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Strategyproof and Budget Balanced Mechanisms for Assembly *

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Abstract

Assembly is an exchange problem with one buyer and multiple sellers. The buyer wants to purchase multiple items and sellers hold one item each. In applications like land acquisition, the buyer is required to purchase contiguous land plots to realize a project worth any value. This paper characterizes strategyproof, individually rational and budget balanced mechanisms for the assembly problem when the valuations of the agents are private information. It also examines several mechanisms in this class.

JEL Classification: C78, D82

1 Introduction

In an assembly problem, a buyer wants to acquire a number of items that she can assemble for a project; there are multiple sellers in the market with one unit each. An interesting application of this is the Land Acquisition (LA) problem: it involves the buyer realizing a positive valuation from purchasing at least a specific number of contiguous plots. Physical contiguity of land plots can be modelled using graphs where each node denotes a plot owned by a seller and physical adjacency of a pair of plots is represented by an edge. The buyer realizes a positive valuation if she is able to purchase a fixed number of connected plots, which is represented as a path of fixed length in the graph described. Assembly problems arise in many different contexts other than land acquisition, as described by Heller (2008), who refers to these as "gridlock" economies.

If the valuations of the agents were publicly known, then the efficient trading rule would prescribe trade whenever the buyer's valuation is greater than the minimum sum of valuations of sellers with required number of items. When valuations are private, buyer strategically tends to understate her valuation while sellers tend to overstate their

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valuation. Consequently, some efficient trade opportunities may be lost. The role of a mechanism designer in this context is to come up with mechanisms that satisfy a set of desirable criteria, e.g., truthful reporting, positive payoff from participation, efficiency, and no external budgetary support.

Mechanism design for assembly problem has flavors of the familiar literature on mechanism design in the tradition of Myerson and Satterthwaite (1983), who characterize the class of mechanisms satisfying such properties for bilateral exchange. However, unlike the case of the bilateral trade problem, it may be possible to design satisfactory mechanisms in the sense of Myerson and Satterthwaite (1983) in assembly problems, provided the number of sellers is greater than the number of plots required. See Sarkar (2017) for a characterization. But implementation of such a mechanism would require the mechanism designer to have knowledge of the prior on valuations, and thus it would be susceptible to Wilson (1987)'s critique. A common approach to prior-free mechanism design is to apply the well-known VCG mechanism (see Sarkar (2017, 2018)). The VCG mechanism satisfies several of the criteria considered desirable for the success of a mechanism, including efficiency, truthfulness and voluntary participation. But unfortunately, it runs into payment deficits, particularly when the number of sellers is small, and requires external budgetary support. This paper, in contrast, characterizes the class of truthful mechanisms that entail voluntary participation and do not require external budgetary support. Further, it examines some useful mechanisms in this class.

The next section briefly reviews the literature in the area. The third section introduces the notation and preliminary concepts. Section 4 presents the characterization of the class of truthful, individually rational and budget balancing mechanisms for the LA problem. We then discuss a number of mechanisms and their properties in this class.

2 Literature

There are two major strands in the literature on assembly problems with strategic agents, one that considers bargaining models where valuations of the agents are publicly known, and the mechanism design models where agents' valuations are their private information.

Even in assembly problems with complete information, sellers may engage in "holdout", i.e., strategic delays unless they get significant share of the surplus that can be generated from successful trade. This may result in outcomes like trade with an inefficient subset of sellers or complete breakdown of negotiations. Several papers have attempted to characterize holdout in assembly problems with land assembly as a prominent application (Asami, 1985; O'Flaherty, 1994; Cai, 2000, 2003; Menezes and Pitchford, 2004; Miceli and Segerson, 2012; Roy Chowdhury and Sengupta, 2012; Göller and Hewer, 2015; Xiao, 2018).

The bilateral trade problem is a special case of the assembly problem involving only one seller with an indivisible item; it has been extensively studied in the mechanism design literature. Myerson and Satterthwaite (1983) characterized the class of mechanisms for this problem that satisfy *Bayesian incentive compatibility*, *interim individual rationality* and *budget balance*. They also showed that no mechanism in this class can be *ex post efficient* and characterized the optimal mechanism in this class which maximizes sum of ex ante payoffs for the two agents. Sarkar (2021) characterizes the class of mechanisms for the land acquisition problem that satisfy the criteria put forward by Myerson and Satterthwaite (1983), and provides the optimal mechanism in this class. In an earlier paper, Sarkar (2017) showed the existence of ex post efficient mechanisms in this class if the number of sellers is greater than the minimum number of items the buyer intends to purchase.

The mechanisms in the class characterized by Myerson and Satterthwaite (1983) and its extensions for the assembly problem are dependent on common knowledge of the prior on valuations and are subject to the critique of Wilson (1987). An alternative is to replace the requirement of Bayesian incentive compatibility with the stronger notion of dominant strategy incentive compatibility, which is also referred to as strategyproofness. It is well-known that efficient and strategyproof mechanisms, e.g., the VCG mechanisms (see Krishna (2002)'s Chapter 5, and also Sarkar (2018)) do not satisfy budget balance (Hurwicz, 1975). Consequently, some efficient trade possibilities have to be dropped in order to attain budget balance with strategyproof mechanisms: e.g., the posted price mechanism in the bilateral trade model by Hagerty and Rogerson (1987), or the strategyproof double auction by McAfee (1992) in multilateral trade problems with unit demand.

Given the impossibility of obtaining efficiency in Myerson and Satterthwaite (1983), many authors have investigated asymptotic efficiency of *second best* mechanisms (e.g., Satterthwaite and Williams (1989b,a); Gresik and Satterthwaite (1989); McAfee (1992); Rustichini et al. (1994); Cripps and Swinkels (2006); Fudenberg et al. (2007); Williams (1991) and Satterthwaite and Williams (2002) among others.) See the review article by Jackson (2000) for elucidation. Sarkar (2021) has shown that in the assembly problem, the possibility of asymptotic efficiency of Bayesian incentive compatible mechanisms is intricately linked with the existence of *critical sellers* who belong to every feasible path that the buyer can possible purchase.

Among the recent papers on mechanism design for assembly problems. Kominers and Weyl (2011), Grossman et al. (2019), Plassmann and Tideman (2010), and Chaturvedi (2020) offer various second-best solutions to the mechanism design problem when the number of sellers is equal to the number of items required by the buyer. Kominers and Weyl (2012) have discussed the combinatorial problem involved in resolution of hold-out.Chaturvedi and Kanjilal (2021) also implement a mechanism in experimental settings. Our paper covers an important mechanism suggested by Ghatak and Ghosh (2011) and one of its shortcomings as noted by Singh (2012).

3 Preliminaries

We will use a model of assembly motivated by the land assembly problem where the buyer requires to purchase a set of contiguous plots. There are n sellers, indexed by i, each holding one unit of an indivisible good (plot). The n items are located on a graph

$\Gamma = (N, E)$ where N denotes the set of nodes (plots) and E denotes the set of edges. A pair of nodes is connected by a direct edge if the corresponding items can be assembled. A subgraph of Γ which is a sequence of connected nodes is called a path, denoted $\mathcal{P}$. A path P is feasible if it contains k nodes where $k \leq n$. A seller is critical if he is in every feasible path in Γ. See Figure 1 for an illustration (also see Gupta and Sarkar (2018) for a comprehensive description of possible graph structures in this context). If the graph contains exactly one feasible path, then all sellers in the feasible path are critical; if the graph contains more than one feasible path, then the graph cannot have more than $k-1$ critical sellers. Notice that an assembly problem where the buyer is not concerned with contiguity of items can be described as the case corresponding to a complete graph, where all nodes are connected. FOr such graphs, the buyer can assemble any set of k items. The buyer, indexed by 0, has the following valuation pattern: his valuation is v_0 if he purchases items which constitute a path of length k units of the good ($k \leq n$) or more, and 0 otherwise. All valuations are positive and private information to the corresponding agents.

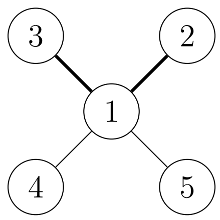

Figure 1: A feasible path in the star graph where $n = 5$ and $k = 3$; seller 1 is critical.

In a Bayesian model, we usually assume that the valuations of the agents are distributed according to some commonly known prior. We may assume that $v_0 \in [\underline{v}_0, \bar{v}_0]$ and distributed according to some cumulative distribution function with positive density on the entire support: $v_0 \sim G(v_0)$; similarly, $v_i \in [\underline{v}, \bar{v}]$ and $v_i \overset{iid}{\sim} F(v_i)$, $i = 1, \ldots, n$. To make things interesting, we do not want values such that trade is always beneficial or it is always bad. Therefore, we may assume that $k\bar{v} > \underline{v}_0$ and $k\underline{v} < \bar{v}_0$. These distributions can be summarized by a *common prior*, i.e., a joint distribution function p over all valuation profiles which is common knowledge. The mechanism design literature in the tradition of Myerson and Satterthwaite (1983) has mostly considered the case of independently distributed valuations. The collection $\langle \Gamma, k, p \rangle$ is referred to as an assembly problem.

The buyer and the sellers report their individual valuations or any signal thereof to an auctioneer who decides on a mechanism to allocate the goods in question. Such a mechanism consists of an *allocation rule* and a *transfer rule*. We will consider only *direct mechanisms* where all agents directly report their valuations.

Definition 1 **(Allocation Rule)** *An allocation is any realization of a random vector* $X = (X_0, X_1, \ldots, X_n)$ *where* $X_i, i \neq 0$ *takes value -1 if trade takes place between the i-th seller and the buyer, and 0 otherwise,* X_0 *is 1 if there exists* $P \in \mathcal{P}$ *such that* $X_i = -1$

for all $i \in P$, and 0 otherwise. Let the set of 2^n possible allocations $\mathbb{X}$. A (deterministic) allocation rule $P : [\underline{v}_0, \bar{v}_0] \times [\underline{v}, \bar{v}]^n \to \mathbb{X}$ specifies an allocation for every reported profile of valuations.

Definition 2 **(Transfer Rule)** *Any rule $t : [\underline{v}_0, \bar{v}_0] \times [\underline{v}, \bar{v}]^n \to \mathbb{R}^{n+1}$, relating the valuations of the agents to transfers t_j gives a transfer rule.*

Definition 3 **(Direct Mechanism)** *Any pair (P, t) is a direct mechanism.*

When defining utility of an agent, we will distinguish among the different stages of private information agents have (Holmstrom and Myerson, 1983). The relevant stage in our context is "ex post", i.e., when agents have reported their valuations and the allocations and transfers have been made.

Definition 4 **(Utility)** *The (ex post) utility of agent j with valuation v_j reporting $\hat{v}_j$ under mechanism (P, t) is*

$$U_j(\hat{v}_j, v_{-j}|v_j) = v_j P_j(\hat{v}_j, v_{-j})) - t_j(\hat{v}_j, v_{-j}) \tag{1}$$

where v_{-j} denotes the n dimensional random vector representing the valuations of agents other than j.

It follows therefore, that the utility of agent j under honest reporting is

$$U_j(v_j, v_{-j}) = v_j P_j(v_j, v_{-j}) - t_j(v_j, v_{-j}) \tag{2}$$

Definition 5 **(Strategyproofness)** *A direct mechanism is Dominant Strategy Incentive Compatible or strategyproof if honest reporting is a weakly dominant strategy for every agent, i.e.,*

$$U_j(v_j, v_{-j}) \geq U_j(\hat{v}_j, v_{-j}|v_j) \quad \forall j, v_j, v_{-j}, \hat{v}_j \tag{3}$$

Definition 6 **(Individual Rationality)** *A given mechanism is (ex post) individually rational if for each agent,*

$$U_j(v_j, v_{-j}) \geq 0 \quad \forall j, v_j, v_{-j} \tag{4}$$

Definition 7 **(Budget Balance)** *A mechanism (P, t) satisfies (strong) budget balance if*

$$\sum_j t_j(v_j, v_{-j}) = 0 \quad \forall v_j, v_{-j}. \tag{5}$$

There is also a weak version of this condition where the equality above is replaced by a weak inequality.

The stage where only own valuations are private information to the respective agents is "interim". Here agents calculate payoffs by taking expectation over the unknown valuations of other agents.

Definition 8 **(Interim Utility)** *The interim utility of an agent j with valuation v_j reporting $\hat{v}_j$ in mechanism (P, t) is the expectation of utility in (1) taken over the valuations of all other agents:*

$$EU_j(v_j, \hat{v}_j) = v_j E_{-j}(P_j(\hat{v}_j, v_{-j})) - E_{-j}(t_j(\hat{v}_j, v_{-j})) \tag{6}$$

j and $E_{-j}(\cdot)$ denotes expectation taken over valuations other than that of the i-th agent.

It follows therefore, that the expected utility of agent j under honest reporting is

$$EU_j(v_j) = v_j E_{-j}(P_j(v_j, v_{-j})) - E_{-j}(t_j(v_j, v_{-j}))$$

The notion of incentive compatibility corresponding to interim payoffs is *Bayesian Incentive Compatibility.*

Definition 9 **(Bayesian Incentive Compatibility)** *A direct mechanism is Bayesian Incentive Compatible or BIC if honest reporting forms a Bayes-Nash equilibrium (Harsanyi, 1967-1968), i.e.,*

$$EU_j(v_j) \geq EU_j(v_j, \hat{v}_j) \quad \forall j, \forall \hat{v}_j \tag{7}$$

.

The participation condition is also modified accordingly.

Definition 10 **(Interim Individual Rationality)** *A given mechanism is interim individually rational if for each agent j and for every v_j,*

$$EU_j(v_j) \geq 0 \tag{8}$$

When no agent has realized a valuation, the appropriate concept of utility is the ex ante one.

Definition 11 **(Ex Ante Utility)** *The ex ante utility of an agent j in mechanism (P, t) is the expectation of utility in (1) taken over all possible valuation profiles:*

$$AU_j = E_v(P_j(v_j, v_{-j})) - E_v(t_j(v_j, v_{-j})) \tag{9}$$

The relevant participation condition becomes as follows.

Definition 12 (Ex Ante Individual Rationality) *A given mechanism is ex ante individually rational if for each agent j,*

$$AU_j \geq 0 \tag{10}$$

We can also distinguish between ex-post, interim and ex-ante concepts of efficiency.

Definition 13 (Efficiency) *An allocation rule P is ex post efficient if it results in the highest sum of realized valuations at every profile: for all v,*

$$\sum_j v_j P_j(v) \geq \sum_j v_j P'_j(v) \text{ for any allocation rule } P'. \tag{11}$$

An allocation rule P is interim efficient if it results in the highest sum of interim expected valuations at every profile: for all v,

$$\sum_j v_j E_{-j} P_j(v) \geq \sum_j v_j E_{-j} P'_j(v) \text{ for any allocation rule } P'. \tag{12}$$

An allocation rule P is ex ante efficient if it results in the highest sum of ex ante expected valuations at every profile for all v,

$$\sum_j E_v(v_j P_j(v)) \geq \sum_j E_v(v_j P'_j(v)) \text{ for any allocation rule } P' \tag{13}$$

The following statements (Holmstrom and Myerson, 1983) can be easily verified:

- A dominant strategy incentive compatible mechanism is also Bayesian incentive compatible, but the converse is not true in general.
- Ex-post individual rationality implies interim individual rationality, which in turn implies ex-ante individual rationality, but none of the converses are true in general.
- Ex-post efficiency implies interim efficiency, which in turn implies ex ante efficiency, but the converses are not true in general.

The set of efficient allocations for an assembly problem is described below. Note that every path $\mathcal{P}$ is a subgraph of Γ. Let us refer to the set of nodes in $\mathcal{P}$ as $N(\mathcal{P})$. Given a vector of valuations $v_0, \ldots, v_n$, observe that

$$\sum_j v_j P_j(v) = \begin{cases} v_0 + \sum_{i=1}^n v_i X_i \text{ if } \exists \mathcal{P} \subset \Gamma : |N(\mathcal{P})| = k, X_j = -1 \forall j \in N(\mathcal{P}); \\ \sum_{i=1}^n v_i X_i \text{ otherwise} \end{cases}$$

The first term refers to the case when trade takes place with sellers on a feasible path of length k, while the second term refers to the case when trade does not take place with sellers on a feasible path. Notice that X_is are negative for every seller i, and v_0 is positive. Therefore the efficient rule which maximizes this sum is specified as follows: trade takes place with only the sellers on the feasible path with the lowest sum of reported seller valuations if this sum is less than the buyer's reported valuation; no trade takes place otherwise. We will denote the efficient rule as P^*.

For a profile of valuations let $\mathcal{P}^*(v) \subset \Gamma$ represent a path in Γ with the minimum sum of valuations, i.e.,

$$\sum_{j \in N(\mathcal{P}^*(v))} v_j \leq \sum_{j \in N(\mathcal{P}(v))} v_j, \quad \forall P(v) \subset \Gamma$$

The efficient rule P^* can then be described as follows:

$$\begin{aligned} P_0^*(v) &= \begin{cases} 1 \text{ if } v_0 > \sum_{j \in N(\mathcal{P}^*(v))} v_j; \\ 0 \text{ otherwise,} \end{cases} \\ P_i^*(v) &= \begin{cases} -1 \text{ if } v_0 > \sum_{j \in N(\mathcal{P}^*(v))} v_j, i \in N(\mathcal{P}^*(v)); \\ 0 \text{ otherwise,} \end{cases} \end{aligned} \tag{14}$$

For illustration, consider Figure 1 with 5 sellers and $k = 3$. Let the valuations of the sellers holding item represented by nodes 1 through 5 be 1, 2, 3, 4 and 5 respectively. The efficient rule will allow trade between the buyer and the sellers holding items 1, 2 and 3, provided the valuation of the buyer is more than 6.

4 Result

Our main result characterizes the class of mechanisms for assembly problems which satisfy dominant strategy incentive compatibility (DSIC), ex-post individual rationality (EIR), and (strong) budget balance (BB). The result states that such mechanisms involve specifying for each agent a function of the valuations of other agents. Trade takes place at a profile only if two conditions are met. One, the reported valuations of all sellers in some feasible path (and possibly more sellers) must be less than the value of the corresponding functions at this profile. Secondly, consider the values of the functions which exceed the reported valuations of the corresponding sellers — the buyer's reported valuation must be greater than the sum of the values of such functions at this profile.

Theorem **1** *A mechanism for the land acquisition problem (P, t) is DSIC, BB and EIR if and only if for every agent i there exist positively valued functions g_i of valuations of* *agents other than i, such that :*

$$P_0(v) = \begin{cases} 1 \text{ if } v_0 > \sum_{i:v_i \le g_i(v_{-i})} g_i(v_{-i}) \text{ and } \exists \mathcal{P} \subset \Gamma : v_i \le g_i(v_{-i}) \forall i \in N(\mathcal{P}); \\ 0 \text{ otherwise} \end{cases}$$

$$P_i(v) = \begin{cases} -1 \text{ if } v_i \le g_i(v_{-i}), v_0 > \sum_{i:v_i \le g_i(v_{-i})} g_i(v_{-i}) \text{ and } \exists \mathcal{P} \subset \Gamma : v_i \le g_i(v_{-i}) \forall i \in N(\mathcal{P}); \\ 0 \text{ otherwise} \end{cases}$$

$$t_0(v) = \begin{cases} \sum_{i:v_i \le g_i(v_{-i})} g_i(v_{-i}) \text{ if } v_0 > \sum_{i:v_i \le g_i(v_{-i})} g_i(v_{-i}) \text{ and } \exists \mathcal{P} \subset \Gamma : v_i \le g_i(v_{-i}) \forall i \in N(\mathcal{P}); \\ 0 \text{ otherwise} \end{cases}$$

$$t_i(v) = \begin{cases} -g_i(v_{-i}) \text{ if } v_i \le g_i(v_{-i}), v_0 > \sum_{i:v_i \le g_i(v_{-i})} g_i(v_{-i}) \text{ and } \exists \mathcal{P} \subset \Gamma : v_i \le g_i(v_{-i}) \forall i \in N(\mathcal{P}); \\ 0 \text{ otherwise} \end{cases} \tag{15}$$

Proof: The sufficient part is relatively easy: at every profile, the buyer's payoff is weakly increasing in her valuation report while the sellers' payoffs are weakly decreasing in corresponding valuation reports. Consequently, it is a weakly dominant strategy for all agents to report truthfully. Budget balance and individual rationality is maintained by construction. We prove the necessary part in seven steps.

Step 1: An ex-post budget balanced mechanism must have the property

$$\sum_{j \in \{0,1,\ldots,n\}} t_j(v) = 0 \quad \forall v. \tag{16}$$

Therefore,

$$t_0(v) = -\sum_{j \in \{1,\ldots,n\}} t_j(v) \quad \forall v. \tag{17}$$

Step 2: By EIR,

$$v_j P_j(v) - t_j(v) \ge 0 \quad \forall j, v \tag{18}$$

For the sellers $j \in \{1, \ldots, n\}$, $P_j(v) \in \{0, -1\}$. Therefore, for $j \in \{1, \ldots, n\}$, we must have

$$t_j(v) \le 0 \quad \forall v \tag{19}$$

Step 3: By steps 1 and 2,

$$t_0(v) \ge 0 \quad \forall v. \tag{20}$$

Step 4: By steps 2 and 3, for all j and v,

$$t_j(v) = 0 \quad \text{if } P_j(v) = 0 \tag{21}$$

Step 5: Let us suppose that a (P, t) is DSIC. Then for any two types of agent j, say, v_j and v'_j, we must have:

$$v_j P_j(v_j, v_{-j}) - t_j(v_j, v_{-j}) \ge v_j P_j(v'_j, v_{-j}) - t_j(v'_j, v_{-j}) \tag{22}$$

$$v'_j P_j(v'_j, v_{-j}) - t_j(v'_j, v_{-j}) \geq v'_j P_j(v_j, v_{-j}) - t_j(v_j, v_{-j}) \tag{23}$$

From (22), we get

$$v_j P_j(v_j, v_{-j}) - v_j P_j(v'_j, v_{-j}) \geq t_j(v_j, v_{-j}) - t_j(v'_j, v_{-j}) \tag{24}$$

From (23), we get

$$t_j(v_j, v_{-j}) - t_j(v'_j, v_{-j}) \geq v'_j P_j(v_j, v_{-j}) - v'_j P_j(v'_j, v_{-j}) \tag{25}$$

From (24) and (25), we get:

$$v_j P_j(v_j, v_{-j}) - v_j P_j(v'_j, v_{-j}) \geq v'_j P_j(v_j, v_{-j}) - v'_j P_j(v'_j, v_{-j})$$
$$\Rightarrow (v_j - v'_j)[P_j(v_j, v_{-j}) - P_j(v'_j, v_{-j})] \geq 0$$

which implies that $v_j \geq v'_j$ if and only if $P_j(v_j, v_{-j}) \geq P_j(v'_j, v_{-j})$. Also note that, if $P_j(v_j, v_{-j}) = P_j(v'_j, v_{-j})$, then by (22), $t_j(v'_j, v_{-j}) \geq t_j(v_j, v_{-j})$, and by (23), $t_j(v_j, v_{-j}) \geq t_j(v'_j, v_{-j})$, so that $t_j(v_j, v_{-j}) = t_j(v'_j, v_{-j})$.

Step 6: Consider seller i. For him, P_i is -1 or 0 depending on the reports of other agents. Therefore, for each v_{-i} we can partition the range of his valuation in two sets:

$$\{v_i | P_i(v_i, v_{-i}) = -1\}, \quad \{v_i | P_i(v_i, v_{-i}) = 0\}$$

Therefore we can define $g_i(v_{-i}) = \inf\{v_i | P_i(v_i, v_{-i}) = 0\}$, so that $P_i(v) = -1$ if $v_i \leq g_i(v_{-i})$ and 0 otherwise. Now, let $t_i^1(v_{-i})$ be the payment when $P_i(v_i, v_{-i}) = -1$ and let $t_i^0(v_{-i})$ be the payment when $P_i(v_i, v_{-i}) = 0$. Let $v_i > g_i(v_{-i})$ and $v'_i \leq g_i(v_{-i})$. By (22),

$$-t_i^0(v_{-i}) \geq -v_i - t_i^1(v_{-i}) \tag{26}$$

By (23),

$$-v'_i - t_i^1(v_{-i}) \geq -t_i^0(v_{-i}) \tag{27}$$

Therefore, if $v_i = g_i(v_{-i}) + \epsilon$ and $v'_i = g_i(v_{-i}) - \epsilon$, we have,

$$-g_i(v_{-i}) + \epsilon - t_i^1(v_{-i}) \geq -t_i^0(v_{-i})$$
$$\geq -g_i(v_{-i}) - \epsilon - t_i^1(v_{-i})$$

Taking $\epsilon \to 0$, we get,

$$-g_i(v_{-i}) - t_i^1(v_{-i}) = -t_i^0(v_{-i})$$

Since by step 4, $t_i^0(v_{-i}) = 0$, we must have $t_i^1(v_{-i}) = -g_i(v_{-i})$.

Step 7: For the buyer, steps 4 and an argument similar to step 6 yield the expressions for t_0^0 and t_0^1. Note that the sum of transfers must be independent of the valuation of the buyer at every profile where trade takes place.

■

The implications of the structure of this class of mechanisms are discussed in the next section.

5 Discussion

5.1 Efficiency

It is easy to see that no strategyproof mechanism satisfying budget balance and ex post individual rationality can be efficient by comparing the efficient allocation rule (14) and the allocation rule specified in (15). The g functions corresponding to sellers who trade at any efficient allocation must involve v_0 in a way that their sum cannot be independent of v_0. However, as will be discussed below, certain mechanisms in this class may realize almost every efficient trade as the number of sellers become large.

5.2 Posted Price Mechanisms

An important sub-class of the mechanisms discussed above use constant g functions. These are referred to as posted price mechanisms. These are the only strategyproof, individually rational and budget balanced mechanisms in the bilateral trade problem. In this problem, our theorem suggests that we must have

$$g_0(v_1) = g_1(v_0) \geq 0$$

which is only possible when these functions are equal to a fixed non-negative real number at every profile. More generally, the constants can be so chosen to allow trade only when it is efficient. For instance, in a problem where the buyer wants to purchase 2 items from 3 sellers, a posted price mechanism may be specified as follows: trade takes place whenever the sum of the two lowest reported valuations is less than 2c and the buyer's reported valuation is greater than 2c, no trade takes place otherwise. The buyer would pay c to each of the successful sellers. Notice that this mechanism only allows trade at profiles where $v_0 > 2c \geq v_{[1]} + v_{[2]}$, where $v_{[j]}$ represents the j-th lowest valuation reported. It is efficient to trade at such profiles. But the mechanism will not trade at some profiles where trade is efficient, e.g., $2c \geq v_0 > v_{[1]} + v_{[2]}$.

5.3 Mechanism by Ghatak and Ghosh (2011)

Consider the following mechanism when any subset containing k items out of n ($k < n$) results in a positive valuation for the buyer: trade takes place with the sellers reporting the lowest k valuations if the buyer's reported valuation v_0 is greater than $kv_{[k+1]}$; the buyer pays $kv_{[k+1]}$ and each of the k successful sellers receive $v_{[k+1]}$. This mechanism is in the class characterized by (15). It realizes trade at all profiles where $v_0 > kv_{[k+1]}$ where it is efficient, but forgoes efficient trade opportunities when $kv_{[k+1]} \geq v_0 > \sum_{i=1}^{k} v_{[i]}$. If the valuations of the sellers are distributed independently and identically over some bounded interval and the buyer's valuation is also bounded, as assumed in the Bayesian model referred to on p. 4, we can show that the probability of trade approaches 1 (see Sarkar (2018) p. 43 for an argument). Although Ghatak and Ghosh (2011) suggested this mechanism in the context of land assembly, as pointed out by Singh (2012), this mechanism may not be strategyproof in the presence of critical sellers. This point is elaborated below.

5.4 Mechanism design in the presence of critical sellers

Consider the LA problem where the buyer wants to purchase a set of items (land plots) that are represented by a path of length k in a graph with n nodes. Recall that a set of sellers are critical if they lie on every such feasible path. In the following example, where $n = 4$ and $k = 3$, sellers 2 and 3 are critical in this sense. Suppose the valuations of the sellers 1 through 4 are 1, 2, 3 and 4 respectively, and buyer's valuation is 10. Here the $k+1$-th lowest valuation is 4. The mechanism by Ghatak and Ghosh (2011) described above is no longer strategyproof: seller 3 can report 6, in which case it will be the highest valuation, and realize a payment of 6, instead of 4.

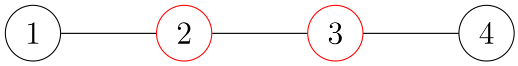

Figure 2: A line graph with critical sellers 2 and 3; $k = 3$

It is useful to revisit the class of mechanisms in (15) in this context once again: when no contiguity requirement is present, the mechanism presented in the previous subsection is strategyproof as the sellers corresponding to the highest $n-k$ valuations are excluded from trade. Thus the transfer functions for every seller is independent of his own valuation. On the other hand, when contiguity requirement is present, this mechanism fails to satisfy strategyproofness as critical sellers must be present in any successful trade. This problem can be overcome by specifying a posted-price mechanism, where the g function for every seller is a constant. Another way out is to spell out seller-specific transfers as functions of valuations reported by other sellers, e.g., $g_1(v_2, v_3, v_4)$, $g_2(v_3, v_4, v_5)$ etc. in the example presented above.

5.5 Optimality

In a Bayesian model, mechanisms are often compared in terms of the ex -ante welfare (sum of ex-ante utilities). For instance, Myerson and Satterthwaite (1983) showed that the double-auction mechanism due to Chatterjee and Samuelson (1983) maximizes ex-ante welfare in the bilateral trade model when valuations of the buyers are independently distributed over $[0, 1]$ interval. Their exercise essentially consisted of characterizing a condition on allocation rules for mechanisms satisfying Bayesian incentive compatibility, interim individual rational and budget balance, and maximizing the ex-ante welfare over all mechanisms satisfying this condition. The resulting mechanism is called the optimal mechanism. Notice that mechanisms described in (15) also satisfy the conditions of Myerson and Satterthwaite (1983). Consequently, the ex-ante welfare in these mechanisms are obtainable when valuations are distributed over bounded intervals with pre-defined densities. Further, it is bounded above by the ex-ante welfare obtainable in the optimal mechanism for the model.

In these Bayesian settings with known prior, one useful approach to construct suitable mechanisms is to consider parametric forms of g functions described in (15), and find the parameter values that maximize ex-ante welfare. For instance, if we consider the class of posted-price mechanisms, where the g functions are constants, we can find the values of these constants which maximize ex-ante welfare. Consider the case with only one seller. The posted price mechanism with price $\frac{1}{2}$ maximizes ex-ante welfare among all DSIC, BB and IR mechanisms when values are distributed iid $U[0, 1]$. Similarly, if these g functions are affine, with constant coefficients, the values of these constant coefficients which maximizes ex-ante welfare can be derived.

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