Patent licensing in the presence of a differentiated good

Jiyun Cao
The School of Economics, Nankai University
& Collaborative Innovation Center for China Economy,
Tianjin, China

Uday Bhanu Sinha
Email: uday@econdse.org
Department of Economics
Delhi School of Economics

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Jiyun Cao*

The School of Economics, Nankai University and Collaborative Innovation Center for China Economy, Tianjin, China

and

Uday Bhanu Sinha

Department of Economics, Delhi School of Economics, New Delhi, India

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Abstract: The existing literature has considered licensing of a patented innovation either in a homogenous good industry or in a differentiated goods industry. We consider the licensing problem between two firms i.e., licensor and licensee producing the homogenous goods when there is a third firm producing a differentiated good in the market. We find that when the costs of non-innovators are not high, the optimal licensing contract depends on the degree of product differentiation and the innovator has more incentive for innovation when it is an insider than when it is an outsider of this market.

Keywords: licensing, two-part tariff, Cournot oligopoly, homogenous and differentiated goods, incentive for innovation.

JEL Classifications: D43, D45, L13

Correspondence to: : Uday Bhanu Sinha, Department of Economics, Delhi School of Economics, University of Delhi, Delhi 110009, INDIA. Fax: (91-11) 27667159; Email: uday@econdse.org.

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1. Introduction

The literature on patent licensing under oligopolistic competition assumes the characteristics of the goods produced by different firms as either homogenous (Wang (1998), Kamien and Tauman (2002), Sen and Tauman (2007), Poddar and Sinha (2010)); or differentiated (Muto (1993), Fauli-Oller and Sandonis (2002), Mukherjee and Balasubramanian (2001)).

As far as we know there is no paper which brings the market competition between homogenous and differentiated goods together and analyses the problem of licensing in such market. To capture this idea in the simplest possible way we consider three firms competing under Cournot in a market where two firms produce homogenous goods and the third firm produces a differentiated good. Then we focus on the problem of technology transfer if one of the homogenous producers is an innovator having a cost reducing process innovation that can be licensed only to the other homogenous good producer. Thus, we consider the optimal licensing contract between two homogenous good producers in the presence of a third firm producing a differentiated good.

In this framework we provide a rational for the existence of variety of licensing contracts with the help of a single model.¹ We extend Wang (1998) by introducing a differentiated product and analyse how the degree of product differentiation interacts with the licensing contract between the two firms producing homogenous goods. We show if the costs of non-innovators are not high, the optimal licensing contract involves only royalty when the degree of differentiation is high and two-part tariff when it is low and finally a fixed fee if all goods are homogenous in the limit. Then we also compare the incentive for innovation for the innovator as an insider (a competitor) to that of being an outsider (not a competitor).

Sections 2 and 3 consider respectively the situations of the innovator as an insider

and as an outsider. Section 4 looks into the incentives for innovation. Section 5 concludes.

2. The Inside Innovator

Consider three firms in an industry with the same marginal production cost, say $c$. Firms 0 and 1 produce the homogenous goods while firm 2 produces a differentiated good. The inverse demand functions of two differentiated goods are $P_1 = 1 - q_0 - q_1 - \gamma q_2$ and $P_2 = 1 - q_2 - \gamma(q_0 + q_1)$, where $q_i$ is the output of firm $i$ ($i = 0, 1, 2$), $P_1$ and $P_2$ are the prices of two differentiated goods, $\gamma$ ($0 < \gamma \leq 1$) measures the degree of product differentiation.

Now, suppose firm 0 attains a cost reducing innovation which can reduce the marginal cost by $\lambda c$, where $\lambda$ ($0 < \lambda \leq 1$) is a constant. Firm 0 can license its innovation to only firm 1. This is because the innovation is not compatible for adoption in firm 2’s production process as firm 2 produces differentiated goods by using a different production technology.\footnote{The other reason could be that firm 2 resides in another country with weak patent protection, or due to a high international licensing cost.}

The game is as follows. At stage 1, firm 0 offers firm 1 a take-it-or-leave-it two-part tariff licensing contract stipulating an upfront fixed fee ($F$) plus a per-unit royalty ($r$). Firm 1 accepts the offer if its payoff is not less than its reservation profit. At stage 2, firms compete like Cournot and profits are realized.

2.1 No Licensing

If there is no agreement at stage 1, the profits of three firms are $\pi_0 = [1 - q_0 - q_1 - \gamma q_2 - (1 - \lambda)c]q_0$, $\pi_1 = (1 - q_0 - q_1 - \gamma q_2 - c)q_1$ and $\pi_2 = [1 - q_2 - \gamma(q_0 + q_1) - c]q_2$ respectively. The first order conditions give the equilibrium outputs of three firms as $q_0^{nl} = \frac{(2 - \gamma)(1 - c) + (4 - \gamma^2)\lambda c}{2(3 - \gamma^2)}$, $q_1^{nl} = \frac{(2 - \gamma)(1 - c) - (2 - \gamma^2)\lambda c}{2(3 - \gamma^2)}$ and $q_2^{nl} = \frac{(3 - 2\gamma)(1 - c) - \gamma \lambda c}{2(3 - \gamma^2)}$ respectively. $q_i^{nl} > 0$ ($i = 0, 1, 2$) requires $c < \bar{c}$, where $\bar{c} = \frac{2 - \gamma}{2(1 + \lambda) - \gamma(1 + \gamma \lambda)}$.\footnoteref{2}
For $c \geq \bar{c}$, firm 1 stays out of market, hence $q_{1}^{n,l} = 0$. Thus, the equilibrium outputs of firms 0 and 2 are $q_{0}^{n,l} = \frac{(2-\gamma)(1-c)+2\lambda c}{4-\gamma^2}$ and $q_{2}^{n,l} = \frac{(2-\gamma)(1-c)-\gamma c}{4-\gamma^2}$ respectively. $q_{2}^{n,l} > 0$ requires $c < \bar{c}$, where $\bar{c} = \frac{2-\gamma}{2-\gamma+\gamma^2}$.

Therefore, the equilibrium profits of firms 0 and 1 under no licensing are as follows. $\pi_{0}^{n,l} = \left\{ \begin{array}{ll} \frac{(2-\gamma)(1-c)+(4-\gamma^2)\lambda c}{4(3-\gamma^2)^2} & \text{for } 0 < c < \bar{c} \\ \frac{(2-\gamma)(1-c)}{(4-\gamma^2)^2} & \text{for } \bar{c} \leq c < \bar{c} \end{array} \right.$

$\pi_{1}^{n,l} = \left\{ \begin{array}{ll} \frac{(2-\gamma)(1-c)-(2-\gamma^2)\lambda c}{4(3-\gamma^2)^2} & \text{for } 0 < c < \bar{c} \\ 0 & \text{for } \bar{c} \leq c < \bar{c} \end{array} \right.$

2.2 Licensing

If firms 0 and 1 reach a contract at stage 1, the respective profits of three firms are $\pi_{0} = [1 - q_{0} - q_{1} - \gamma q_{2} - (1 - \lambda)c]q_{0} + r q_{1} + F$, $\pi_{1} = [1 - q_{0} - q_{1} - \gamma q_{2} - (1 - \lambda)c - r]q_{1} - F$ and $\pi_{2} = [1 - q_{2} - \gamma(q_{0} + q_{1}) - c]q_{2}$. Similarly, we get the respective outputs of three firms as $q_{0} = \frac{(2-\gamma)(1-c)+2\lambda c+(2-\gamma^2)r}{2(3-\gamma^2)}$, $q_{1} = \frac{(3-2\gamma)(1-c)-2\gamma c+\gamma r}{2(3-\gamma^2)}$, $q_{2} = \frac{(2-\gamma)(1-c)+2\lambda c+(2-\gamma^2)r}{2(3-\gamma^2)}$

Under a take-it-or-leave-it licensing contract, the equilibrium fixed fee is $F = (q_{1})^2 - \pi_{1}^{n,l}$, yielding $\pi_{1}^{l} = \pi_{1}^{n,l}$. Thus, firm 0 determines $r$ to maximise its profit $\pi_{0} = (q_{0})^2 + r q_{1} + (q_{1})^2 - \pi_{1}^{n,l}$. As $\frac{\partial \pi_{0}}{\partial r} = \frac{(1-\gamma^2)[(2-\gamma)(1-c)+2\lambda c]-2(2-\gamma^2)r}{2(3-\gamma^2)^2}$, the optimal royalty under the constraint of $r \leq \lambda c$ is

$r^{l} = \left\{ \begin{array}{ll} \bar{r} & \text{for } c > c_{0} \\ \lambda c & \text{for } c \leq c_{0} \end{array} \right.$, where $\bar{r} = \frac{(1-\gamma^2)((2-\gamma)(1-c)+2\lambda c)}{2(2-\gamma^2)^2}$, $c_{0} = \frac{2-\gamma-2\gamma^2+\gamma^3}{2-\gamma-2\gamma^2+\gamma^3+2\lambda} \leq \bar{c}$.

For $c \leq c_{0}$, $r^{l} = \lambda c$ implies that the licensing doesn't affect firm 1’s marginal cost, hence outputs of all firms, making all firms compete under licensing as they do under no licensing. For $c > c_{0}$, given $r^{l} = \bar{r}$, the equilibrium respective outputs of

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3 In this paper, we do not consider the case of $c \geq \bar{c}$ for which firm 2 producing differentiated goods will also stay out of market, making firm 0 a monopolist.

4 In order to save space, we do not show the profit of firm 2 throughout the paper.
three firms are \( q_0^{IJ} = \frac{(2 - \gamma)(1 - c) + 2 \lambda c}{4} \), \( q_1^{IJ} = \frac{\gamma^2[(2 - \gamma)(1 - c) + 2 \lambda c]}{4(2 - \gamma^2)} \) and \( q_2^{IJ} = \frac{(4 - 2 \gamma - \gamma^2)(1 - c) - 2 \gamma \lambda c}{4(2 - \gamma^2)} \), all positive for \( c < \hat{c} \). Thus, the equilibrium profit of firm 0 is

\[
\pi_0^{IJ} = \begin{cases} 
\frac{(2 - \gamma)^2(1 - c)^2 + 2(2 - \gamma)(7 - 2\gamma)(1 - c) \lambda c + (4 + 2\gamma^2 - \gamma^4) \lambda^2 c^2}{4(3 - \gamma^2)^2}, & \text{for } 0 < c \leq c_0 \\
\frac{(2 - \gamma)^2(5 - 4\gamma + 4\gamma^4)(1 - c)^2 + 4(2 - \gamma)(13 - 10\gamma^2 + 2\gamma^4)(1 - c) \lambda c + 2(10 - 4\gamma^4 + \gamma^6) \lambda^2 c^2}{8(2 - \gamma^2)(3 - \gamma^2)^2}, & \text{for } c_0 < c < \hat{c}.
\end{cases}
\]

For \( c < \hat{c} \), the licensing is always profitable as firm 0 can get better even if it offers a royalty alone licensing contract to firm 1. For \( \bar{c} \leq c < \hat{c} \), we also have \( \pi_0^{IJ} > \pi_0^{nJ} \) indicating that firm 0 has incentive to license to firm 1. This would certainly have adverse effect on firm 0’s payoff from market by enhancing competition. However, this adverse effect is shared by firm 2, making it overweighed by licensing revenue.

Now, let us look the impact of the degree of product differentiation on the optimal licensing contract. Since \( \frac{\partial c_0}{\partial \gamma} = -\frac{2(1 + 4\gamma - 3\gamma^2)\lambda}{(2 - \gamma - 2\gamma^2 + \gamma^4 + 2\lambda)^2} < 0 \) and \( \frac{\partial \hat{c}}{\partial \gamma} = -\frac{2\lambda}{(2 - \gamma + \gamma\lambda)^2} < 0 \), both \( c_0 \) and \( \hat{c} \) are decreasing with \( \gamma \). Combined with \( c_0|_{\gamma=0} = \hat{c}|_{\gamma=1} = \frac{1}{1 + \lambda} \), we plot \( c_0 \) and \( \hat{c} \) in figure 1. The region determined by \( 0 < c < \hat{c} \) and \( 0 \leq \gamma \leq 1 \) in figure 1 is divided into two areas by \( c_0 \). The optimal licensing contract is a royalty alone in the below area, a two-part tariff in the upper area, and only a fixed fee for \( \gamma = 1 \). Thus, we can infer that for \( \frac{1}{1 + \lambda} < c < \hat{c} \), the optimal licensing contract is a two-part tariff for \( 0 \leq \gamma < \hat{\gamma} \), where \( \hat{\gamma} \) is a value of \( \gamma \) making \( \hat{c} = c \); for \( 0 < c \leq \frac{1}{1 + \lambda} \), it is a royalty alone for \( 0 \leq \gamma \leq \gamma^* \), a two-part tariff for \( \gamma^* < \gamma < 1 \), and only a fixed fee for \( \gamma = 1 \), where \( \gamma^* \) is a value of \( \gamma \) making \( c_0 = c \). Thus, we have the following proposition.

Proposition 1. If \( 0 < c \leq \frac{1}{1 + \lambda} \), the optimal licensing contract is a royalty alone for \( 0 \leq \gamma \leq \gamma^* \), a two-part tariff for \( \gamma^* < \gamma < 1 \), and only a fixed fee for \( \gamma = 1 \). If \( \frac{1}{1 + \lambda} < c < \hat{c} \), the optimal licensing contract is a two-part tariff for \( 0 \leq \gamma < \hat{\gamma} \).
The licensing with \( r < \lambda c \) has adverse impact on the licensor’s market profit by enhancing the competition. This adverse impact increases with the licensor’s pre-licensing market share which decreases with \( \gamma \) and increases with its rivals’ costs. The licensor has to balance this adverse effect with its licensing revenue. When its rivals’ costs are not high \((0 < c \leq \frac{1}{1+\lambda})\), the licensor will choose a royalty alone (two-part tariff) as its pre-licensing market share is high (intermediate) for \( 0 \leq \gamma \leq \gamma^* \) \((\gamma^* < \gamma < 1)\), and only a fixed fee as its pre-licensing market share is low for \( \gamma = 1 \). When its rivals’ costs are high \((\frac{1}{1+\lambda} < c < \hat{c})\), the licensor will always choose a two-part tariff as its pre-licensing market share is also intermediate.

3. The outside innovator

In this section, we discuss the situation where the innovator is out of the market. Accordingly, we have \( q_0 = 0 \) no matter licensing occurs or not.

3.1 No licensing

If there is no contract at stage 1, the profit of firm 0 is \( \pi_0^{n,O} = 0 \), while the profits of firms 1 and 2 are \( \pi_1 = (1 - q_1 - \gamma q_2 - c)q_1 \) and \( \pi_2 = (1 - q_2 - \gamma q_1 - c)q_2 \).
respectively. The first order conditions give their equilibrium outputs as \( q_{1}^{n,O} = q_{2}^{n,O} = \frac{1-c}{2+y} \), making \( \pi_{1}^{n,O} = \pi_{2}^{n,O} = \left(\frac{1-c}{2+y}\right)^2 \).

3.2 Licensing

If firms 0 and 1 reach a contract at stage 1, the profits of three firms are \( \pi_{0} = r q_{1} + F \), \( \pi_{1} = [1 - q_{1} - \gamma q_{2} - (1 - \lambda)c - r]q_{1} - F \) and \( \pi_{2} = [1 - q_{2} - \gamma q_{1} - c]q_{2} \) respectively. The profit-maximum outputs of firms 1 and 2 at stage 2 are \( q_{1} = \frac{(2-\gamma)(1-c) + 2 \lambda c - 2r}{4 - y^2} \) and \( q_{2} = \frac{(2-\gamma)(1-c) - \gamma \lambda c + yr}{4 - y^2} \) respectively.

Under a take-it-or-leave-it licensing contract, the equilibrium fixed fee is \( F = (q_{1})^2 - \pi_{1}^{n,O} \), yielding \( \pi_{1}^{L,O} = \pi_{1}^{n,O} \). Thus, firm 0 determines \( r \) to maximise its profit \( \pi_{0} = \frac{[(2-\gamma)(1-c) + 2 \lambda c - 2r]r}{4 - y^2} + \left[\frac{(2-\gamma)(1-c) + 2 \lambda c - 2r}{4 - y^2}\right]^2 - \left(\frac{1-c}{2+y}\right)^2 \). We get \( \frac{\partial \pi_{0}}{\partial r} = -\frac{[(2-\gamma)(1-c) + 2 \lambda c]y^2 + 4r(2-y^2)}{(4-y^2)^2} \leq 0 \), indicating the optimal royalty is \( r^O = 0 \), making \( q_{1}^{L,O} = \frac{(2-\gamma)(1-c) + 2 \lambda c}{4 - y^2} \), \( q_{2}^{L,O} = \frac{(2-\gamma)(1-c) - \gamma \lambda c}{4 - y^2} \), and \( \pi_{0}^{L,O} = \frac{4[(2-\gamma)(1-c) + \lambda c] \lambda c}{(4-y^2)^2} \).

Proposition 2. For outside patentee the optimal licensing contract is always fixed fee irrespective of the degree of product differentiation.

4. Comparison of the incentive for innovation

In this section, we compare the firm 0’s incentives for innovation when it is an outsider with when it is an insider. The incremental payoff of firm 0 as an outside innovator due to the innovation is simply its post-innovation licensing revenue, i.e. \( \Delta \pi_{0}^{O} = \pi_{0}^{L,O} - 0 = \frac{4[(2-\gamma)(1-c) + \lambda c] \lambda c}{(4-y^2)^2} \).

When firm 0 is an insider, its post-innovation payoff is \( \pi_{0}^{L,I} \) as shown in subsection 2.2. Through standard calculations similar to subsection 2.1 except that the marginal production cost of firm 0 without innovation is \( c \), we get firm 0’s pre-innovation profit
is $\pi_0^I = \frac{(2-\gamma)^2(1-c)^2}{4(3-\gamma^2)^2}$. Thus, the incentive for innovation for firm 0 as an insider is 

$$\Delta \pi_0^I = \pi_0^{II} - \pi_0^I$$

as follows.

$$\Delta \pi_0^I = \begin{cases} 
\frac{2(2-\gamma)(7-2\gamma^2)(1-c) + (4+2\gamma^2-\gamma^6)c|c|}{4(3-\gamma^2)^2}, & \text{for } 0 < c \leq c_0 \\
(2-\gamma)^2(1-c)^2(1-c)^2 + \frac{4(2-\gamma)(13-10\gamma^2+2\gamma^4)(1-c)\lambda c + 2(10-4\gamma^4+\gamma^6)\lambda^2 c^2}{8(2-\gamma^2)(3-\gamma^2)^2}, & \text{for } c_0 < c < \bar{c} \\
\frac{[(2-\gamma)(1-c)+2\lambda c]^2 - (2-\gamma)^2(1-c)^2}{8(2-\gamma^2)} - \frac{(2-\gamma)^2(1-c)^2}{4(3-\gamma^2)^2}, & \text{for } \bar{c} \leq c < \hat{c} 
\end{cases}$$

By comparing $\Delta \pi_0^I$ with $\Delta \pi_0^O$, we get $\Delta \pi_0^I > \Delta \pi_0^O$, indicating the following proposition.

**Proposition 3.** The innovator as an insider realizes higher incremental payoff and consequently has higher incentives to innovate than as an outsider.

The incentive for innovation for firm 0 as an outsider equals $(q_1^{I,O})^2 - \pi_1^{n,O}$. For $\bar{c} \leq c < \hat{c}$, $(q_1^{I,O})^2$ equals $\pi_0^{n,I}$. Combined with $\pi_0^{n,O} > \pi_0^I$ and $\pi_0^{II} > \pi_0^{n,I}$, $\Delta \pi_0 = \pi_0^{II} - \pi_0^I > \pi_0^{n,I} - \pi_0 > (q_1^{I,O})^2 - \pi_1^{n,O} = \Delta \pi_0^O$ holds. For $c < \bar{c}$, the output of homogenous goods produced by firms 0 and 1 is higher than that produced by only firm 1 when firm 0 is an outsider. Thus, the gains from cost reduction when firm 0 as an outsider offers only a fixed fee are less than those when firm 0 as an insider also licenses with a fixed fee contract, which is not more than the incremental payoff of firm 0 as an insider if it offers its optimal licensing contract.

5. **Concluding remarks**

We have considered a simple model with three firms in the market where two of them can produce the homogenous good and the third firm produces a differentiated good. We find that in the presence of a differentiated good, if the costs of non-innovators are not high, the optimal licensing contract between two homogenous good producers can involve only royalty when the degree of differentiation is high and two-part tariff when it is low and finally a fixed fee if all goods are homogenous in the limit.
patentee is an outsider the optimal licensing contract is always fixed fee. It is also shown that the incentive for innovation is higher when the innovator is an insider than when it is an outsider.

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