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EQUILIBRIA UNDER NEGLIGENCE LIABILITY: *HOW THE STANDARD CLAIMS FALL APART*

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Abstract. In many accident contexts, the expected accident harm depends on the care levels and activity levels chosen by the parties involved. In an important and seminal contribution Shavell (1980) extended the scope of economic analysis of liability rules by providing a model that allows for the care and activity level choices. The subsequent works have extended the model to predict outcomes under various liability rules and also to compare their efficiency properties. These works make several claims about the existence and efficiency of equilibria under different liability rules, in most cases without providing formal proof. In this paper, examine the claims in the literature using the standard model as it is. Yet, contrary to the existing claims, we show that none of the negligence based liability rules induces an equilibrium in most accident contexts admissible under the standard model. Moreover, we show that the rules of strict liability for injurer, and no-liability for injurer, generally are much more efficient than the standard negligence rules as well as the rules that require sharing of accident loss between the parties. We show that the standard model is inherently flawed. The social optimization problem induced by it generally does not have a solution, or has solutions not discoverable by the first order conditions. Consequently, predictions emanating from the model do not gel with the real world use of liability rules.

1. Introduction

In many tort settings, the probability of an accident and the harm caused by the accident depend on the care exercised by the parties involved as well as their activity levels. In a pioneering contribution, Shavell (1980) extended the scope of economic analysis of liability rules by providing a model that allows for the care and activity level choices by the parties. Further, assuming that care levels are verifiable but the activity levels are not, he proved that the rule of no liability for the injurer can lead to a first best outcome. Shavell's model has served as a basis for much of the subsequent work.

Shavell (1987), Endres (1989), Miceli (1997 p. 29), Cooter and Ulen (2004, pp. 332-33), a Dari-Mattiacci (2002), Delhaye (2002), Goerke (2002), Parisi and Fon (2004), Singh (2006), Shavell (2007a and 2007b), Singh (2009), Parisi and Singh (2010), Dari-Mattiacci, Lovat and Parisi (2014) among others, have worked with or developed models based on Shavell (1980).[1] Most of these works have examined the efficiency properties of strict liability, no-liability, and the standard negligence criterion-based rules; namely, the rule of simple negligence, the rule of negligence with a defense of contributory negligence, the rule of negligence with a defense of comparative negligence, and the rule of strict

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[1]For works dealing with the doctrinal foundations of liability and the related issues See Polinsky (1980), Landes and Posner (1987), Arlen (1990), Miceli (1997), Hylton (2001), Jain and Singh (2002), Feldman and Singh (2009), Cooter and Ulen (2004), BarGil and Ben Shahar (2003), Singh (2003, 2007).

liability with a defense of contributory negligence. In an important contribution, Dari-Mattiacci, Lovat and Parisi (2014) has extended the analysis to examine rules that permit sharing of liability between non-negligent parties.

In this paper, we reexamine the existing claims about the existence of equilibria under liability rules and their efficiency properties. Specifically, we use the standard model along with its assumptions to dispute the following interdependent claims in the existing literature.

First, under the rule of negligence (with or without defense of contributory negligence), the injurer's activity level will be excessive, i.e., greater than the first best level of his activity. However, the victim will make efficient choices, given the inefficient activity choice by the injurer. Analogously, under the rule of strict liability with defense of contributory negligence, the victim's activity level will be inefficiently high, but the injurer's choices will be efficient.

Second, the rule of strict liability with a defense of contributory negligence is more efficient than the rule of negligence, if reduction in the injurer's activity level is relatively important for reducing the accident loss. And, the rule of negligence is more efficient than the rule of strict liability with defense of contributory negligence if reduction in the victim's activity level is relatively important for reducing accident loss.

Third, equilibrium do exist under the standard liability rules. This claim follows from the first two claims - otherwise, it will be pointless to talk about the outcome under liability rules. Some works have specifically argued that the negligence criterion based liability rules induce equilibria in which the injurer and the victim opt for care levels that are appropriate from the view point of first-best efficiency. [2]

Fourth, the sharing of liability between the parties can be desirable as it can enhance the efficiency of liability rules, by incentivizing the parties to moderate their levels of activity.[3] This, by implication, rules out desirability of the rule of strict liability and no liability on the ground of efficiency.

The literature has made these claims without providing formal proofs, to the best of our knowledge. On their face, the claims seem plausible and perhaps that is why the literature has not bother to verify them mathematically.

In this paper, we examine the above claims rigorously. Their intuitive appeal notwithstanding, we show that the above claims do not hold even if we strictly follow the standard model including for identification of the first best, setting of due care standards for the parties, etc. In particular, we show that for most accident contexts consistent with the standard models, there do not exist Nash equilibria under any of the negligence based rules. Moreover, the rule of strict liability and/or the rule of no-liability is significantly more efficient than any of the standard negligence based rules. Specifically, the rules splitting liability between vigilant parties are shown to be inferior to the rule of strict liability and the rule of no liability.[4]

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[2]See Miceli (1997 p. 29), Cooter and Ulen (2004, pp. 332-33), and research papers Dari Mattiacci (2002), Parisi and Fon 2004, also see Delhaye (2002).

[3]See Singh (2006) and Dari-Mattiacci, Lovat, and Parisi (2014).

[4]These problems do no arise under the assumption of constant activity levels. For analysis with different contexts with constant activity levels See Polinsky (1980), Landes and Posner (1987), Arlen (1990), Miceli (1997), Hylton (2001), Jain and Singh (2002), Feldman and Singh (2009), Cooter and Ulen (2004), BarGil and Ben Shahar (2003), Singh (2003, 2005, 2007a and 2007b).

Finally, we examine the nature of the social objective function under the existing versions of the standard model. We show that the social optimization problems induced by these models generally do not have a solution, or have multiple solutions none of which is discoverable by the first order conditions.

Section 2 summarizes the standard model of analysis including the standard approach towards the first best and the due care levels for the parties. Section 3 provides illustrative examples and proofs for how the claims in the existing literature do not hold. In Section 4, we prove our results disputing the existing claims. In this section, we also review the literature in view of our findings in this paper. Section 5 explains the nature of underlying problems with the standard model. Section 6 we explain why the standard model does not gel with the real-world use of liability rules and discuss the questions that need to be addressed by the future research on the subject. Technical details and analysis are provided in the Appendix.

2. The Standard Framework of Analysis

2.1 **The Basics**

Here we described the standard model that is used in the subsequent sections. Following the standard model, assume there are two people, X and Y; both are private benefit maximizing and risk-neutral. They engage in activities that creates a risk of accidents. If an accident takes place there is one injurer, person X, and one victim, person Y. All the accident costs fall initially on the victim, Y. After an accident has taken place, a court adjudicates any dispute between X and Y.

Generally in this paper each person chooses two things: a level of care and an activity level. (In example 1, however, the activity levels are fixed, and X and Y are only allowed to choose care levels.) The care levels for X and Y are x and y respectively. These variables are observable by X and Y, and by the court. The activity levels for X and Y are s and t, respectively. These variables are observable by X and Y. However, *the court cannot observe either activity level.* We require that all 4 variables, s, x, t and y, are non-negative. Care levels x and y are measured in dollars. Activity levels are measured in some other units, depending on the nature of the activity.

For example, X may be the driver of a large old truck in rough condition, and Y may be the driver of an expensive new BMW. If they collide, all the damages will fall on Y. They share the same roads. Each party can vary its level of care (controlling their speed, obeying traffic signals, remaining sober, etc, all of which are translated into x or y, measured in dollars.) The levels of care are observable by both parties, and by the court. Each party can also vary its activity level, s and t for X and Y, respectively. The activity level might be miles driven, for example. The severity of any one accident might depend on x and y, but the probability of an accident in any time interval might depend on s and t, as well as on x and y.

The injurer X has a benefit function which depends on his activity level s and his care level x. His benefit is measured in dollars. In general this may be written as $u(s, x)$. Following the literature, u is a decreasing function of care level x. The benefit function u is often assumed to be an always increasing and strictly concave function of s. However, some works have assumed that u starts as an increasing function of s but becomes a declining function after some point.[5] We analyze both scenarios. Similarly, the victim Y has a benefit function $v(t, y)$ that depends on his activity level t and his care level y. This is measured in dollars; it is a strictly concave function of activity level t, and either always increasing in t or increasing at first but eventually declining. $v(t, y)$ is a decreasing function of care level y. Both benefit functions have the usual smoothness properties and are public knowledge. That is, the injurer's benefit function $u(s, x)$ is known to the victim and to the court, and the victim's benefit function $v(t, y)$ is known to the injurer and to the court. It is standard to assume that the benefit functions of the injurer and victim have the following simple form:[6]

$$u(s, x) = u(s) - xs \qquad \text{and} \qquad v(t, y) = v(t) - yt.$$

With these functions, the injurer's care x can be interpreted as money spent on care per unit of his activity level s, and his total spent on care per unit time is therefore xs. Similar comments apply to the victim's care.

2.2 **Damages and Social Benefit**

If an accident occurs, the victim suffers a loss, denoted by D. The victim's loss is observable and known to all. We use π to denote the *probability* of an accident. Following the literature, the expected accident loss is given by: $E(D) = \phi(s, t)\pi(x, y)D(x, y)$, with ϕ an increasing function of both s and t, assumed to be zero when $s = t = 0$. It is common to assume that $\phi(s, t) = st$ or $\phi(s, t) = s+t$, with π and D decreasing functions of x and y.[7]. Therefore, we will work with the two forms of the function $E(D) = \phi(s, t)\pi(x, y)D(x, y)$: $E(D) = st\pi(x, y)D(x, y)$ and $E(D) = (s+t)\pi(x, y)D(x, y)$. In our section on examples (Section 3 below) we will use simplifying functional forms for the functions $\pi(x, y)$ and $D(x, y)$ which are consistent with the standard models.

Finally, the *net social benefit* (NSB) from the activities of X and Y equals the sum of their benefit functions $u(s, x)$ and $v(t, y)$, minus expected damages. This gives

$$NSB = u(s, x) + v(t, y) - E(D).$$

2.3 **The Standard Approach**

Under the *standard* model and assumptions discussed above, the net social benefit becomes

$$NSB = u(s) - xs + v(t) - yt - \phi(s, t)\pi(x, y)D(x, y).$$

The *standard approach* in the literature is to assume that the social goal, and goal of the court, is to maximize net social benefit. That is, the social objective is to solve:

$$\max_{s,x,t,y} \{u(s) - xs + v(t) - yt - \phi(s, t)\pi(x, y)D(x, y)\}.$$

[5]E.g., see Miceli 1997, Parisi and Fon 2004, Shavell (1980) and (1987) Shavell 2007 a and b, Parisi and Singh 2009, Dari-Mattiacci, Lovat, and Parisi (2014).

[6]See Shavell 1980 and 1987, Miceli 1997, Parisi and Fon 2004, Shavell (2007), Parisi and Singh (2009), Dari-Mattiacci at el (2014)).

[7]See, e.g., Landes and Posner (1987), Shavell (1980) and (1987); and Miceli (1997). This form arises as a special case for models in Endres (1989), Parisi and Fon (2004), Parisi and Singh (2010), and Dari-Mattiacci, Lovat, and Parisi (2014)

It is often assumed the the above optimization problem has a unique solution, say (s^*, x^*, t^*, y^*), identifiable by solving the following first-order conditions:

$$\partial NSB/\partial s = u'(s) - x - \phi_s(s,t)\pi(x,y)D(x,y) = 0; \tag{1}$$
$$\partial NSB/\partial x = -s - \phi(s,t)[\pi_x(x,y)D(x,y) + \pi(x,y)D_x(x,y)] = 0; \tag{2}$$
$$\partial NSB/\partial t = v'(t) - y - \phi_t(s,t)\pi(x,y)D(x,y) = 0; \tag{3}$$
$$\partial NSB/\partial y = -t - \phi(s,t)[\pi_y(x,y)D(x,y) + \pi(x,y)D_y(x,y)] = 0. \tag{4}$$

This approach towards the social objective and identification of the socially efficient solution is standard in the literature. E.g., see Shavell (1980, 1987), Miceli (1997), Cooter and Ulen (2004), and Parisi and Fon (2004), Parisi and Singh (2010), Dari-Mattiacci, Lovat, and Parisi (2014). The profile (s^*, x^*, t^*, y^*) is called the *first best* profile of activity and care levels.

There are two different approaches to the benefit functions $u(s)$ and $v(t)$ in the literature; in both approaches $u(0) = v(0) = 0$; in approach 1 the functions are always increasing, but in approach 2 the functions increase up to maximima and then decline.[8] More specifically, the standard model

Version 1 assumes $u(0) = 0$, $u'(s) > 0$, and $u''(s) < 0$ for all $s \geq 0$, and similarly for $v(t)$.

Version 2 assumes $u(0) = 0$, $u''(s) < 0$, but $u'(s) > 0$ holds only up to some $\hat{s} > 0$; with $u'(s) < 0$ thereafter, and similarly for $v(t)$.

2.4 The Standard Liability Rules

Under the standard approach, the profile of activity and care levels identified by the first order conditions, i.e., (s^*, x^*, t^*, y^*), serves as a natural *focal point* for the mainstream modeling of negligence liability rules.[9] The court sets due care standards by solving the first-order condition equations for (s^*, x^*, t^*, y^*). The court cannot observe s and t, but it can observe x and y, and it knows how all the variables enter the benefit functions and the $E(D)$ function. Accordingly, under standard negligence based liability rules, the court sets care standards for negligence-based rules at x^* and y^* for X and Y, respectively. That is, party X will be found *negligent* if and only if $x < x^*$, and similarly, party Y will be found *negligent* if and only if or $y < y^*$.

When an accident occurs the victim Y initially incurs a loss. Afterward, a court adjudicates the dispute between X and Y. That is, the court determines what part of the loss will fall on each of the two parties (or possibly on others). A *standard negligence-based liability rule* determines shares of damages to fall on each of the two parties, contingent on x, x^*, y and y^*. More formally, it determines weights (w_X, w_Y), where w_X is the fraction of the loss to fall on the injurer X, and w_Y is the fraction to fall on the victim Y. For a normal rule the weights satisfy $0 \leq w_X, w_Y \leq 1$, and $w_X + w_Y = 1$. (A rule using *punitive damages*, for example, would not fit this definition, since the use of punitive

[8]These approaches are special cases in Goerke (2002) and Dari-Mattiacci, Lovat, and Parisi (2014) where focus is on general form of NSB function. Also see Hindley and Bishop (1983) or de Meza (1986).

[9]For a detailed game-theocratic discussion on the role of focal points in coordinating activities of players see Basu (2018).

damages often implies that the injurer's weight w_X is greater than 1, and the victim's weight w_Y is less than 0. Nor would a rule that placed some or all of the losses on persons other than the injurer and victim, for then $w_X + w_Y < 1$.)[10]. Here are examples of standard negligence-based liability rules:

1. Simple negligence. This rule says $w_X = 1$ and $w_Y = 0$ (all the loss is placed on the injurer) if and only if $x < x^*$ (the injurer is negligent). Otherwise, $w_X = 0$ and $w_Y = 1$ (all the loss stays with the victim).

2. Negligence with a defense of contributory negligence. This rule says $w_X = 1$ and $w_Y = 0$ (all the loss is placed on the injurer) if and only if $x < x^*$ (the injurer is negligent) and $y \geq y^*$ (the victim is non-negligent). Otherwise, $w_X = 0$ and $w_Y = 1$ (all the loss stays with the victim).

3. Strict liability with a defense of contributory negligence. This rule says $w_X = 1$ and $w_Y = 0$ (all the loss is placed on the injurer) if and only if $y \geq y^*$ (the victim is non-negligent). Otherwise, $w_X = 0$ and $w_Y = 1$ (all the loss stays with the victim).

4. Comparative Negligence. This rule says $w_X = 1$ and $w_Y = 0$ (all the loss is placed on the injurer) if and only if $x < x^*$ (the injurer is negligent) and $y \geq y^*$ (the victim is non-negligent); $w_X = 0$ and $w_Y = 1$ (all the loss is placed on the victim) if and only if $x \geq x^*$ (the injurer is non-negligent); and when $x < x^*$ and $y < y^*$ (both are negligent) the loss is split in proportion to their degrees of negligence.

Besides, the above rules we consider the following negligence liability based rules.

5. 50/50 split liability when both are negligent. This rule says $w_X = 1$ and $w_Y = 0$ (all the loss is placed on the injurer) if and only if $x < x^*$ (the injurer is negligent) and $y \geq y^*$ (the victim is non-negligent); $w_X = 0$ and $w_Y = 1$ (all the loss is placed on the victim) if and only if $x \geq x^*$ (the injurer is non-negligent); and $w_X = 1/2$ and $w_Y = 1/2$ (the loss is split 50/50) when $x < x^*$ and $y < y^*$ (both are negligent).

6. 50/50 split liability when both are non-negligent. This rule says $w_X = 1$ and $w_Y = 0$ (all the loss is placed on the injurer) if and only if $x < x^*$ (the injurer is negligent); $w_X = 0$ and $w_Y = 1$ (all the loss is placed on the victim) if and only if $y < y^*$ (the victim is negligent) and $x \geq x^*$ (the injurer is non-negligent); and $w_X = 1/2$ and $w_Y = 1/2$ (the loss is split 50/50) when $x \geq x^*$ and $y \geq y^*$ (both are non-negligent).

An important property of the standard negligence-based liability rules (rules 1-4 above) is this: When an accident occurs, if a court finds that one of the parties is negligent while the other is not, it places all the damages on the negligent party. Both of our 50/50 split liability rules (5 and 6) also satisfy this property. In contrast two real-world negligence rules do not satisfy these properties. These are:

7. *Strict liability for injurer.* This rule says $w_X = 1$ and $w_Y = 0$ (all the loss is placed on the injurer) for any x, x^*, y, or y^* (always).

8. *No liability for injurer.* This rule says $w_X = 0$ and $w_Y = 1$ (all the loss is placed on the victim) for any x, x^*, y, or y^* (always).

[10]Note that w_X depends on only care levels x, x^*, y and y^*, and not on activity levels, which are unobservable to the court. See Shavell (1980, 1987), Miceli (1997), Cooter and Ulen (2004), Parisi and Fon (2004), and Dari-Mattiacci, Lovat, and Parisi (2014)

2.5 **Individual Choices and Payoffs**

Given that the shares of accident losses are determined by liability rules, the injurer and victim act accordingly. Each wants to choose an activity level and a care level to maximize his benefit function *net of the expected damages placed on him by the liability rule.* In general, the injurer wants to choose s and x to maximize: $u(s, x) - w_X E(D)$. Under the standard case assumptions he wants to maximize

$$u(s) - xs - w_X \phi(s, t)\pi(x, y)D(x, y).$$

In our examples section (Section 3 below), we will describe our simplifying assumptions on the $\pi(x, y)$ and $D(x, y)$ functions, and describe and solve the injurer's (s, x) choice problem. Similarly, in general, the victim wants to choose t and y to maximize: $v(t, y) - w_Y E(D)$. Under the standard case assumptions he wants to maximize

$$v(t) - yt - w_Y \phi(s, t)\pi(x, y)D(x, y).$$

In our examples section, we describe and solve choice problems for the parties, using our simplifying assumptions on the $\pi(x, y)$ and $D(x, y)$ functions. Note that the sum of the net benefit functions of the injurer and the victim (in the general case) is

$$u(s, x) + v(t, y) - (w_X + w_Y)E(D) = u(s, x) + v(t, y) - E(D) = NSB,$$

since the liability weights sum to 1.

3. Examples

In the next section we prove that none of the standard negligence liability rules has a Nash equilibrium. Moreover, we show that the strict liability and no-liability rules are more efficient than the liability sharing rules. As a prelude our main results in the next section, here present some illustrative examples. These examples provide several counter-intuitive insights on functioning of the standard liability rules.

As in the standard negligence liability rules, (x^*, y^*) is used to set due care standards. Our examples show what happens when the parties X and Y make th reacting to the legal rule and to each other. For illustration we use a dynamic process, called a *Cournot dynamic process*, in which X and Y take turns moving. However, it should be noted that the process is used only for illustrative purpose. Our results do not depend on use of this process.

It is plausible to assume that under a negligence liability rule, (s^*, x^*, t^*, y^*), specifically the due care levels, (x^*, y^*), provide a 'focal' point for the parties.[11] Therefore, let the dynamic process starts at (s^*, x^*, t^*, y^*), and then allow X and Y to make a sequence of moves, starting with X, followed by Y, followed by X, and so on. When it's X's turn to move, he responds to the legal rule, (how will his choice of x affect w_X?), and he responds to what Y did at the previous stage. When it's Y's turn to move, he responds to the legal rule (how will his choice of y affect w_Y?) and to what X did at the previous stage.

We show how such a process will lead to a cycle. We also show how bad the cycle is: Is it long or short? Does it involve large or small variations in $E(D)$? Does it create huge variability in the parties' benefit functions, net of the damage they must pay?

[11]On role of focal points in games see Basu (2018)

To keep things simple we will assume that $D = 50$, a constant. In Examples 1, 6, and 9, the benefit functions are of the simple form $u(s) - xs = s^{1/2} - xs$ and $v(y) - yt = t^{1/2} - yt$. For all examples except Examples 6 and 7, the $\phi(s,t)$ function is $\phi(s,t) = st$. In Examples 6 and 7, $\phi(s,t) = s + t$. For the accident probability $\pi(x,y)$, we use $\pi(x,y) = 1/(1 + x + y)$. Therefore for everything except Examples 6 and 7, expected damages are given by $E(D) = 50st/(1 + x + y)$. Summing up, for Examples 1, 6 and 9, we will use the following specification of the net social benefit function:

Specification 1: $$NSB = s^{1/2} - xs + t^{1/2} - yt - \frac{50st}{1+x+y}.$$

A slight complication of the benefit functions in Specification 1 is that, for a given x or y, they are unbounded in the activity levels s and t. We will take care of this problem by putting constraints on s and t. When we consider Specification 1, we will use the constraints $s \leq 1{,}000{,}000$ and $t \leq 1{,}000{,}000$. As is shown in the subsequent sections, our results do not depend on these constraints. Our specification of $u(s)$ and $v(t)$ functions is in line with version 1 of the standard model described in Section 2.3 above.

In our Specification 2 examples we will functions $u(s)$ and $v(t)$ consistent with version 2 of the standard model. In particular, we add terms like $-\delta_1 s$ and $-\delta_2 t$ to $s^{1/2} - xs$ and $t^{1/2} - yt$, respectively, with $\delta_1, \delta_2 > 0$. This makes the benefit functions bounded in the activity levels. For Examples 2, 3, 4, 5, 7 and 10, we will use the following specification of NSB:

Specification 2: $$NSB = s^{1/2} - xs - \delta s + t^{1/2} - yt - \delta t - \frac{50st}{1+x+y}.$$

For ease of illustration for the most part we will work with $\delta_1 = \delta_2 = 0.01$. However, we also produce an example and arguments when $\delta_1 \neq \delta_2$.

Summing up, our examples deals with the accident settings as described below.

	Specification	$\phi(s,t)$	Liability Rule	What Happens?
Example 1	Spec. 1	st	Simple negligence	Cycle but no Nash Equilibrium
Example 2	Spec. 2	st	Simple negligence	Cycle but no Nash Equilibrium
Example 3	Spec. 2	st	negligence with defense of contributory negligence	Cycle but no Nash Equilibrium
Example 4	Spec. 2	st	50/50 split, both negligence	Cycle but no Nash Equilibrium
Example 5	Spec. 2 $\delta 1 \neq \delta 2$	st	Simple negligence	Cycle but no Nash Equilibrium
Example 6	Spec. 1	s+t	Simple negligence	Cycle but no Nash Equilibrium
Example 7	Spec. 2	s+t	Simple negligence	Cycle but no Nash Equilibrium
Example 8	Spec. 1	st	50/50 split when both non-negligent	Nash equilibrium starting at SBO
Example 9	Spec. 2	st	50/50 split when both non-negligent	Nash equilibrium starting at SBO

In all of our examples except for Examples 8 and 9, the starting point for a sequence of moves between person X and person Y is a solution to the first order condition for NSB maximization, i.e., (s^*, x^*, t^*, y^*). We abbreviate this "Solution FOC." In each example we provide a table showing relevant numbers. Line 1 in each table in Examples 1 through

8, shows Solution FOC. In Examples 8 and 9 line 1 does not show a solution to the first order conditions for NSB maximization, rather it shows a second-best optimum, or SBO i.e., $(s^{**}, x^*, t^{**}, y^*)$, as defined above.

After we begin, as is shown on line 2 of each table except the last two examples, the injurer X reacts to solution FOC: He takes as given Y's choice of (t^*, y^*) as in the solution FOC, and chooses a benefit maximizing pair (s, x) for himself. In making his choice, X uses his benefit function, the expected damages function, and the liability rule which governs how expected damages are distributed. On line 2 of each table we will show the (s, x) numbers that X chooses in **bold**, and on the same line we also show the (t^*, y^*) numbers that Y is assumed to start with in non-bold.

In the tables for Examples 1 through 8, line 2 shows how the injurer X reacts to the variables in line 1, Solution FOC. He takes as given victim Ys variables (t*,y*), and chooses a benefit maximizing pair (s,x) for himself. In making his choice, Y uses his benefit function, the expected damages function, and the liability rule which governs how expected damages are distributed. On the line 3 of each table we show the (t, y) numbers that Y chooses in **bold**, and the (s, x) numbers to which he responds, in non-bold.

This dynamic process goes on, with X's move, followed by Y's move, followed by X's move, until it reaches a Nash equilibrium, or a repeating cycle, or some other strange result.

In each table for each example, and on each line, we also show the liability assignment given the variables of that line. ("All X" means that given the (s, x, t, y) variables of the line - and only x and y matter for liability - and given the liability rule, all the damages will fall on X; "All Y" means all the damages will fall on Y; "50/50" means half the damages will fall on X and half on Y.) Also in each table on each line we show the net benefit amounts for X and for Y; that is, each party's benefits net of any damages for which that party is liable at that stage.

In the notes beneath each table in each example we show the liability rule (e.g., for simple negligence, "X is liable if and only if $x < x^*$".) We also show any special constraints on the variables. A general constraint, used in all the examples, is that s, x, t, and y are all always non-negative. We will not repeat this in each of the examples. However, in the Specification 1 cases, Examples 1, 6 and 9, we indicate s and t are both constrained to be less than or equal to 1,000,000. Under each table for each example, following the liability rule and constraints, is a list of notes for each of the lines in the table.

It should finally be noted that the tables in this paper provide only the relevant details. All of the tables in this paper are done in extensive Excel spreadsheets, which are available on request.

For Specification 1, the first-order conditions for maximizing NSB, corresponding to equations (1), (2), (3), and (4) above, are as follows:

$$(1') \qquad (1/2)s^{-1/2} - x - \frac{50t}{1+x+y} = 0,$$

$$(2') \qquad -s + \frac{50st}{(1+x+y)^2} = 0,$$

$$(3') \qquad (1/2)t^{-1/2} - y - \frac{50s}{1+x+y} = 0,$$

$$(4') \qquad -t + \frac{50st}{(1+x+y)^2} = 0.$$

After some manipulation this gives:

$$(1'') \qquad s = (1/4)(1+2x+y)^{-2},$$

$$(2'') \qquad 50t = (1+x+y)^2,$$

$$(3'') \qquad t = (1/4)(1+x+2y)^{-2},$$

$$(4'') \qquad 50s = (1+x+y)^2.$$

From (2″) and (4″), we conclude that $s = t$. Then from (1″) and (3″) we conclude that $x = y$. This system of equations has only one non-negative real solution, given by $x = 0.355473$ and $s = 0.058547$. We can now easily finish solving the first-order conditions. We find the following:

$$(s^*, x^*, t^*, y^*) = (0.058547, 0.355473, 0.058547, 0.355473).$$

The (s^*, x^*, t^*, y^*) calculated in this fashion is shown on line 1 of the table for examples based on Specification 1.

Example 1 Assumptions: Simple negligence, Specification 1.
Result: No equilibrium but a Cycle!

Consider the following table based on the simple negligence rule and Specification 1. For this case we discuss all the steps in details. For the other cases discussed through remaining examples, the process is similar and hence we will discuss only the main points.

Line 1 of the table shows the first-order conditions solution, i.e., (s^*, x^*, t^*, y^*). This profile serves a the focal point for the remaining steps. At line 2 X chooses s and x. With respect to his choice of x, because of the liability rule he will not increase it to any level above x^* because x is costly and there would be no gain: under simple liability all damages fall on Y once X is choosing any x greater than or equal to x^*. So, he will either keep his care level at $x^* = 0.355473$ or reduce it. We have calculated what (s, x) would be the best choice for him if he (1) opts to have all damages fall on Y by choosing $x = x^* = 0.355473$, or if he (2) opts to have all damages fall on himself by choosing a smaller x. It turns out option 1 is better. So he sets $x = x^*$ and then solves for the best s to go along with x^*; this leads to $s = 1.978455$. At line 3, Y chooses t and y, in response to x^* and $s = 1.978455$ chosen by X. All damages will fall on Y because X has chosen $x = x^*$, and in response Y chooses the t and y shown in bold.

At line 4, it is again X's turn. He again makes the choice between (1) opting to have all the damages fall on Y by choosing $x = x^* = 0.355473$, or (2) opting to have all damages fall on himself by choosing a smaller x. This time he finds option 2 is better.

His first-order maximization conditions point to a negative x, which is not allowed. He choses $x = 0$ instead, and the large $s = 17374$ shown - in any case, he is better off choosing $x = 0$ and $s = 17374$, given the t and y from line 3.

At line 5 Y chooses. Since X has just chosen $x = 0$, by simple liability all damages will fall on X, no matter what Y does. Therefore Y sets his care level $y = 0$, and chooses the largest t allowed. If we use the constraint $t \leq 1,000,000$, Y will choose $t = 1,000,000$; otherwise he will raise activity level beyond limits.

Example 1 Table

	s	x	t	y	Liability	X's benefit Net of Damages	Y's benefit Net of Damages
1	0.058547	0.355473	0.058547	0.355473	All Y	0.22115	0.12098
2	**1.978455**	**0.355473**	0.058547	0.355473	All Y	0.70329	-3.16388
3	1.978455	0.355473	**0.000728**	**8.590526**	All Y	0.70329	0.01349
4	**17374.66**	**0.000000**	0.000728	8.590526	All X	65.9065	0.0207
5	17374.66	0.000000	**1000000**	**0.000000**	All X	-8.69E+11	1000.000
6	**1.978455**	**0.355473**	1000000	0.000000	All Y	0.70329	-72979213
7	1.978455	0.355473	**0.000728**	**8.590526**	All Y	0.70329	0.013486

Notes:
X is liable if and only if $x < x^*$.
Constraint: $s, t \leq 1,000,000$.
Line 1 solves first order conditions to get (s^*, x^*, t^*, y^*).
Line 2: X chooses s and x. He chooses to keep x at x^*, and increase s to 1.978.
Line 3: Y chooses t and y. FOC's give interior solution: $t = .000728$ and $y = 8.5905$.
Line 4: X chooses s and x. FOC implies negative x, so $x = 0$ is used. FOC implies $s = 17374.6$.
Line 5: Y chooses t and y. Since $x = 0$, X pays damages; therefore Y sets $y = 0$, and wants t large.
Line 6: X chooses s and x. He must choose $x = 0.355473$ to escape huge damages. FOC gives s= 1.978.
Line 7: Y chooses t and y. Back to line 3! Cycle!

At line 6, X chooses. He sets $x = x^*$ to escape the huge damages, and solves a first-order condition to get $s = 1.978455$. At line 7 it is Y's turn to choose. But Y's choice at line 7 is exactly the same as his choice had been at line 3. We are in a cycle![12]

We conclude that a Cournot dynamic process, starting from a reasonably chosen initial point, may lead to a bizarre cycle rather than to a Nash equilibrium. Also, note that the logic of the cycle in lines 3 to 7 will remain even if we relax the constraint $t \leq 1,000,000$.

In fact, as is proved in the next subsection for above Specifications 1 and 2 there cannot exist a Nash equilibrium under negligence liability, regardless of whether parties make their choices sequentially as described above, or otherwise.

[12]It is easy to check that if we fix $s = t = 1$ a cycle will not be possible; instead, a unique Nash equilibrium will exits under negligence rule.

Example 2 Assumptions: Simple negligence, Specification 2. Result: No equilibrium but a Cycle!

We now proceed with our 2nd example. In this and subsequent examples we will try to be more brief than we were in example 1.

This example is much like example 1 above, except that it is based on Specification 2, taking $\delta = 0.01$. That is, the benefit functions are modified so that they have maxima in s and t. The benefit functions are now: $u(s) - xs = s^{1/2} - xs - .01s$ and $v(y) - yt = t^{1/2} - yt - .01t$. With this assumption it should no longer be necessary to constrain s and t in order to find a maximum for NSB.

Example 2 Table

	s	x	t	y	Liability	X's benefit Net of Damages	Y's benefit Net of Damages
1	0.058295	0.353629	0.058295	0.353629	All Y	0.22025	0.12072
2	**1.890707**	**0.353629**	0.058295	0.353629	All Y	0.68751	-3.00768
3	1.890707	0.353629	**0.000763**	**8.369300**	All Y	0.68751	0.01381
4	**1262.607**	**0.000000**	0.000763	8.369300	All X	17.76659	0.02123
5	1262.607	0.000000	**2500.000**	**0.000000**	All X	-1.58E+08	25.0000
6	**1.890707**	**0.353629**	2500.000	0.000000	All Y	0.68751	-174571
7	1.890707	0.353629	**0.000763**	**8.369300**	All Y	0.68751	0.01381

Notes:
X is liable if and only if $x < x^*$.
Line 1 solves first order conditions to get (s^*, x^*, t^*, y^*).
Line 2: X chooses s and x. X stays with $x = .3536$, but increases s to s= 1.8907.
Line 3: Y chooses t and y. FOC's give interior solution: $t = .000763$ and $y = 8.3693$.
Line 4: X chooses s and x. FOC implies negative x, so $x = 0$ is used. FOC implies s= 1262.607.
Line 5: Y chooses t and y. Since $x = 0$, X pays damages; Y sets $y = 0$, and uses FOC to get $t = 2500$.
Line 6: X chooses s and x. He sets $x = 0.3536$ to avoid huge accident losses, then chooses s= 1.89071.
Line 7: Y chooses t and y. Back to line 3! Cycle!

For Specification 2, the first-order conditions for maximizing NSB, corresponding to equations (2) and (4) above, are the exactly the same as (2′) and (4′). However, the first-order conditions corresponding to equations (1) and (3) above will be:

$$(1''') \qquad (1/2)s^{-1/2} - x - 0.01 - \frac{50t}{1+x+y} = 0,$$

$$(3''') \qquad (1/2)t^{-1/2} - y - 0.01 - \frac{50s}{1+x+y} = 0,$$

For Specification 2, the system of the first-order conditions has only one non-negative solution. Now we have the following:

$$(s^*, x^*, t^*, y^*) = (0.058295, 0.353629, 0.058295, 0.353629).$$

That is, the line 1 first-order condition solutions are close to what we had in example 1, but not exactly the same. What is the outcome for example 2? Once again, no Nash equilibrium, and instead another bizarre cycle, shown on lines 3 through 7 of the Example 2 table.

Next, consider the rule strict liability with a defense of contributorily negligence. This rule is the mirror image of the simple negligence. If we swap party X with party Y and vice-versa, then whatever claim is valid for party X under simple negligence, a similar claim will be true for party Y under strict liability with a defense of contributorily negligence. In view of the symmetry of the functional forms, it is easy to produce an example similar to Example 1 to show existence of cycle under the rule strict liability with a defense of contributorily negligence. As is proved in the next subsection, there cannot exist a Nash equilibrium under this rule as well.

Next we show the existence of a cycle under the rule of negligence with a defense of contributory negligence.

Example 3 Assumptions: Negligence with a defense of contributory negligence, Specification 2. Result: No equilibrium but a Cycle!

We now proceed with our 3rd example. This example is much like example 2 above, i.e., is based on Specification 2, except the liability rule used here is negligence with a defense of contributory negligence. What happens under this rule? Once again, another bizarre cycle, shown on lines 3 through 7 of the following Example 3 Table.

Example 3 Table

	s	x	t	y	Liability	X's benefit Net of Damages	Y's benefit Net of Damages
1	0.058295	0.353629	0.058295	0.353629	All Y	0.22025	0.12072
2	**1.890707**	**0.353629**	0.058295	0.353629	All Y	0.68751	-3.00768
3	1.890707	0.353629	**0.000763**	**8.369300**	All Y	0.68751	0.01381
4	**1262.607**	**0.000000**	0.000763	8.369300	All X	17.76659	0.02123
5	1262.607	0.000000	**1.890707**	**0.353629**	All X	-88155.64	0.687515
6	**1.890707**	**0.353629**	1.890707	0.353629	All Y	0.687515	-104.006
7	1.890707	0.353629	**0.000763**	**8.369300**	All Y	0.687515	0.01381

Notes:
X is liable if and only if $x < x^*$ and $y \geq y^*$.
Line 1 solves first order conditions to get (s^*, x^*, t^*, y^*).
Line 2: X chooses s and x. X stays with $x = .3536$, but increases s to $s = 1.8907$.
Line 3: Y chooses t and y. FOC's give interior solution: $t = .000763$ and $y = 8.3693$.
Line 4: X chooses s and x. FOC implies negative x, so $x = 0$ is used. FOC implies $s = 1262.607$.
Line 5: Y chooses t and y. Since $x = 0$, X pays damages as long as $y \geq y^*$; otherwise Y pays. Y is better off setting $y = 0.353629$ than using the FOC to set $t = 1.890707$.
Line 6: X chooses s and x. He sets $x = 0.3536$ to avoid huge accident losses, then chooses $s = 1.89071$.
Line 7: Y chooses t and y. Back to line 3! Cycle!

Working along the lines described in Example 3 above, but substituting Specification 1 for Specification 2, it is easy to produce another cycle for Specification 1. As is explained below, between the two specifications, Specification 2 is more plausible. Therefore, in the following, we will use it more frequently. However, corresponding to a cycle shown for Specification 2, a similar or worse cycle can be shown to exist for Specification 1.

Example 4 Assumptions: 50/50 split liability when both negligent, Specification 2. Result: No equilibrium but a Cycle!

Now we turn to an example in which the liability rule involves the sharing of liability by the 2 parties. In particular, the benefit functions and the expected damages function is as in Specification 2, with (delta) = 0.01 for both parties. And the liability rule is of 50/50 split liability when both parties are negligent. More formally: X bears all the loss when he is negligent and Y is not; that is, when $x < x^*$ and $y^* \leq y$. Y bears all the loss when X is non-negligent; that is, when $x^* \leq x$. When both X and Y are negligent, that is, when $x < x^*$ and $y < y^*$, the loss is split 50/50. Turning to the table:

Example 4 Table

	s	x	t	y	Liability	X's benefit Net of Damages	Y's benefit Net of Damages
1	0.058295	0.353629	0.058295	0.353629	All Y	0.220245	0.12072
2	**1.890707**	**0.353629**	0.058295	0.353629	All Y	0.687515	-3.00768
3	1.890707	0.353629	**0.000763**	**8.369300**	All Y	0.687515	0.01381
4	**1262.607**	**0.000000**	0.000763	8.369300	All X	17.76659	0.02123
5	1262.607	0.000000	**1.890707**	**0.353629**	All X	-88155.64	0.68751
6	**1.890707**	**0.353629**	1.890707	0.353629	All Y	0.687515	-104.006
7	1.890707	0.353629	**0.000763**	**8.369300**	All Y	0.687515	0.01381

Notes:
X is liable when X is negligent ($x < x^*$) and Y is non-neg; Y is liable when X is non-neg. However when both X and Y are negligent, liability is split 50/50.
Line 1 solves first order conditions to get (s^*, x^*, t^*, y^*).
Line 2: X chooses s and x. He chooses to keep x at x^*, and increase s to 1.890707.
Line 3: Y chooses t and y. FOC's give interior solution: $t = .000763$ and $y = 8.369300$.
Line 4: X chooses s and x. FOC implies negative x, so $x = 0$ is used. FOC implies $s = 1262.607$
Line 5: Y chooses t and y. Since $x = 0$, Y sets $y = 0.353629$ to avoid damages, and uses FOC to get $t = 1.890707$.
Line 6: X chooses s and x. Back to s and x of line 2.
Line 7: Y chooses t and y. Back to line 3.
Cycle! And they never enter the 50/50 split region.

In the table line 1 again shows the (s^*, x^*, t^*, y^*) that solves the first-order conditions. In line 2 X chooses (s, x) to maximize his utility, taking Y's (t, y) from line 1 as given; in line 3 Y reacts to what X did in line 2, and so on. The result? We once again have a cycle, in lines 3 to 6.

Example 5. **Assumptions: Simple negligence, Specification 2, but with different δ's. Result: No equilibrium but a Cycle!**

Example 5 is a variation of Example 2. All the assumptions are the same as in example 2, except that now the $.01t$ factor in Y's benefit function, which guarantees he wants a finite amount of t, is smaller than it was by a factor of 10. The benefit functions are now: $u(s) - xs = s^{1/2} - xs - .01s$ and $v(t) - yt = t^{1/2} - yt - .001t$. This gives the following specification of the net social benefit function:

$$NSB = s^{1/2} - xs - 0.01s + t^{1/2} - yt - 0.001t - \frac{50st}{1+x+y}.$$

The liability rule is simple negligence. The Example 5 conclusion is like the Example 2 conclusion: A bizarre cycle and no equilibrium.

Example 5 Table

	s	x	t	y	Liability	X's benefit Net of Damages	Y's benefit Net of Damages
1	0.058408	0.349958	0.058408	0.358958	All on Y	0.22065	0.12084
2	**1.929460**	**0.349958**	0.058408	0.358958	All on Y	0.69453	-3.07663
3	1.929460	0.349958	**0.000747**	**8.472134**	All on Y	0.69453	0.01366
4	**1286.024**	**0.000000**	0.000747	8.472134	All on X	17.93059	0.02100
5	1286.024	0.000000	**250000.0**	**0.000000**	All on X	-1.61E+10	250.000
6	**1.929460**	**0.349958**	250000.0	0.000000	All on Y	0.69453	-17865675
7	1.929460	0.349958	**0.000747**	**8.472134**	All on Y	0.69453	0.01366

Notes:
X is liable if and only if $x < x^*$.
Line 1 solves first order conditions to get (s^*, x^*, t^*, y^*).
Line 2: X chooses s and x. X stays with $x = .3500$ but increases s to $s = 1.9295$.
Line 3: Y chooses t and y. FOC's give interior solution: $t = .000747$ and $y = 8.4721$.
Line 4: X chooses s and x. FOC implies negative x, so $x = 0$ is used. FOC implies $s = 1286.024$.
Line 5: Y chooses t and y. X pays damages; Y sets $y = 0$; FOC gives $t = 250{,}000$.
Line 6: X chooses s and x. Sets $x = 0.3500$ to escape huge accident losses; uses FOC to get $s = 1.9295$.
Line 7: Y chooses t and y. Back to line 3! Cycle!

Before proceeding further let us consider a case where $\phi(s,t) = s + t$. Therefore in this example expected damages are given by $E(D) = 50(s+t)/(1+x+y)$. Now, the NSB function takes the following form:

$$NSB = s^{1/2} - xs + t^{1/2} - yt - \frac{50(s+t)}{1+x+y}.$$

This specification also leads to strange outcomes.To see why, consider simple negligence for the liability rule. We can continue to use the benefit functions we have used in Example 1, and we continue to constrain s and t to be less than or equal to 1,000,000.

We have also added a constraint on the size of x and y. The crucial difference in this example is that we have changed the $\phi(s,t)$ function from st to $s+t$.

Example 6. Assumptions: Simple negligence, Specification 1, but with $\phi(s,t) = s+t$. Result: No equilibrium but a Cycle!

Example 6 Table

	s	x	t	y	Liability	X's benefit Net of Damages	Y's benefit Net of Damages
1	0.002770	4.500000	0.0027701	4.500000	All Y	0.040166	0.012465
2	**0.012346**	**4.500000**	0.0027701	4.500000	All Y	0.055556	-0.035413
3	0.012346	4.500000	**2.50E-07**	**1000.00**	All Y	0.055556	-0.000364
4	**100.2004**	**0.000000**	2.50E-07	1000.00	All X	5.005000	0.000250
5	100.2004	0.000000	**1000000**	**0.000000**	All X	-5.00E+07	1000.000
6	**0.012346**	**4.500000**	1000000	0.000000	All Y	0.055556	-9089909
7	0.012346	4.500000	**2.50E-07**	**1000.00**	All Y	0.055556	-0.000364

Notes:
X is liable if and only if $x < x^*$. Constraints: $s, t \leq 1,000,000$. $x, y \leq 1,000$
Line 1 solves first order conditions to get (s^*, x^*, t^*, y^*).
Line 2: X chooses s and x. X chooses $x = 4.5$ to avoid liability, and FOC gives $s = 0.1235$.
Line 3: Y chooses t and y. Y chooses t = 2.5E-7 and y very large, e.g. y = 1,000.
Line 4: X chooses s and x. He sets $x = 0$ and becomes liable. FOC quickly gives s = 100.2004.
Line 5: Y chooses t and y. X is liable since $x = 0$. Y wants to max $t^{\frac{1}{2}} - yt$. Solution: set $y = 0$ and t large.
Line 6: X chooses s and x. He sets $x = 4.5$ to avoid liability and solves FOC for $s = 0.0123457$
Line 7: Y chooses t and y. Back to line 3! Cycle!

What happens in this example? No Nash equilibrium, but another bizarre cycle in lines 3 to 7. Note the huge variations in the net benefits for X and Y. These are caused by huge variations in the payoffs.

Example 7. Assumptions: Simple negligence, Specification 2, but with $\phi(s,t) = s+t$. Result: No equilibrium but a Cycle!

Example 7 is just like Example 6, except it substitutes Specification 2 for Specification 1. Once again, the result is a big cycle.

Example 7 (Specification 2)

	s	x	t	y	Liability	X's benefit Net of Damages	Y's benefit Net of Damages
1	0.002764	4.500000	0.002764	4.500000	All Y	0.040109	0.012467
2	**0.012291**	**4.500000**	0.002764	4.500000	All Y	0.055432	-0.035167
3	0.012291	4.500000	**2.50E-07**	**1000.00**	All Y	0.055432	-0.000361
4	**69.560214**	**0.000000**	2.50E-07	1000.00	All X	4.170138	0.000250
5	69.560214	0.000000	**2500**	**0.000000**	All X	-128470	25
6	**0.012291**	**4.500000**	2500	0.000000	All Y	0.055432	-22702.4
7	0.012291	4.500000	**2.50E-07**	**1000.00**	All Y	0.055432	-0.000361

Notes:
X is liable if and only if $x < x^*$. Constraints: $x, y \leq 1,000$
Line 1 solves first order conditions to get (s^*, x^*, t^*, y^*).
Line 2: X chooses s and x. X chooses $x = 4.500000$ to avoid liability, and FOC gives $s = 0.012291$.
Line 3: Y chooses t and y. Y chooses $t = 2.50\text{E-}07$ and y very large, e.g. $y = 1{,}000.000$.
Line 4: X chooses s and x. He sets $x = 0$ and becomes liable. FOC quickly gives $s = 69.56021$.
Line 5: Y chooses t and y. X is liable since $x = 0.000000$. Y wants to max $t^{\frac{1}{2}} - yt - .01t$. Solution: set $y = 0$ and $t = 2500$.
Line 6: X chooses s and x. He sets $x = 4.500000$ to avoid liability and solves FOC for $s = 0.012291$
Line 7: Y chooses t and y. Back to line 3! Cycle!

Example 8 . Assumptions: 50/50 split liability when both are non-negligent, Specification 1, Start dynamic process at SBO.

Examples 8 is based on 50/50 split liability when both parties are non-negligent. Specifically, this liability rule works like this: X bears all the loss when he is negligent;that is, when $x^* > x$ and $y^* \leq y$. Y bears all the loss when he is negligent and X is not; that is, when $y < y^*$ and $x^* \leq x$. When both X and Y are non-negligent, that is, when $x^* \leq x$ and $y^* \leq y$, the loss is split 50/50. Let s^{**} and t^{**} be the (Nash equilibrium) pair of activity levels that X and Y would choose as mutually best responses, assuming there is one. In the Example 8 table below, we examine what happens when we start the dynamic process at line 2, i.e., at the 'second-best optimum', $(s^{**}, x^*, t^{**}, y^*)$.

The functions used are as in Specification 1. The results are shown in the Example 8 table. As usual we start at the (s^*, x^*, t^*, y^*), found by solving the first-order conditions for NSB maximization. However, now we start the dynamical process from what Dari-Mattiacci at el (2014) call the second best optimal (SBO). The SBO takes x^* and y^* as fixed, lets X and Y choose s and t, respectively, keeping their liability shares fixed at 1/2. Note that on lines 1 through 4, both parties are non-negligent, and so damages are split 50/50. The result is a Nash equilibrium at $(0.089838, 0.355473, 0.089838, 0.355473)$.

Example 8

	s	x	t	y	Liability	X's benefit Net of Damages	Y's benefit Net of Damages
1	0.058547	0.355473	0.058547	0.355473	50/50	0.171067	0.171067
2	**0.089838**	0.355473	**0.089838**	0.355473	50/50	0.149865	0.149865
3	**0.089838**	**0.355473**	0.089838	0.355473	50/50	0.149865	0.149865
4	0.089838	0.353629	**0.089838**	**0.355473**	50/50	0.149865	0.149865

Notes:
Line 1 solves first order conditions to get (s^*, x^*, t^*, y^*).
Lines 2: "Second-best optimum" SBO takes x^* and y^* as fixed, lets X and Y choose s and t, respectively.
Line 3: X chooses, starting at SBO. Best choice is to stay put.
Line 4: Y chooses. Best choice is to stay put.
Nash equilibrium!

In other words, if the rule is 50/50 split liability when both are non-negligent, there exists a Nash equilibrium under both specifications.

Example 9. **Rule: 50/50 split liability when both are non-negligent, Specification 2, Start dynamic process at SBO.**

Example 9 is just like Example 8, except it substitutes Specification 1 for Specification 2. The line 1 shows the solution to the first-order conditions, (s^*, x^*, t^*, y^*) on line 1.

Example 9 Table

	s	x	t	y	Liability	X's benefit Net of Damages	Y's benefit Net of Damages
1	0.058295	0.353629	0.058295	0.353629	50/50	0.170483	0.170480
2	0.089379	0.353629	0.089379	0.353629	50/50	0.149482	0.149480
3	**0.089379**	**0.353629**	0.089380	0.353629	50/50	0.149482	0.149480
4	0.089379	0.353629	**0.089380**	**0.353629**	50/50	0.149482	0.149480

Notes:
X is liable when X is negligent $(x < x^*)$, and that Y is liable when Y is negligent but X is not. However, when both X and Y are non-negligent, liability is split 50/50.
Line 1 solves first order conditions to get (s^*, x^*, t^*, y^*).
Lines 2: "Second-best optimum." The SBO takes x^* and y^* as fixed, lets X and Y choose s and t, respectively.
Line 3: X chooses, starting at SBO. Best choice is to stay put.
Line 4: Y chooses. Best choice is to stay put.
Nash equilibrium!

What's the result in Examples 8 and 9? Here we see that if we start the dynamic process at the 'second-best optimum', line 2, we stay at the second-best optimum. Again, we have a Nash equilibrium.

4. Results and Comparisons

The above examples show that under negligence based liability rules, the parties can end up in a cycle. In fact, below we prove that under the standard negligence rules there cannot exist a Nash equilibrium, regardless of whether parties make their choices sequentially as described above, or they choose care and activity levels simultaneously, or in any other manner. The formal claims are proved for Specifications 1 and 2. However, as is discussed below, these extends to a much wider class of accident contexts under the standard models.

Claim 1. There is no Nash equilibrium under the standard rule negligence.

Here is why the claim holds. Let's start with Specification 1. Obviously, under simple negligence there cannot be a Nash equilibrium in which party X opts for $x > x^*$. So consider the case when X decides to opt for x^*. Now, as is shown in Example 1, his best activity choice is $s = 1.978455$. Given these choices by X, party Y's best response is to choose $t = 0.000728$ and $y = 8.5905$. But given these choices by Y, x^* and $s = 1.978454$ is not a best response for X - as is shown in the table, X is better off choosing $x = 0$ and $s = 17374.7$. This means that a Nash equilibrium cannot have party X choosing x^*.

Finally, consider the case, $x < x^*$. This would make X negligent under simple negligence liability. So, all damages will fall on X, no matter what Y does. Therefore Y will set his care level $y = 0$, and will choose the largest t possible. That is, the choice of Y will be as in line 5 of the Example 1 table. But we have already shown that given this choice by Y, X is better off choosing x^* and $s = 1.978454$. Therefore, a Nash equilibrium is not possible at $x < x^*$.

A similar argument using the Example 2 table shows that a Nash equilibrium is not possible under Specification 2.

Next, consider the rule strict liability with a defense of contributorily negligence. This rule is the mirror image of the simple negligence. If we swap party X with party Y and vice-versa, then whatever claim is valid for party X under simple negligence, a similar claim will be true for party Y strict liability with a defense of contributorily negligence. In view the symmetry of the functional forms, arguing along the above lines one can prove the following claim.

Claim 2. There is no Nash equilibrium under the standard rule of strict liability with a defense of contributorily negligence.

Next, we have the following claim.

Claim 3. There is no Nash equilibrium under the standard rule of negligence with a defense of contributory negligence.

Here is why the claim holds. As under simple negligence, it still makes no sense for party X to opt for any $x > x^*$. If X decides to opt for $x^* = 0.353629$, as shown in the Example 3 table, his best activity choice is $s = 1.890707$. Given these choices by X, the party Y's best response is to choose $t = 0.000763$ and $y = 8.369300$. But given these choices by Y, party X can increase his payoff by choosing $x = 0$ and $s = 1262.607$. This means that a Nash equilibrium cannot have party X opting x^*. The only possibility is a choice of $x < x^*$.

Suppose there is a Nash equilibrium in which party X chooses a $x < x^*$. As to the choice of y by party Y, there are two possibilities: $y < y^*$ or $y \geq y^*$. In the former case, i.e., when $y < y^*$, party Y liable is under the rule of Negligence with a defense of contributory negligence, regardless of the choices made by party X. In such a scenario, party X' best response is to choose $x = 0$ and $s = 2500$, deriving a net payoff of 25.[13] However, given these choices by party X, the party Y can do better by choosing $y = 0.353629$, i.e., y^* along with $t = 1.890707$ is a better choice compared to any other pair whatever pair of (t, y), in which $y < y^*$. However, $y^* = 0.353629$. Therefore, there cannot be a Nash equilibrium in which party X chooses some $x < x^*$ and party Y opts for some $y < y^*$.

Finally consider the case where party X chooses some $x < x^*$ but party Y opts for some $y \geq y^*$. But, this would mean that all damages will fall on X, as long as Y keeps his $y \geq y^*$. In the region $y \geq y^*$, the unique best choice for party Y is to choose $y = 0.353629$, i.e., y^* along with $t = 1.890707$. However, given these choices of Y, as is shown in line 6 of Example 3 table, X's uniquely best choice is x^* and $s = 1.890707$. Again, a Nash equilibrium with $x < x^*$ is not possible.

In fact, arguing along the lines of the above claim, for this rule we can make the following claim about the rule of 50/50 split liability when both are negligent.

Claim 4. There is no Nash equilibrium under the standard rule of 50/50 split liability when both are negligent.

It is straightforward to see that just like in case of the above discussed rules, under the rule of 50/50 split liability when both are negligent, there cannot be an equilibrium in which X opts for $x > x^*$ and Y opts for $y < y^*$, or X opts for $x < x^*$ and Y opts for $y > y^*$. Also from Example 4 Table, $x = x^*$ and $y < y^*$ cannot be part of Nash equilibrium. Similarly, it can be seen that $y = y^*$ and $x < x^*$ cannot be part of Nash equilibrium Therefore, a Nash equilibrium is possible only in the region $x < x^*$ and $y < y^*$. In this region, using Specification 1 and solving for mutually best responses result in symmetric choices: $s = 0.086245$ and $x = 0.234187$ by X and $t = 0.086245$ and $y = 0.234187$ by Y. But, it turns out that X is better of deviating to care level x^*. Similar is the case for Specification 2. So, there is no Nash equilibrium under the rule.

The claim about non-existence of equilibrium under the rule of 50/50 split between negligent parties extends to the rule of comparative negligence. That is, under Specifications 1 and 2 there is no *symmetric* Nash equilibrium under the standard rule of comparative negligence. However, due to the complexity of the calculations involved, we have not be able to rule out possibility of an asymmetric Nash equilibrium with both parties negligent.

The problem of non-existence of Nash equilibrium is not restricted to the above specifications only. The problem does not go away even if we further modify the functional forms in these specifications; for instance by substituting $0.01s$ and $0.01t$ with $0.001s$ and $0.001t$, respectively. As the Example 5 demonstrates, even if we use non-symmetric benefit functions, equilibrium continues to elude us.

[13] When $x = 0$, FOC implies $s = 2500$.

Next we turn to the relative efficiency of the rules discussed above. The rule of strict liability for the injurer is more efficient than any of the negligence-based rules 1-5 described in Section 2.4 above. Formally, we have the following claim.

Claim 5. The rule of strict liability for the injurer is more efficient than any of the negligence-based rules 1-6 described above, including the rule of 50/50 split liability when both parties are non-negligent.

The above claim holds since there is no Nash equilibrium under the negligence-based rules including the rule of 50/50 split liability when both parties are negligent. Moreover, as the proof provided in the Appendix I shows, the value of the NSB under the strict liability rule is as high as it can under any liability rule. Next, consider the rule of 50/50 split takes place when both parties are non-negligent. Consider the equilibrium under the rule for Specification 2. Adding up the payoffs of the two parties, value of the NSB at the equilibrium is (0.299). However, the value of NSB under the strict liability rules is 25! Similarly, for specification 1 the equilibrium value of NSB at under the 50/50 split between non-negligent parties is much less than under the strict liability. Similar is the case via-a-vis the no-liability rule. For detailed arguments see Appendix I.

The outcome under the rule of no liability is a mirror image of the outcome under the rule of strict liability, so we have the following claim.

Claim 6. The rule of no liability for the injurer is more efficient than any of the negligence based rules 1-6, including the rule of 50/50 split liability when both parties are non-negligent.

In fact, it turns out that the all or nothing rules - rule of strict liability and the rule of no liability for the injurer - are more efficient than any of the negligence-based rules 1-6 can be, even if the due care standards were set at levels different from x^* and y^*, respectively. Moreover, in Appendix II we discuss why for a wide class of accident contexts these rules are more efficient than negligence based rule.

Literature Reexamined

Here we describe how our findings are very different from the claims in the existing literature, even though the NSB functions examined by us follow directly from the standard models. Version 1 of the standard model, as described in Section 2.3 above, is used in Miceli 1997, Cooter and Ulen (2004), and Singh (2006), and Shavell (2007 a and b), among others. Version 2 as described in Section 2.3 above is used in Shavell (1980, 1987 page 27-29 and 45), and Dari-Mattiacci, Lovat, and Parisi (2014). It arises as a special case of the models in Endres (1989), Parisi and Fon 2004, and Parisi and Singh (2010). These works have argued that under each of the standard negligence based rules (rules 1-4 mentioned in Section 2.4) an equilibrium exists. Specifically, this literature uses the solution of first order conditions to set due care standards, and makes the following unproved claims about the outcomes under various rules: Under the rule of negligence, with and without defense of contributory negligence, injurers will opt for the due care levels but their activity levels will be excessive. Given these choice of the injurer, the choices of the victims will be efficient. However, under the rule of strict liability with defense of contributory negligence, the victims will take optimal care but their activity levels will be excessive; given this, the injurers' behaviors will be efficient.

The specifications of NSB function used by us meets all requirements of standard models. Moreover, we have followed standard procedure towards the identification of the first best and the setting of due care standards for the parties. Yet, as is shown previous section, equilibrium does not exist under any of the standard negligence based rules. Since our specifications of the NSB function follow from the standard models, this means that claims in the literature about existence of equilibrium do not hold for generalizations of our specifications 1 and 2, that is, version 1 and 2 of the standard model.

As to the benefits of splitting accident loss between the parties, Singh (2006) and Dari-Mattiacci, Lovat, and Parisi (2014) have argued that the sharing of liability between the non-negligent parties is desirable, as it can enhance the efficiency of liability rules by incentivizing the parties to moderate their activity levels.[14]

Our Examples 8 and 9 allow sharing of liability between non-negligent parties and do have Nash equilibria. Moreover, the outcomes under the rule of 50/50 split liability when both are non-negligent is seemingly along expected lines. In equilibrium, each party adheres to the due care standard and opts for moderate activity levels as shown in table lines 2-4. But there are surprises here also. As shown above, the rule of strict liability for the injurer, and the rule of no liability for the injurer, are both more efficient than the negligence-based 50/50 split liability rules. That is, loss sharing rule is significantly less efficient than the all-or-nothing rules, i.e., the rule of strict liability and the rule of no-liability for the injurer. Under these rules, there is no sharing of liability. Yet, in many contexts, these rules turn out to be much more efficient than any of the negligence rules 1-5.

A related strand of literature suggests that efficiency of rule of negligence can be improved by raising the due care standard for injurers, see Goerke (2002), Dari-Mattiacci, Lovat, and Parisi (2014).[15] The basic idea is this: An increase in due care standard (beyond the first best level) increases the per-activity level cost for the injurer. Consequently he has incentive to keep his activity level moderate, rather than raising it to excessive level under the standard rule. The same logic applies to the due care level for the victim and his activity level under the rule of strict liability with defense of contributory negligence.

Contrary to these claims, in the next section we show that for a large class of accident contexts, economic efficiency is improved by reducing the injurer's due care standard to absolute zero. Similarly, efficiency of the rule of strict liability with a defense of contributory negligence is increased by reducing the due care standard for the victim to zero.

[14]Splitting of liability between the vigilant parties can potentially dilute the incentives of parties to adhere to the due care levels. See Parisi and Fon (2004) and Parisi and Singh (2010). However, when activity levels are constant, it is possible to design rules that allow sharing of liability between both negligent and vigilant parties, in proportion their degrees of negligence/vigilance, and yet maintain incentives for efficient care. See Feldman and Singh (2009) and (2011); Singh (2007a), (2007b).

[15]Shavell 2007 makes this argument for unilateral accidents.

5. What went wrong?

To repeat, the NSB function examined by us, through specification 1 and 2 and all nine examples, meet all the requirements of the standard models. Moreover, we have strictly followed the standard procedure for identification of the first best, and for setting the due care standards for the parties. Yet, as we have shown above, the prevalent claims about the equilibrium outcomes under negligence based liability rules and their efficiency properties, do not hold.

What is even problematic is this: The equilibrium values of NSB under strict liability for the injurer, and also under no liability for the injurer, are much higher than the NSB values at the solution to the first order conditions for the social optimization problems. For Specification 2, the value of NSB at (s^*, x^*, t^*, y^*) is just 0.340966, while at the equilibrium under strict liability it is 25.00035! (See Appendix). For Specification 1 the difference is even more bizarre. The value of NSB at (s^*, x^*, t^*, y^*) is just 0.34213, while at the Nash equilibrium under no-liability, it is 1,000! (see Appendix) It is clear that the first-order conditions for the maximization of NSB have been misleading us.

We have tried variations of specifications 1 and 2, e.g., by using different values of D and δ, replacing $\phi(s,t) = st$ with $\phi(s,t) = \sqrt{st}$ and $\phi(s,t) = s + t$, etc, but the above problems persist. The first order conditions, do not identify a global maxima. Therefore, we have to ask: What exactly has gone wrong with the standard models?

Consider again the problem of maximizing $NSB = s^{1/2} - xs + t^{1/2} - yt - 50st\pi(x,y)$. The reason we had problems maximizing this is that *there are no maxima*. Here's why: set $x = y = t = 0$. Then the NSB function reduces to $NSB(s,0,0,0) = s^{1/2}$, which is *unbounded* without a maximum! Similarly, $NSB(0,0,0,t) = t^{1/2}$, is *unbounded*. The maxima in our examples resulted from the constraints that we had imposed, i.e., $s, t \leq 1,000,000$. Take away these constraints, and there are no maxima.

Since the first order conditions to do not identify a global maximum, means that all if a mamixmum exists it must lie on one or the other boundary. With $s, t \leq 1,000,000$ constraints, there are two corner global maxima; one requires $x = 0$ along with $s = 1,000,000$, and the other requires $y = 0$ along with $t = 1,000,000$.

Now, it is easy to see why the value of social benefit function is highest under the no-liability or the strict liability rule. Under no-liability, X will set $x = 0$ and maximum possible level of $s = 1,000,000$. As is shown in the Appendix, the outcome under no-liability approximates a global maximimum arbitrarily closely. A similar argument applies to the rule of strict liability.

It is straightforward to see that these problems persist even if we replace $s^{1/2}$ and $t^{1/2}$ with functions like $s^{1/k}$ and $t^{1/k}$, for $k > 1$. In fact, this "bad behavior" extends to the Standard Model 1 itself. It takes $NSB = u(s) - xs + v(t) - yt - st\pi(x,y)D(x,y)$. If we set $x = y = t = 0$, then $NSB = u(s) + v(0)$. The function $u(s)$ is monotonically increasing. Clearly, there can be no maxima if $u(s)$ or $v(t)$ is unbounded. So, the standard approach to finding the first best cannot work. As is shown in the Appendix, even if these functions are bounded above, the first order conditions generally fail to identify the global maxima.

Specification 2 suffers from similar problems. Note that under Specification 2, we get $NSB(s,0,0,0) = s^{1/2} - \delta s$. It can be made arbitrarily large by choosing sufficiently small δ. In other words, it is easy to find a range of δ for which global maxima cannot be interior.

In fact, the standard models 1 and 2 are inherently prone to the above problems. To see, suppose t is zero. Fixing $t = 0$, means that the care becomes costless for the victim, so it makes sense to choose very high y and set $x = 0$. Moreover, expected accident costs becomes zero. That is, under the standard models, restricting one of the activities to zero level, has two direct and significant social benefits. First, the total cost of care for one party can be reduced to zero. Second, the expected accident loss also becomes zero, even if the injurer opts for very high level of activity and very little care. As is shown below, for a large set of accident contexts and functional forms consistent with the standard models, the global maxima lie at one or the boundary. Therefore, the standard approach of using the first order condition to identify the first best and using the solution to set due care levels is misleading in general.

There is yet another serious problem with the existing literature, even if we completely disregard the above issues. To explain, suppose the NSB function is 'well-behaved' with a unique solution identifiable by the first order conditions. That is, assume that (s^*, x^*, t^*, y^*) is indeed a unique global maximum, and the due care standards for the injurer and victim are set at x^* and y^*, respectively. The literature argues/assumes that under the rule of negligence, the injurer would choose x^* as care level. Specifically, it is assumed that a downward deviation from x^* (corresponding to deviation line 5 in Example 1 table) cannot be in the interest of injurer. But, to our knowledge, not a single work has provided proof behind such a critical belief.

In view of the above problems, our findings are not surprising.

6. Concluding Remarks

In this paper, contrary to claims in the existing literature, we have shown that for a large set of accident contexts admissible under the standard model: (1) There are no Nash equilibria under standard negligence based liability rules; (2) the rules of strict liability for the injurer and no liability for the injurer are more efficient than the standard negligence based rules, such as simple negligence or negligence with a defense of contributory negligence; (3) all-or-nothing rules are more efficient than the rules that require sharing of liability between non-negligent parties (presumably to incentivise them to moderate their activities).

Furthermore, we have shown that even when the individual optimization problems are well defined, the social optimization problem does not possess properties assumed in the literature. For large class of functions consistent with the standard model, the standard approach towards identification of the first best does not work. Formally speaking, under standard model no interior maxima exists in many cases. Either a global maximum does not exist or it requires very high activity with zero care from one of the parties, along with almost zero activity with very high care from the other party. This, in turn, is responsible for dominance of all-or-nothing rules -rules of strict liability and no liability - over the negligence liability based rules.

Moreover, we have shown that under the standard models, efficiency of negligence based rules can significantly be increased if the due care standard for one party is reduced to zero.

However, in the real world we observe that negligence based rules are much more prevalent than the all-or-nothing rules of strict liability and no-liability of injurer. Besides, the due care standards used by the negligence based rules are significantly greater than zero.[16] In other words, the above mentioned implications of the standard models do not gel with the prevalence of negligence liability rules in the real world.

Therefore, several questions arise. The first question is for what types of accidents contexts, the standard models is appropriate? The second question is: What additional conditions on the components of the existing models will result in a net social benefit function whose maximum can be found by using the usual first order condition approach? We have tried several different combination of functional forms but the answers do not seem to be straightforward.

The next questions is: Even if the social optimization problem could be made 'well behaved' with an interior solutions, would a Nash equilibrium necessarily exist under negligence based rules? Going by our analysis, the answer is not necessarily 'yes.'

Therefore, we need to ask: What model can validate the mainstream claims about existence of equilibria under the negligence based liability rules and their efficiency properties?

To sum up, the existing literature on the subject has contributed a great deal by providing an intuitive and easy to work with models of care and activity levels. However, further research is required to answer the questions and issues arising from our study. Our findings are expected to be useful guide for the future research on this subject.

APPENDIX

Appendix I: Proofs of Claims 5 and 6:

First, we prove the claims for standard negligence based liability rules that use x^* and y^* to set the due care standards as described above. Towards the end, discuss the rules that can due care standard at other care levels.

For Specification 1 with constraints $s, t \leq 1,000,000$, there are two global maxima: $(s, x, t, y) = (1.25E{-}9, 7070, 1,000,000, 0)$; and $(1,000,000, 0, 1.25E{-}9, 7070)$.[17] At each of these points the value of $NSB = 1000.0000176$. Note that these two NSB maxima require either very high activity and zero care from the injurer, along with almost zero activity and very high care from the victim, or vice-versa. For this NSB maximization problem, there are no interior solutions.

The first of the two global maximum is achieved as a Nash equilibrium under the rule of strict liability for the injurer. Since X pays damages; therefore Y sets $y = 0$ and t as large as possible, i.e., 1,000,000. Now by solving optimization problem for X, his the best response to choose $x = 7070$, $s = 1.25E^{-9}$, $y = 0$ and $t = 1,000,000$. Specifically, under this rule the equilibrium is: $(s, x, t, y) = (1.25E - 9, 7070, 1000000, 0)$, and it is a NSB maximum with $NSB = 1000$. That is, the rule of strict liability almost attains the first best. By symmetry, under the rule of no liability for the injurer the equilibrium will be at $(s, x, t, y) = (1000000, 0, 1.25E{-}9, 7070)$; and the value of NSB at this equilibrium will again be $NSB = 1000$.

In contrast, under the standard negligence liability based rules number 1-5 there is no Nash equilibrium. The actual choices by the parties will vary from contexts to context

[16]When due care is set at zero level or at very high level, the negligence rule reduces to the rule of no-liability and the rule of strict liability, respectively. For details see Singh (2004).

[17]Use of Mathematica throws up several solutions to the global maximization problem with $NSB =$ 1000.00002 approximately. All of the global maxima seem to converge to these two points.

and, as can be seen from the example tables, can lead to violent fluctuations in the individual payoffs and hence the value of NSB. In any case, the actual choice by the parties and hence the value of NSB almost always is different from their first best levels.

As far as the standard rule of comparative negligence is concerned, the only possibility of an asymmetric Nash equilibrium has both parties to be negligent. Specifically, each party spends more than 0 on care but less than 0.355437. Note that the global maxima require one party to take zero care and the other one to take very high care. This mean that even under the unlikely scenario of an equilibrium under the comparative negligence the actual choices will be very different from the first best.

Finally, consider the 50/50 split rule when both parties are non-negligent. Under this rule there is a Nash equilibrium. However, at the equilibrium point the value of $NSB = 0.2989637$ is much less than $NSB = 1000$ under the rule of strict liability.

The outcome under the rule of no-liability is will be a mirror image of the outcome under the strict liability. By symmetry, under the rule of no liability for the injurer the equilibrium will be at $(s, x, t, y) = (1000000, 0, 1.25E - 9, 7070)$; and the value of NSB at this equilibrium will again be $NSB = 1000$. That is,the rule of no-liability almost achieves the second of the global maxima $(1,000,000, 0, 1.25E - 9, 7070)$ and hence is more efficient than any of the other rules.

Under Specification 2 with $\delta = 0.01$ also there are two global maxima: $(s, x, t, y) = (5.01E-7, 352.551, 2499.96, 0)$, and $(s, x, t, y) = (2499.96, 0, 5.01E-7, 352.551)$. At these points $NSB = 25.0004$. Now, under strict liability for the injurer, the Nash equilibrium is: $(s, x, t, y) = (5.01E - 7, 352.553, 2500, 0)$, resulting in NSB=25.00035. Again, the strict liability approximates the second of the global maxima. The first one is approximated as the Nash equilibrium outcome under the rule of no liability for the injurer. In contrast, under the negligence liability rules either there is no equilibrium or it is far away from the first best.

Next, consider any other negligence rule with any arbitrarily chosen due care standards, say $\bar{x}$ and $\bar{y}$ for the injurer and the victim, respectively. The simple rule of negligence can approximate the first best profile $(1,000,000, 0, 1.25E - 9, 7070)$ as an outcome, if $\bar{x}$ is be kept at 0. But, this means that the rule is essentially the rule of no-liability for injurer. Similarly, the other maximum $(1.25E - 9, 7070, 1,000,000, 0)$ can be achieved under any rule of negligence with defense of contributory negligence by setting $\bar{y} = 0$, but then such a rule is nothing but the rule of strict liability for the injurer. In fact, the maximum can be achieved under any rule of negligence with defense of contributory negligence by setting $\bar{y} = 0$ and $\bar{x}$ 'very high'.[18] Again, any such rule essentially is the rule of strict liability.

Finally, the above reasoning suggests that whenever NSB has corner maxima, the rule of strict liability or the rule of no-liability will be more efficient than the standard rules of negligence. Either a negligence rule will mimic rule of strict liability or the rule of no-liability rule, or it will not be able to approximate the first best, and hence will be as efficient as the two all-or-nothing rules.

[18]It is well known that when $\bar{x}$ is set very high, the injurer prefers to bear liability rather than comply with the high due care level.

Appendix II: Problems with the standard models

Here we show that the standard models are not appropriate for many accident contexts. To start with we use our specifications to show the exact nature of the problems. Thereafter we show how these problems extend to the standard models.

To see the problem with Version 1, let us start with Specification 1 which is a special case of that approach. As shown in Section 3.2, the system of first order conditions has only one non-negative solution given by $(s^*, x^*, t^*, y^*) = (0.058547, 0.355473, 0.058547, 0.355473)$. However, the point $(0.058547, 0.355473, 0.058547, 0.355473)$ is in fact neither a global maximum for NSB, nor a local maximum. It is not even a global or local minimum point for NSB. Given $(x^*, y^*) = (0.355473, 0.355473)$, $(s^*, t^*) = (0.058547, 0.058547)$ is actually a *saddle point* in s/t space for the NSB function. See Plot 1. In other words, the first-order conditions led us astray.

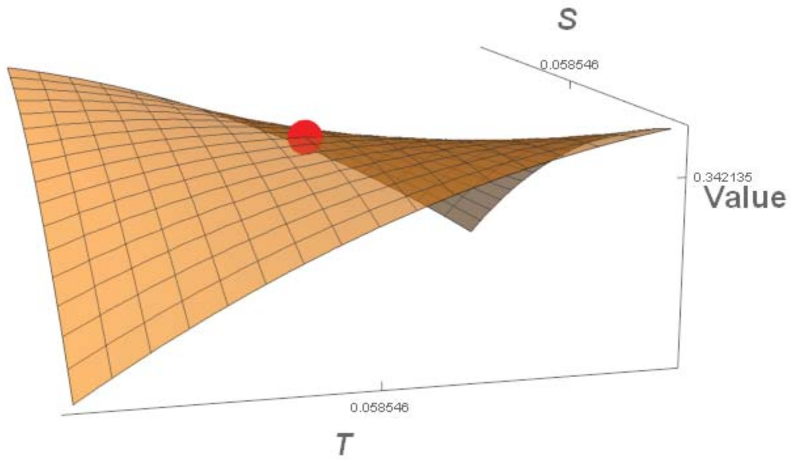

Figure 1
Value of NSB at (x^*, y^*) about (s^*, t^*) in (s, t) space.

But the problem is worse than that. As is shown above, $NSB(s, 0, 0, 0) = s^{1/2}$, which is *unbounded*. The maxima in our examples based on Specification 1 resulted from the constraints that we had imposed, i.e., $s, t \leq 1,000,000$. Take away these constraints, and there are no maxima. With these constraints, there are two global maxima: $(s, x, t, y) = (1.25E-9, 7070, 1,000,000, 0)$; and $(1,000,000, 0, 1.25E-9, 7070)$.[19] These two maxima lie at boundaries for s and t. There is no interior solution.

This conclusion is valid for any constraints optimization of the NSB in specification 1. Since the NSB function is continuous, therefore on a compact constraint set, a global maximum exits. But, note that the first order conditions completely fail to identify even a local maximum, therefore a maximum has to lie on a boundary, regardless of the size the constrains set as long as it is nonempty and convex. It is straightforward to see that the problems of Specification 1 continue to hold even if we replace $s^{1/2}$ and

[19]Use of Mathematica throws up several solutions to the global maximization problem with $NSB =$ 1000.00002 approximately. All of the global maxima seem to converge to these two points.

$t^{1/2}$ with functions like $s^{1/k}$ and $t^{1/k}$, for $k > 1$. In fact, this "bad behavior" extends to version 1 of the standard model as such. Version 1 assumes $NSB = u(s) - xs + v(t) - yt - st\pi(x,y)D(x,y)$. In this more general model, if we set $x = y = t = 0$, then $NSB(s,0,0,0) = u(s)$. Similarly, $NSB(0,0,t,0) = u(t)$. The functions $u(s)$ and $v(t)$ are monotonically increasing. Clearly, as long as $u(s)$ or $v(t)$ is unbounded, no maximum can exists and use of the first order conditions will be misleading.

Next, let us consider the case when $u(s)$ and $v(t)$ are bounded. In view of the above discussion, a global maximum will not be interior as long as the upper bound for either of the functions is sufficiently large. Even for reasonably low upper bound on $u(s)$ and $v(t)$ is not guaranteed. For instance, consider monotonic but bounded functions, $u(s)\frac{\sqrt{s}}{\sqrt{1+s}}$ and $\frac{\sqrt{t}}{\sqrt{1+t}}$. Clearly, $u(s) \leq 1$ and $v(t) \leq 1$ for all s and t. Still, there is no interior maximum. There two global maxima, the point $(3180.99, 0, 3.93946 \times 10^{-7}, 397.81)$ and its mirrior. At the points $NSB = 1.00016$.

Next, we consider non-monotonic and bounded above functions, such as $u(s) = s^{1/2} - xs$ and $v(t) = t^{1/2} - yt$, where $x, y > 0$. But this is exactly what we have for specification 2, which gives us

$$NSB = s^{1/2} - xs - \delta s + t^{1/2} - yt - \delta t - \frac{50st}{1+x+y}.$$

Let $\delta = 0.01$. Clearly, the NSB function is bounded. A global maximum does exist. Now, first order conditions give $(s^*, x^*, t^*, y^*) = (0.058295, 0.353629, 0.058295, 0.353629)$ (see Table 3) and this point satisfies the following property: given (s^*, x^*), the point (t^*, y^*) maximizes NSB, and vice-versa. See Plots 2 and 3 However, the point $(0.058295, 0.353629, 0.058$ is neither a maximum nor a minimum for NSB. As in the case of Specification 1, here too (s^*, x^*, t^*, y^*) is a saddle point in s/t space. See Plot 4.

Moreover, there are two global maxima for this NSB function, the points $(s,x,t,y) = (5.01E-7, 352.551, 2499.96, 0)$, and $(s,x,t,y) = (2499.96, 0, 5.01E-7, 352.551)$. Neither is an interior point. The value of NSB at each of global maxima is approximately 25.0004.

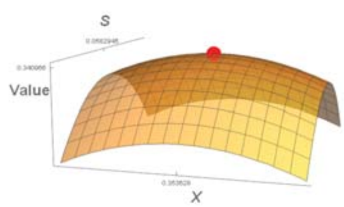

FIGURE 2
Value of NSB at (t^*, y^*) about (s^*, x^*) in (s,x) space.

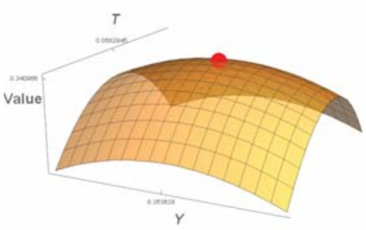

FIGURE 3
Value of NSB at (s^*, x^*) about (t^*, y^*) in (t,y) space.

In fact, specification 2 suffers from problems similar to specification 1. Under Specification 2, $NSB(s,0,0,0) = s^{1/2} - \delta s$. Let $\bar{s}(\delta)$ solve $\max_s\{s^{1/2} - \delta s\}$. That is, $\bar{s}(\delta) = \frac{1}{4\delta^2}$.

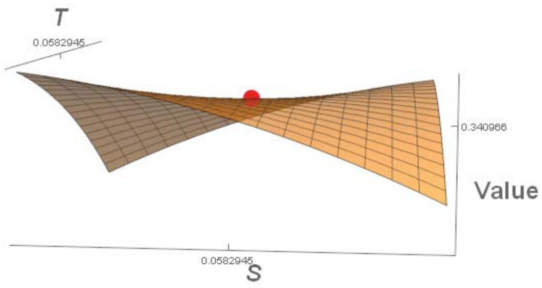

FIGURE 4
Value of NSB at (x^*, y^*) about (s^*, t^*) in (s, t) space.

Clearly, $NSB(\bar{s}, 0, 0, 0) = \bar{s}^{1/2}(\delta) - \delta s$ can be made arbitrarily large by choosing sufficiently small δ. The analysis above shows that even when $\bar{s}$ is moderate ($\bar{s} = 2500$), the first best is not interior. The problem only becomes worse when $\bar{s}$ is large.

Things do not change if $s^{1/2}$ and $t^{1/2}$ are replaced with $s^{1/k}$ and $t^{1/k}$, for any $k > 1$. For Specification 2, we have tried several variations; e.g., by replacing terms $-0.01s$ and $-0.01t$ with terms like $-0.001s$ and $-0.001t$. The above problems continue to persist - there is no interior global maxima. For both specifications, we have also examined the outcomes by replacing $\phi(s, t) = st$ with $\phi(s, t) = \sqrt{st}$ but qualitative nature of results does not change.

It should be noted that for each specification, as marginal gain from s, t approaches ∞ as $s, t \to 0$, we can rule out the maxima at boundary where $s, t = 0$. Therefore, a maximum will have either $x = 0$ or $y = 0$.

Summing up, the outcomes under Specifications 1 and 2 show similar features. Under each specification, a global maximum for NSB requires either: very high activity with zero care for the injurer, along with almost zero activity with very high care for the victim, or vice-versa. Moreover, none of the maxima is interior. Also, as is shown in Appendix I, as long as there is a corner global maximum, the rules of strict liability for the injurer and no liability for the injurer are more efficient than any of the negligence based rules 1-6.

In fact, these problems extend to both versions of the standard models. To see this fix $t = 0$. This means the care costs for the victim is zero, even when y is very high. So, it makes sense to choose extremely high y but set $x = 0$ to bring down injurer's costs of care. Still, expected accident costs remains zero since $t = 0$. That is, under both versions of the standard model, fixing $t = 0$ has two direct and significant social benefits. First, the total cost of care for can be reduced to zero. Second, the expected accident loss also becomes zero, even if the injurer opts for very high level of activity and very little care. Our analysis shows that for a large set of functional forms, consistent with the above specifications, these two gains dominate the opportunity cost, i.e, keeping the net gains to the victim at $v(0) - 0y = 0$. This logic applies to the positive but arbitrarily small levels of t.

It seems, under the standard models the problem will persist as long as $\lim_{s,t\to 0} \phi(s, t) = 0$ holds. This problem does not arise when $\phi(s, t) = s + t$. So, let us proceed with $\phi(s, t) = s + t$.

In Examples 6 and 7 we examined case where instead of assuming the standard $\phi(s,t) = st$, we assumed $\phi(s,t) = s + t$. These examples had the usual problems. See Tables 9 and 10.

In fact, it gets worse. Consider a specification corresponding to Specification 1 above, except now we will let $\phi(s,t) = s + t$ instead of $\phi(s,t) = st$. So, the net social benefit function is now

$$NSB = s^{1/2} - xs + t^{1/2} - yt - \frac{(s+t)50}{1+x+y}.$$

The system of first order conditions for this specifications has one non-negative solution: $(0.00277083, 4.5, 0.00277083, 4.5)$. That is, the apparent first-best optimum is now $(s^*, x^*, t^*, y^*) = (0.002770083, 4.5, 0.002770083, 4.5)$. At this point, $NSB = 0.0526316$. However, from Table 9A in the appendix we know that NSB can be as much as 950! In fact, holding $t = x = 0$,

$$NSB = s^{1/2} - s \times 0 + 0 - y \times 0 - \frac{(s+0)50}{1+0+y} = s^{1/2} - \frac{s50}{1+y}.$$

So, by increasing s and y such that $y = 50s - 1$, the NSB can be increased without any upper limit! Again, a global maximum does not exist.

Next, consider a specification corresponding to Specification 2 above, except now we will let $\phi(s,t) = s+t$ instead of $\phi(s,t) = st$. So, choosing $\delta = 0.01$, the net social benefit function is now

$$NSB = s^{1/2} - xs - 0.01s + t^{1/2} - yt - 0.01t - \frac{(s+t)50}{1+x+y}.$$

The system of first order conditions for this specifications has one non-negative solution: $(s, x, t, y) = (.002764, 4.5, .002764, 4.5)$. At this point, $NSB = .0525762$. However, holding $t = x = 0$, the maximum value of NSB is

$$\bar{s}^{1/2}(\delta) - \bar{s}(\delta) \times 0 - \delta\bar{s}(\delta) + 0 - y \times 0 - \delta \times 0 - \frac{(\bar{s}(\delta)+0)50}{1+0+y} = \bar{s}^{1/2}(\delta) - \delta\bar{s}(\delta) - \frac{\bar{s}(\delta)50}{1+y}.$$

So, by choosing arbitrarily small δ and large y, the NSB can be increased without any upper limit, leading to a corner global maximum.

Besides, it can be seen that there cannot be a Nash equilibrium under rules of no-liability and under strict liability.

Consider the rule of no liability. Under specification 1, the injurer wants to choose $x = 0$ and s to maximize: $s^{1/2}$. Clearly, the optimum s does not exist for the injurer. Under specification 2, the injurer wants to choose $x = 0$ and s to maximize: $s^{1/2} - 0.01s$. So, the injurer will choose $\bar{s} = 2500$. Given $x = 0$ and $\bar{s} = 2500$ opted by the injurer, the victim can now vary both t and y. But his objective function

$$t^{1/2} - yt - 0.01t - \frac{50(\bar{s}+t)}{1+0+y}$$

actually has no maximum when t and y are allowed to freely vary! Here's why: Set $t = 0$. Then the victim's objective function becomes

$$0^{1/2} - 0.01 \times 0 - y \times 0 - \frac{(50\bar{s})+0}{1+0+y} = -\frac{50\bar{s}}{1+y}.$$

For any $y \geq 0$ this is a negative number. As y increases, it always increases, approaching 0 from below in the limit. In short, the victim would like to choose an activity level of zero, and a care expenditure (per unit of activity) of plus infinity. If he does this, his bottom line is 0, which is far better than choosing $t > 0$.[20] So, there is no Nash equilibrium under no-liability, and by analogous argument under the rule of strict liability.

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[20]From every algorithm we can find, when $t > 0$ the best case scenario for the victim is a payoff of -.0008

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