases.

3.2 Profit of the Modern Firms

Given the aggregate demand equation as function of n, we can now express the profit earned by each of the monopolist firm who has modernised as a function of n . This $\pi(n)$ function is given below:

$$\begin{aligned} \pi(n) &= \left[Y - (1+v)\left(\frac{Y}{\alpha(\lambda)} + \hat{L}\right)\right] && (11) \\ &= \frac{[\alpha(\lambda) - (1+v)]}{\alpha(\lambda)} Y - (1+v)\hat{L} && (12) \\ &= \frac{[\alpha(\lambda) - (1+v)]}{\alpha(\lambda)} \left(\frac{\bar{L} - n\hat{L}}{1 - n\frac{\alpha(\lambda)-1}{\alpha(\lambda)}}\right) - (1+v)\hat{L} && (13) \\ &= \frac{\bar{L}\left[\alpha(\lambda) - (1+v)\right] - \hat{L}\left[n\alpha(\lambda) + (1-n)(1+v)\right]}{(1-n)\alpha(\lambda) + n}. && (14) \end{aligned}$$

It is also easy to verify that as long as the inequality specified in (Assumption 2) holds, $\frac{d\pi}{dn} > 0$ as well. (This is intutive; profit is positively related to aggreagte income and the n term enters into the profit equation only through aggreagte demand. Thus $\frac{dY}{dn} > 0 \Rightarrow \frac{d\pi}{dn} > 0$).

4 Technology Adoption in Equilibrium

In this section, we take the skilled-unskilled ratio (λ) as given and analyse the technology adoption decision at any point of time t. Technology adoption in equilibrium will depend on two factors: (a) the profitability of operating a modern production unit, which in turn will determine whether a firm has positive incentives to adopt the modern technology or not; and (b) the supply of skilled labour - which is going to act as a constraint on how many firms can actually adopt the modern technology - even if incentives are positive. Notice that these are two independent constraints, which will yield different values of n. The equilibrium n_t will depend on whichever is the binding constraint at period t.

Let us first consider the skilled labour market. Given λ,

$$\text{Supply of skilled labour} = \lambda\bar{L} \tag{15}$$

and,

$$\text{Demand for skilled labour} = n\left[\frac{Y}{\alpha(\lambda)} + \hat{L}\right] \tag{16}$$

Clearing in the skilled labour market then requires:

$$\lambda\bar{L} = n\frac{Y}{\alpha(\lambda)} + n\hat{L} \tag{17}$$

We are interested to know if there exists a labour-market clearing solution for n which lies in the continuum $[0, 1]$. For establishing the existence of such a solution, we turn to the diagrammatic analysis of the skilled labour market clearing condition. From equation 15 let us define:

$$G(n) \equiv \frac{\lambda\bar{L}}{n}$$

and from equation 16, after substituting for Y, we define:

$$H(n) \equiv n\frac{\bar{L} + \alpha(\lambda)(1-n)\hat{L}}{\alpha(\lambda) - [\alpha(\lambda) - 1]\,n}$$

Plotting both these functions against n, we can draw some conclusion about the solutions for n. We begin by analysing the $H(n)$ function:

$$\frac{dH(n)}{dn} \gtreqless 0 \quad \text{according as} \quad \alpha(\lambda) \gtreqless \frac{\bar{L}}{\bar{L} - \hat{L}}$$

$$\frac{dH(n)}{dn} > 0$$

iff

$$\lambda < 1 + \frac{\alpha(\lambda)\hat{L}}{\bar{L}}$$

which indeed holds, given Assumption 2. Thus, we get an upward sloping curve for this function (see Figure 1). Moreover within the continuum $[0, 1]$, $H(n)$ function satisfies the following limit conditions:

$$\lim_{n\to 0} H(n) = \frac{\bar{L}}{\alpha(\lambda)} + F$$

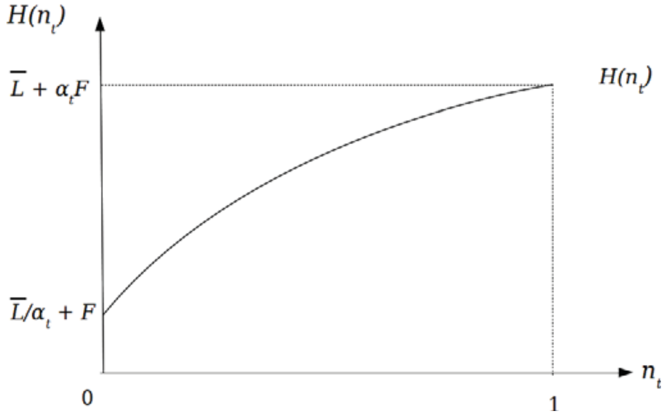

Figure 1:

and

$$\lim_{n\to 1} H(n) = \bar{L} + \alpha(\lambda)F$$

Similarly, for the $G(n)$ function:

$$G\prime(n) = \frac{-\lambda\bar{L}}{n^2}$$

which is always negative. We thus get a downward sloping curve, as shown in Figure 2. Moreover within the continuum $[0,1]$, $G(n)$ function satisfies the following limit conditions:

$$\lim_{n\to 0} G(n) = \infty$$

and

$$\lim_{n\to 1} G(n) = \lambda\bar{L}$$

Now, juxtaposing the functions $H(n)$ and $G(n)$ in the same graph, and noting that $\lim_{n\to 0} H(n) < \lim_{n\to 0} G(n)$, while $\lim_{n\to 1} H(n) > \lim_{n\to 1} G(n)$, we get exactly one intersection point within the continuum $[0,1]$, as illustrated in Figure 3.

Thus, for any given value of the skilled-unskilled ratio (λ), the (skilled) labour market clearing condition will always generate an $\bar{n} \in [0,1]$.[5] More-

[5] In fact the skilled labour market clearing condition (equation 17) generates a quadratic equation in n which can be solved to derive the precise value of $\bar{n}$ that lies between zero and unity. This value is given by:

$$\bar{n}_t = \frac{\alpha(\lambda_t)(\lambda_t\bar{L}+\hat{L}(1-\lambda_t)\bar{L} - (\alpha(\lambda_t)(\lambda_t\bar{L}+\hat{L}(1-\lambda_t)\bar{L}+(1-\lambda_t)\bar{L})^2 - 4\alpha(\lambda_t)^2\lambda\hat{L}\bar{L}}{2(\alpha(\lambda_t))^2\hat{L}\bar{L}\lambda_t}.$$

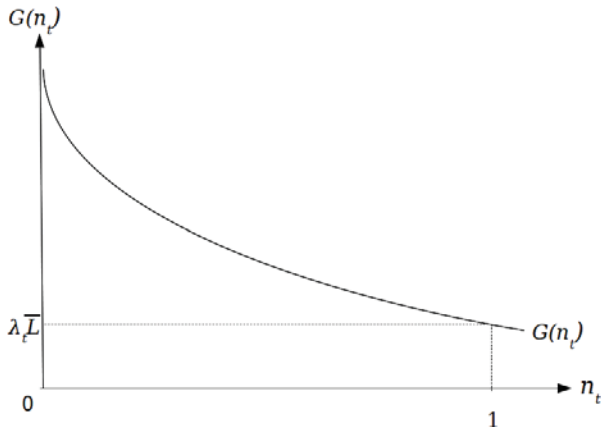

Figure 2:

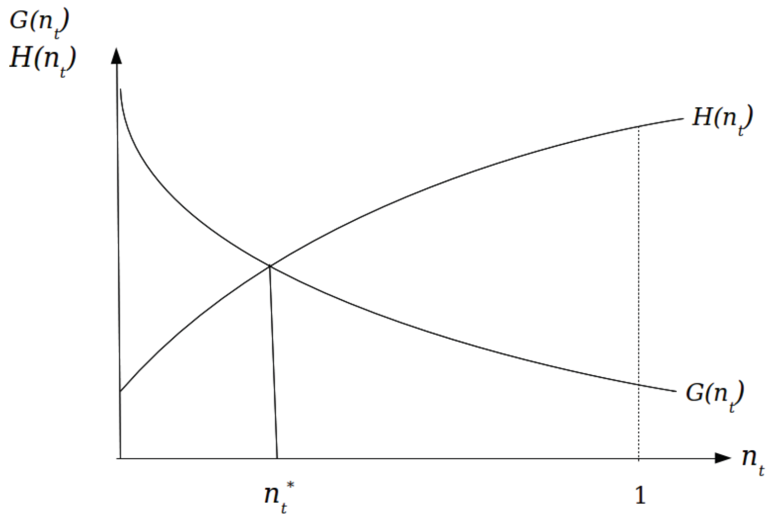

Figure 3:

over this labour market clearing value of n is exactly equal to unity when $\lambda = 1$.

Recall however that even though there exists a $\bar{n}$ that clears the skilled labour market in every period, this may not necessarily be the equilibrium n. For this to be the equilibrium, the concommitant profit level for each of these n modern firm must be non-negative. This brings us to the other constraint, namely the incentive constraint, which we discuss below.

A modern firm will have incentives to operate with the advanced technology only if it earns non-negative profit from this activity. The profitability of each modern firm on the other hand depends on how many other firms are operational as well, as was shown eralier by the $\pi(n)$ expression in equation (11). Thus by setting the $\pi(n)$ expression to zero (and noting that $\frac{d\pi(n)}{dn} > 0$), we shall get another value of n, say $\hat{n}$ which would tell us the minimum number of modernised firms required such that all the modernized firms at or beyond this threshold would earn non-negative profits. This break-even value of n, arising out of the incentive constraint, is given as follows:

$$\hat{n} = \frac{\bar{L}\,[\alpha(\lambda) - (1+v)] - (1+v)\hat{L}}{[\alpha(\lambda) - (1+v)]\,\hat{L}} \tag{18}$$

Thus the equilibium value of n will be determined by the relative magnitutes of $\bar{n}$ and $\hat{n}$. In particular, for any given λ we shall have the following two mutually exclusive cases:

1. $\bar{n} < \hat{n}$. In this case the skilled labour supply becomes binding before the firms can earn positive profit. So no firm would opt for modernization and the economy would be saddled with the cottage technology.

2. $\bar{n} \geqq \hat{n}$. In this case the skilled labour supply condition is not binding at $\hat{n}$. Thus all firms would now be willing to adopt the modern technology as long as at least $\hat{n}$ firms are adopting them. This is the case of multiple equilibria arising out of coordination failure, and adoption of the modern technology would now crucially depend on the expectations of the firms. In a buoyant economy where everybody is optimistic about the general economic environment and conducive profit conditions, the economy would choose to be at $\bar{n}$. On the other hand, expectations of a stagnant economy would induce no firm to adopt the modern technology and the economy will indeed remain saddled with only cottage production.

Notice however that the $\bar{n}$ and $\hat{n}$ themselves depend on λ . Hence we analyse below the scenarios that emerge with various values of λ.

4.1 Proportion of Skilled Labour and Technology Adoption

Recall that $\pi(n)$ is an increasing function of λ. Thus if we plot the profit function, $\pi(n)$, with respect to n, then the profit line shifts up as λ increases. It is concievable that at $\lambda = 0$, the parametric conditions are such that the profit line lies entirely in the 4th quadrant (below the horizontal axis), i.e.,

$$\text{at } \lambda = 0,\ \pi(0) < \pi(1) < 0. \tag{19}$$

Without any loss of generality, we start with this scenario as our benchmark. (All other possibilities will be captured with the subsequent values of λ). With higher values of λ, the profit line shifts upward. Let us define a $\underline{\lambda}$ such that

$$\text{at } \lambda = \underline{\lambda},\ \pi(0) < 0 \text{ and } \pi(1) = 0. \tag{20}$$

Then for all higher λ values, at least one part of the profit line must lie above the horizontal axis. Finally let us define a $\bar{\lambda}$ such that

$$\text{at } \lambda = \bar{\lambda},\ \pi(0) = 0 \text{ and } \pi(1) > 0. \tag{21}$$

We can now examine the profit lines for various values of λ and analyse the economic implications corresponding to each such case. (See Figure 4).

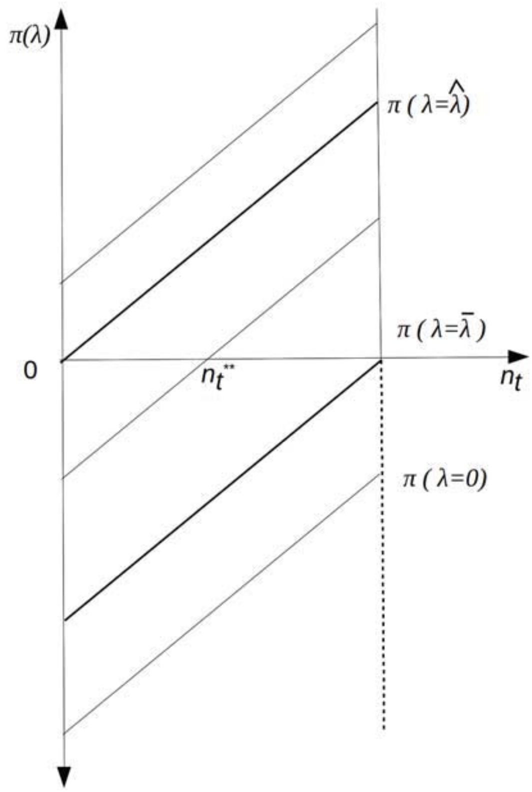

Figure 4:

1. Case 1: $\lambda \in [0, \underline{\lambda}]$

In this case the skilled to unskilled labour ratio is too low and therefore the productivity of modern technology as well as corresponding aggregate demand is too low to make adoption of modern technology viable. The composition of labourforce is clearly unfavourable to adoption of modern technology. This is the case of pure low level technology trap where the equilibrium $n^* = 0$.

2. Case 2: $\lambda \in [\underline{\lambda}, \bar{\lambda},]$

   In this case, adoption of modern technolgy becomes profitable if and only if sufficient number of firms decide to adopt it simultaneously. This complementarity of profits across modern firms creates the possibility of multiple equilibria. However multiple equilibria will not be realized if the skilled labour supply constarint becomes binding before profit level becomes positive. This generates the follwong two subcases.

   (a) Subcase 2a: $\hat{n} > \bar{n}$ implying $\pi(\bar{n}) < 0$.

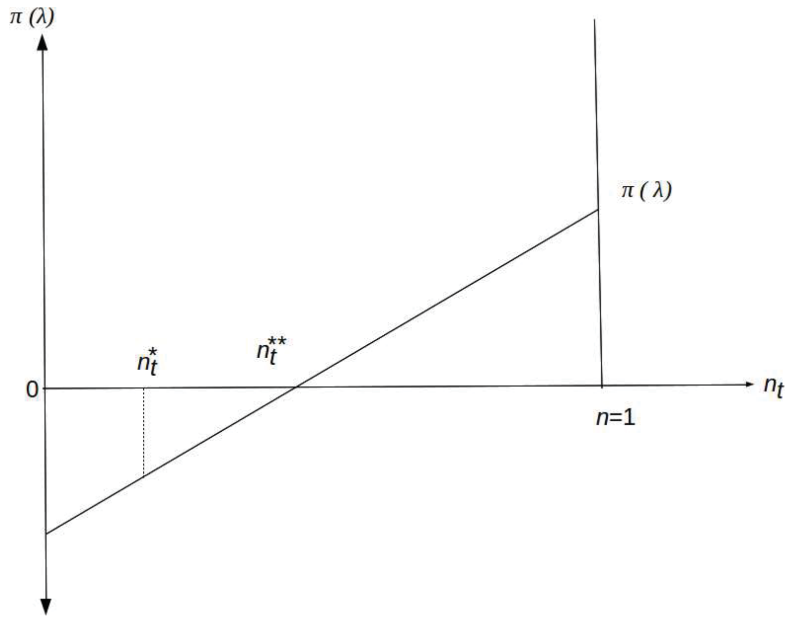

Figure 5:

This case is shown in Figure 5. For any such value of λ lying between $\underline{\lambda}$ and $\bar{\lambda}$, potentially there could be multiple equilibria (as represented by $n = 0$ and $n = \hat{n}$). But this multiple equilibria scenario is not realized because the labour supply constraint becomes binding before $\hat{n}$ is attained. In other words, while there exist incentives to adopt modern technology purely from the profit concerns (provided at least $\hat{n}$ firms also do the same), the binding labour supply constraint prevents the economy to achieve this. Hence the only equilibrium that can be actually attained is the one characterized by cottage production. This is a scenario where even if a firm wishes, it cannot adopt modern technology due to shortage of skilled labour. Possibility of multiple equilibria is therefore only theoretical. The equilibrium n in this case is again represented by $n^* = 0$.

(b) Subcase 2b: $\hat{n} < \bar{n}$ implying $\pi(\bar{n}) > 0$.

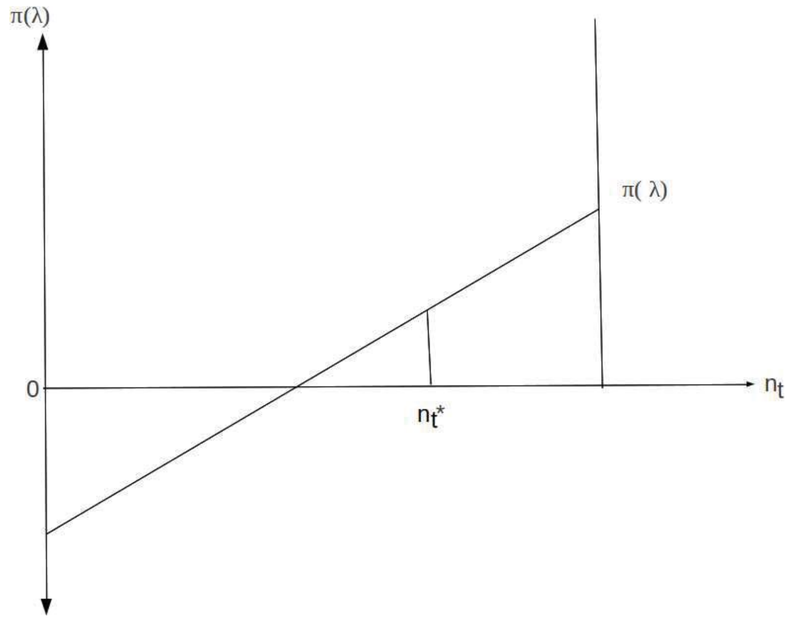

Figure 6:

This case is shown in Figure 6. In this case, at $\bar{n}$ (where the skilled labour constraint becomes binding) the firms are already earning positive profits. In fact there exists profit incentives to go even beyond $\bar{n}$, but shortage of skilled labour does not allow the firms to modernize beyond this point. However, this is the case where possibility of multiple equilibria becomes real. If the business sentiment in the economy is optimistic and expectations of earning positive profits dominate, then all firms that could adopt the modern technology will do so. Hence the equilibrium value of n would be given by $n^* = \bar{n}$. On the other hand, if the production sector is plagued by pessimistic business sentiment, then no firm would be willing to adopt the modern technology, in which case the equilibrium value of n would be given by $n^* = 0$.

3. Case 3: $\lambda \in [\bar{\lambda}, 1]$

   As dictated by the profit considerations, all firms now have incentives to adopt the modern technology i.e. the economy would want to move completely to the modern technology frontier. In such a scenario, the skilled labour constraint becomes the only binding factor. Thus, as long as λ is less than unity, shortage of skilled labour forces the economy to stay at an equilibrium $n^* = \bar{n} < 1$, with only partial industrialization. Once everybody in the economy becomes skilled, (i.e., $\lambda = 1$), the economy attains complete industrilization with every firm adopting the modern technology (i.e., $n^* = \bar{n} = 1$). Notice that business sentimenst and producers' expections cease to play any role in the process of technology adoption here.

   Proposition 1 below summarise all these results.

Proposition 1. *There exist two threshold values of the initial proportion of rich population in the economy, given by $\hat{\lambda}$ and $\bar{\lambda}$ respectively, such that:*

1. *If $\gamma_0 \in [0, \hat{\lambda}]$, the economy gets saddled in a pure low level cottage technology trap, with equilibrium $n^* = 0$.*

2. *If $\gamma_0 \in [\hat{\lambda}, \bar{\lambda}]$, expectation plays a crucial role:*

   a *with pessimistic business expectations, no firm adopts modern technology and at equilibrium $n^* = 0$;*

   b *When optimistic business expectations prevail, as many firms as is permitted by the skilled labour constraint adopt modern technology, and at equilibrium $n^* = \bar{n}$.*

3. *If $\gamma_0 \in [\bar{\lambda}, 1]$, expectation ceases to play any role in determination of equilibrium n. At equilibrium, $n^* = \bar{n}$ and if the economy does not attain complete modernization, it is only because of shortage of skilled labour.*

5 Long Run Dynamics

In our analysis so far we have taken the skilled-unskilled ratio (λ) as given. The skilled-unskilled ratio is of course endogenously determined. Agents decide to acquire or not to acquire skills (subject to their wealth constraints) conditional on the expected returns (in the form of higher wages) and the relative costs. Accordingly, the skilled-unskilled ratio, λ, evolves over time, depending on the skill acquisition decisions of agents. In this section, we analyse the long run dynamics of λ and the concommitant dynamics of technology adoption. We show that the initial distribution of wealth plays a critical role in determining the long run evolution of λ.

Recall that γ_0 represents the initial proportion of rich agents in the distribution (i.e, those who can afford to incur the investment cost (h) of skill formation from inherited wealth). If all the rich people always opted for skill formation then γ_t would be identically equal to λ_t in every period.However, as we argue below, under certain macroeconmic scenarios, the rich agents will not opt for skill formation (even though they can afford to) and therefore γ_t and $\lambda_t$will diverge, with λ_t being less than γ_t.

Let us start with any given γ_0. To begin the discussion, let us first assume that at this γ_0, all the rich agents opt for skill formation (which may or may not be the actual case). As we have already seen in Proposition 1, at any point of time, the decision of firms to adopt modern technology depends crucially on the value of λ_t.In particular, it depends on the relative position of λ vis-a-vis the two threshold levels, $\underline{\lambda}$ and $\bar{\lambda}$. Accordingly, we can consider following three cases:

5.1 Case A: Either (i) $\gamma_0 < \underline{\lambda}$; or (ii) $\gamma_0 \in [\underline{\lambda}, \bar{\lambda}]$ and $\hat{n}(\gamma_0) > \bar{n}(\gamma_0)$

The economy now begins with an initial distribution of income such that even when all the rich agents opt for skill formation, the corresponding skilled-unskilled ratio either lies below $\underline{\lambda}$;or it lies between $\underline{\lambda}$ and $\hat{\lambda}$ but profit for modern firms is still negative. (These correspond respectively to cases 1 and $2a$ in section 4 above). Clearly then modern firms will not operate. If modern firms do not operate, then there is no incentive for the rich agnets to opt for skill formation either. Hence nobody will opt for skill formation. Thus, even though we started with the assumption that the rich opts for skill formation, in equilibrium no forward-looking agent will opt for skill formation. Thus in this case, the initial skilled-unskilled ratio will be given by:

$$\lambda_0 = 0.$$

Thus in this self-fulfilling macroeconomic (temporary) equilibrium, no modern firms operates; nobody acquires skills and the economy operates only with the low level cottage technology.

A question that immediately follows is: is this situation temporary or permanent? In other words, starting from this low γ_0, can the economy climb out the low technology equilibrium over time? How does the proportion of rich people evolve over time, and what impact it would have on corresponding skilled-unskilled ratio? To answer these questions, we have to analyse the bequest dynamics for these two subcases.

As discussed in section 4 above, both these subcases are characterized by no modern firms in the equilibrium, such that:

$$n_t^* = 0,$$

and

$$\pi_t = 0.$$

Also, since modern firms are not operating, nobody earns the modern sector wages; all agents earns the same wage rate, which is the unskilled wage $w_N = 1$.

Now let us consider the end-of-the-period wealth of various categories of households. In the absence of profit income as well as skilled wages in this economy, end-of- the-period wealth of an agent depends on the bequest received and the unskilled wage rate. Thus, end-of- the-period wealth of poor and rich agents are given respectively by:

$$\hat{y}_t^P = 1 + x_t^P;$$

$$\hat{y}_t^R = 1 + x_t^R$$

Accordingly, the bequests left by individuals for their progeny (which constitute the inherited wealth of agents in period $t+1$) can be written as:

$$x_{t+1}^P = (1-\beta)[1 + x_t^P];\ x_0^P < h \tag{22}$$

$$x_{t+1}^R = (1-\beta)[1 + x_t^R];\ x_0^R > h \tag{23}$$

Notice that the dynamic equations representing the bequest dynamics of the 'poor' and the 'rich' are identical. Hence we can calculate the steady state value for bequest left by the 'poor' as well the 'rich', as given below:

$$x^{P*} = x^{R*} = \frac{(1-\beta)}{1-(1-\beta)} \equiv x^*. \tag{24}$$

We now make the following assumption:

$$0 < \frac{(1-\beta)}{1-(1-\beta)} < h. \qquad \text{(Assumption 3)}$$

Assumption 3 implies that in the absence of profit income and/or skilled wage rate, inherited wealth (no matter how large) and unskilled wages together are **not** sufficient to ensure a steady state wealth level that allows one to acquire skill. This implies that children of the initial 'poor' dynasties will be never be able to opt for skill formation. On the other hand, bequest left by the initial 'rich' dynasties, although greater than h at the beginning,will evetually fall below h and will approach a value $x^* < h$ in the long run. This dynamics is represented by Figure 7 below.

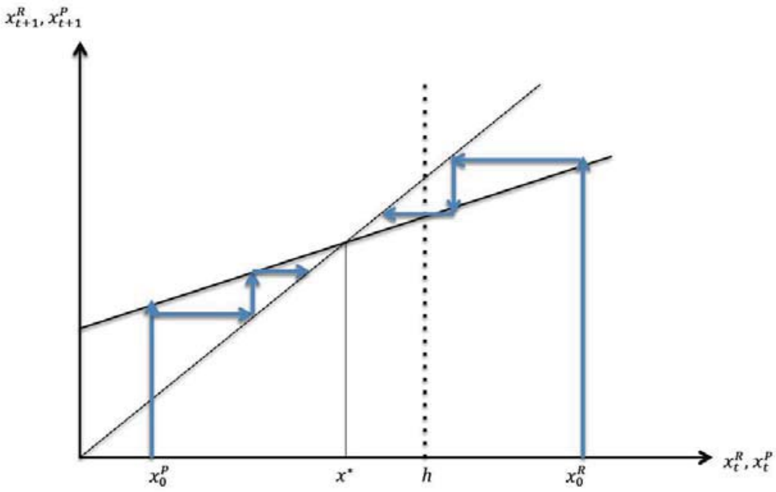

Figure 7: Bequest Dynamics

The bequest dynamics illustrated above allows us to trace the dynamics of γ_t and the concommitant dynamics of λ_t for this case. Notice that the rich dynasties start with an initial wealth level (x_0^R) greater than h; moreover, the wealth level of these dynasties remains above h until a finite time period, say $t = \hat{t}$. Thereafter their inherited wealth level crosses below h. Since we have used the resource cost for education, h, as the threshold for classifying the population into the wealth categories 'rich' and 'poor', by thie definition then, after time period $\hat{t}$, the entire population become 'poor'[6]. This generates the following evolution path for γ_t :

$$\gamma_t = \begin{cases} \gamma_0 \text{ for all } t \text{ such that } 0 \leqq t \leqq \hat{t}; \\ 0 \text{ for all } t > \hat{t}. \end{cases}$$

Clearly, since the γ_t values remains constant either at γ_0 or at zero, the corresponding λ value remains at zero forever. That is,

$$\lambda_t = 0 \text{ for all } t \geqq 0.$$

[6]Although among the 'poor' category now there are two types of agents - those whose ancestors were 'rich' and those whose ancestors were 'poor'. The first category of agents remains relatively more wealthy than the second category in the the short run.

Thus indeed the economy is perpetually stuck in a low skill - primitive technology trap; characterized by extremely low factor productivity.

5.2 Case B: $\gamma_0 \in [\underline{\lambda}, \bar{\lambda}]$ and $\hat{n}(\gamma_0) \leqq \bar{n}(\gamma_0)$

In this case (corresponding to section 4; case 2(b) above) the economy begins with an initial distribution of income such that if all the rich agents opt for skill formation then at the corrsponding skill-unskilled ratio, profits for a modern firm is positive - provided at least $\hat{n}(\gamma_0)$ other modern firms are operating. Thus in a buoyant business environment where all modern firms are optimistic about participation by others, all of them would like to operate. In this case the degree of modernizaton will be limited only by the availability of skilled labour, and in equilibrium $\bar{n}(\gamma_0)$ firms will operate with modern technology. Since at $\bar{n}(\gamma_0)$ the entire skilled labour force is fully employed, all the rich agents who opted for skill formation would find employment at the skilled wage rate w_M.Therefore the forwarding looking agents would indeed opt for skill formation. The corresponding the initial skilled-unskilled ratio will be given by:

$$\lambda_0 = \gamma_0.$$

Thus in this self-fulfilling macroeconomic (temporary) equilibrium, $\bar{n}(\gamma_0)$ number of modern firms operate; all rich agents acquire skills and the economy operates with a mix of modern and cottage technology; full modernization is constrained only by the shortage of skilled labour.

This however is not the only possible scenario. In a pessimistic business environment where the modern firms are sceptical about others' participation, none would like to operate. In this case again no modern firm will operate; hence there is no benefit from acquiring skills. Hence none of the rich agents will opt for skill formation and we would be back to a secnario similar to Case A, with the corresponding skilled-unskilled ratio given by:

$$\lambda_0 = 0.$$

This then represents another self-fulfilling macroeconomic (temporary) equilibrium where no modern firms operates; nobody acquires skills and the economy operates only with the low level cottage technology.

Case B captures the classic multiple equlibria scenario where expectations rule the roost. Notice however that this mutiple equilibria scenario may not persist in the long run. When the agents' expectations are pessimistic and the economy hits the lower bound of $\lambda_0 = 0$, it stays there forever. Once λ takes zero value the economy moves into the realm of Case A, as discussed above. It is then perpetually caught in a low technology long run trap.

What happens in the long run if agents remain optimistic? To analyse this, we once again have to examine the coresponding bequest dynamics. Notice that when agents are optimistic then $\lambda_0 = \gamma_0$. Thus $\bar{n}(\gamma_0)$ firms will operate with modern technology and all the rich agents earn the skilled wage rate $w_M = 1 + v$. Moreover each agent (rich or poor) also earn a positive profit now, given by:

$$\pi_{\bar{n}(\gamma_0)} = \frac{\bar{L}\left[\alpha(\gamma_0) - (1+v)\right] - \hat{L}\left[\bar{n}(\gamma_0)\alpha(\gamma_0) + (1-\bar{n}(\gamma_0))(1+v)\right]}{(\left[1-\bar{n}(\gamma_0)\right]\alpha(\gamma_0) + \bar{n}(\gamma_0)} \tag{25}$$

The corresponding income levels of each categories of agents are then given as follows.

$$y_0^P = 1 + x_0^P + \frac{\bar{n}(\gamma_0).\pi_{\bar{n}(\gamma_0)}}{\bar{L}}$$

$$y_0^R = 1 + v + \left(x_0^R - h\right) + \frac{\bar{n}(\gamma_0).\pi_{\bar{n}(\gamma_0)}}{\bar{L}}$$

Accordingly, the bequest dynamics for each of these categries will be represented by the following set of equations:

$$x_{t+1}^P = (1-\beta)\left[1 + x_t^P + \frac{\bar{n}(\gamma_t).\pi_{\bar{n}(\gamma_t)}}{\bar{L}}\right];\ x_0^P < h \tag{26}$$

$$x_{t+1}^R = (1-\beta)\left[1 + (v-h) + x_t^R + \frac{\bar{n}(\gamma_t).\pi_{\bar{n}(\gamma_t)}}{\bar{L}}\right];\ x_0^R > h \tag{27}$$

Notice that the bequest dynamics of the poor agents in this case differs from Case A by the per capital profit term $\tilde{\pi}_t(\bar{n}(\gamma_t)) \equiv \frac{\bar{n}(\gamma_t).\pi_{\bar{n}(\gamma_t)}}{\bar{L}}$. On the other hand the bequest dynamics of the rich agents in this case differs from Case A by the per capita profit term $\tilde{\pi}_t$ as well as the additional net (skilled) wage income given by $(v-h)$. Thus whether the progenies of the two categories of agents can acquire skill in the long run or not depends crucially on the magnitudes of these two terms. Note that all the bequest lines above have the same slope; they differ only in terms of the intercept term. If the profit level remains constant at some $\tilde{\pi}(\bar{n}(\gamma_t))$, then the corresponding steady state values for each of these dynamic equations will be as follows:

$$\begin{aligned} x^{P*} &= \frac{(1-\beta)\left[1 + \tilde{\pi}(\bar{n}(\gamma_t))\right]}{1-(1-\beta)} \\ x^{R*} &= \frac{(1-\beta)\left[1 + v - h + \tilde{\pi}_t(\bar{n}(\gamma_t))\right]}{1-(1-\beta)} \end{aligned}$$

The phase diagrams for these two cases are given in Figures 8 and 9 respectively. Once again, we make the following two assumptions:

$$(1-\beta)(1+v) > h; \qquad \text{(Assumption 4)}$$

$$\exists\ \hat{\lambda}\epsilon[\underline{\lambda}, \bar{\lambda}] : \frac{(1-\beta)\left[1+\tilde{\pi}(\bar{n}(\hat{\lambda}))\right]}{1-(1-\beta)} = h. \qquad \text{(Assumption 5)}$$

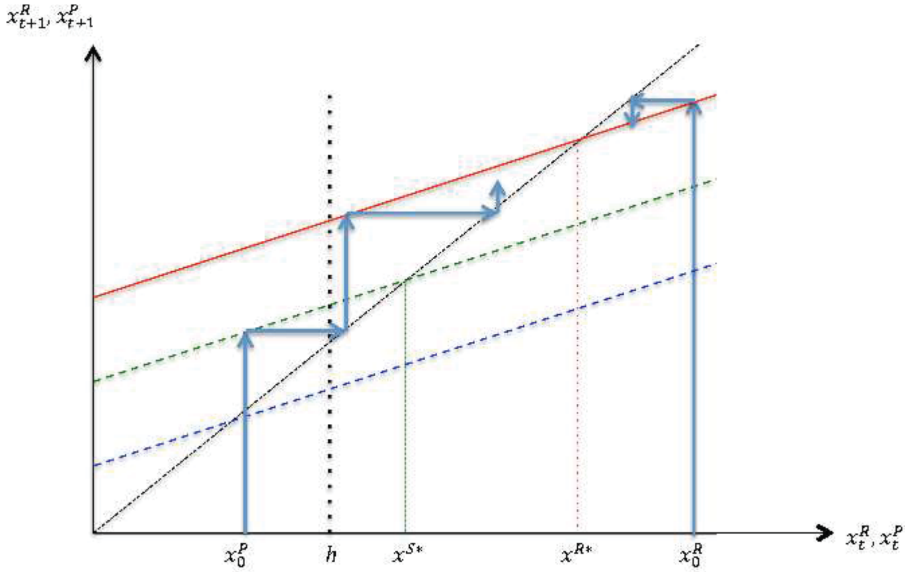

Figure 8:

Assumption 4 implies that the skilled wage rate is large enough such that bequest left from skilled wages alone are sufficient to allow one to acquire skill in the long run. Assumption 6 ensures that for a sufficiently large skiled-unskilled ratio $\hat{\lambda}$, the productivity in modern technology is high enough, such that bequests left from the corresponding profit is large enough to allow one to acquire skill in the long run.

Assumption 5 implies that in Case B, under optimistic business expectations, children of all rich agents will have enough bequests to invest in skill formation and therefore will acquire skills. Thus the correspondind skilled-unskilled ratio will remain at least as high as γ_0. Moreover, if the initial proportion of rich population is high enough and therefore the initial skill-unskilled ratio is high enough, (i.e., if $\gamma_0 = \lambda_0 > \hat{\lambda}$) then even the children of the poor entreprenuers will eventually become rich (by Assumption 6). At that point, under optimistic business environment, the entire population will start investing in skill formation, whereby the proportion of rich and

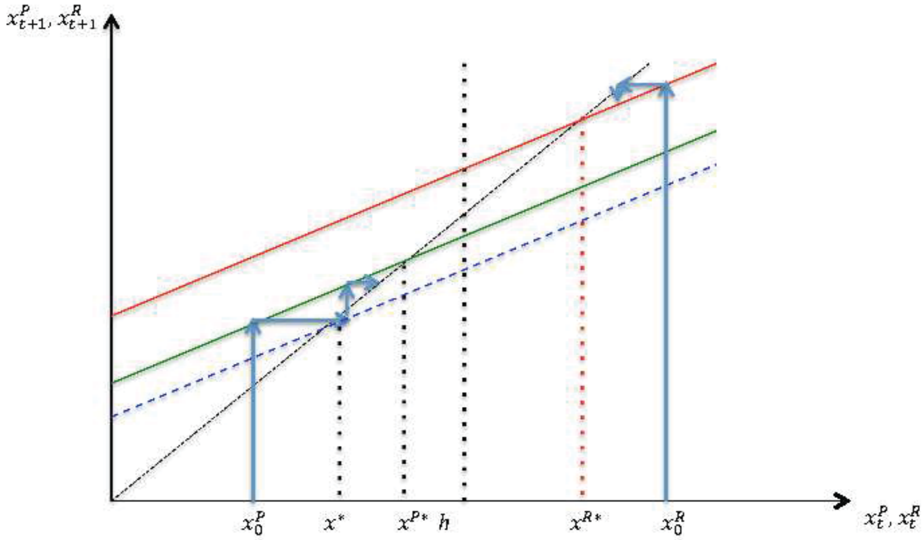

Figure 9:

the corresponding skilled-unskilled ratio will eventually reach unity.Let $\hat{t}'(\gamma)$ denote the time when the wealth level of dynasties which were initially 'poor' crosses over above h.[7] Accordingly, when business environment is optimistic, the dynamics of γ_t and the concommitant dynamics of λ_t for Case B will be as shown below:

$$
\begin{aligned}
\text{When } \gamma_0 \,\epsilon [\underline{\lambda}, \hat{\lambda}],\ \gamma_t &= \lambda_t = \gamma_0 \text{ all } t \ ; \\
\text{When } \gamma_0 \,\epsilon [\hat{\lambda}, \bar{\lambda}],\ \gamma_t &= \lambda_t = \begin{cases} \gamma_0 \text{ for all } t \text{ such that } 0 \leqq t \leqq \hat{t}'(\gamma) \\ \quad 1 \text{ for all } t > \hat{t}'(\gamma). \end{cases}
\end{aligned}
$$

Notice however that for each such non-zero λ value under optimistic expectations, there exists a pessimistic equilibrium where λ goes to zero. Thus an expectation-driven long run low technology trap cannot be ruled out at any point of time.

5.3 Case C: $\gamma_0 \in [\bar{\lambda}, 1]$

This scenario (corresponding to case 3 in section 4 above) depicts an economy that begins with a favourable initial distribution of income such that if all

[7]Notice the finite time period when the 'poor entreprenuer dynasties' would cross over the threshold wealth level h depends on the initial distribution (γ). This is because in the corresponding phase diagram, the length of the intercept with the y-axis varies with different values of γ.

the rich people opt for skill formation that the productivity parameter in the modern technology is high enough to yield positive profit at all levels - even when no other firms opt for the modern technology. Thus each modern firm would like to opearte - irrespective of what other firms are doing. Thus the coordination failure among modern firms, generating multiple equilibria in Case B, is no longer applicable. This also implies that the rich agents will always have incentives for skill formation. Morever, since in this case the γ_0 value is already greater than $\hat{\lambda}$, even the poor agents will eventually become rich. The corresponding dynamics of γ_t and the concommitant dynamics of λ_t here is given as follows:

$$\gamma_t = \lambda_t = \begin{cases} \gamma_0 \text{ for all } t \text{ such that } 0 \leqq t \leqq \hat{t}'(\gamma) \\ 1 \text{ for all } t > \hat{t}'(\gamma). \end{cases} \tag{28}$$

Thus the economy will be fully modernized in the long run and all agents will be skilled. The corresponding factor productivity will be extremely high.

The findings of this section are summarized in Proposition 2 below.

Proposition 2. *The initial distribution of income, as represented by the initial proporion of rich agents in the population, γ_0, plays a crucial role in simultaneous determination of the decisions to go for skill formation and the decisions to adopt modern technology such that:*

1. *If γ_0 is too low, then neither modern firms have incentives adopt modern technology, nor any of the agnets have incentives to opt for skill formation. Consequently, the economy gets caught perpetually in a low technology trap;*

2. *Under a moderate γ_0, the economy exhibits multiple equilibria: with optimistic business expectations, the economy attains partial modernization; with pessimistic business expectations, no firm in the economy adopts modern technology and the economy is back to the low technology trap;*

3. *If γ_0 is sufficiently high then all firms adopt the modern technology and there is complete modernization.*

6 Conclusion

In this paper we have shown how the initial distribution of wealth impinges upon the realized skilled unskilled labour ratio in an economy, which in turn has important implications for the technology adoption decision by firm.

However skilled to unskilled ratio is not the only factor that determines the technology adoption trajectory. Under certain conditions, the economy migh exhibit mutiple equilibria, where expectations play a crucial role.

We also show that there is feed back from skill formation to technology adoption and vice versa, such that the initial inequality impinges on the technology adoption decisions even in the long run. An economy may get stuck at a self-perpetuating low technology equilibrium: because the skilled to unskilled ratio ratio is too low it is not profitable for the firms to adopt the modern technology; and at the same time, because nobody adopts the modern technology there is no demand for skilled labour (i.e., skill formation pays no extra return). as a result forward-looking agents do not opt for skill formationand the skilled to unskilled ratio also remains low (in fact zero). Thus there is self-fulfilling expectations that perpetuates the bad equilibrium even in the long run. .

References

Acemoglu, D. & Zilibotti, F. (2001), 'Productivity differences', *The Quarterly Journal of Economics* **116**(2), 563–606.

Atkinson, A. B. & Stiglitz, J. E. (1969), 'A new view of technological change', *Economic Journal* **79**(315), 573–78.

Basu, S. & Weil, D. N. (1998), 'Appropriate technology and growth', *The Quarterly Journal of Economics* **113**(4), 1025–1054.

Benhabib, J. & Spiegel, M. M. (2005), Chapter 13 human capital and technology diffusion, Vol. 1, Part A of *Handbook of Economic Growth*, Elsevier, pp. 935 – 966.

Galor, O. & Zeira, J. (1993), 'Income distribution and macroeconomics', *Review of Economic Studies* **60**(1), 35–52.

Hall, R. E. & Jones, C. I. (1999), 'Why do some countries produce so much more output per worker than others?*', *The Quarterly Journal of Economics* **114**(1), 83.

Jerzmanowski, M. (2007), 'Total factor productivity differences: Appropriate technology vs. efficiency', *European Economic Review* **51**(8), 2080–2110.

Klenow, P. J. & Rodríguez-Clare, A. (1997), 'Economic growth: A review essay', *Journal of Monetary Economics* **40**(3), 597 – 617.

Murphy, K. M., Shleifer, A. & Vishny, R. W. (1989), 'Industrialization and the big push', *Journal of Political Economy* **97**(5), 1003–26.

Nelson, R. R. & Phelps, E. S. (1966), 'Investment in humans, technological diffusion, and economic growth', *The American Economic Review* **56**(1/2), 69–75.

Parente, S. L. & Prescott, E. C. (1994), 'Barriers to technology adoption and development', *Journal of Political Economy* **102**(2), 298–321.